A Bayesian Approach to 2000 Census Evaluation using A.C.E. Survey Data and Demographic Analysis

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Introduction

Despite enormous efforts, the US Census missed enumerating some US residents and counted others twice or more as part of Census 2000. To evaluate the degree of net overcount/undercount, capture-recapture techniques similar to those used in wildlife population estimation (Seber 1982) can be employed. These techniques typically rely on behavioral assumptions – namely independence of capture and recapture – that are untenable in human populations. However, in human populations demographers can supply alternative sources of population data that allow Census evaluation to be made under less restrictive assumptions (Wolter 1990; Bell 1993; Bell et al. 1996). Incorporating demographic analysis (DA) data into Census evaluation poses numerous difficulties. DA data relies in part on immigration and emigration estimates and consequently is available at only the national level, but evaluation is desired at a variety of small-area and subpopulation estimates. A variety of assumptions can be made that yield different estimates of undercount at subpopulation level, yet are difficult to distinguish empirically; additionally, previous techniques have yielded unstable subpopulation estimates, and do not incorporate uncertainty in demographic analysis data. Thus we develop a Bayesian hierarchical model that incorporates capture-recapture and demographic data to estimate Census over/undercount at subnational levels, shrinks outlying estimates of Census overcount/undercount, avoids spurious "negative cell counts" in overcount/undercount estimation, uses regression techniques to estimate common main effects of subnational covariates, and allows incorporation of uncertainty in demographic analysis data. This approach provides inference about overcount/undercount estimates that incorporates a wide range of contributions of uncertainty, and allows us to consider model averaging across a set of plausible behavioral models.

Background: Capture-Recapture Methods in the 2000 Census

Undercount estimation begins with a follow-up coverage measurement survey, in 2000 termed the Accuracy and Coverage Evaluation or A.C.E. Survey. The A.C.E. began by drawing a probability sample of Census clusters and conducting a detailed independent enumeration of households within in them immediately after the actual Census. Partial 2×2 tables are then formed, based on a probability match between the individuals in the Census for whom matching information is available (the "P-sample") and the persons in the A.C.E. clusters (the "E-sample"), stratified by gender, age, race/ethnicity, region of residence, urbanicity of residence, tenure (owner/renter status), and whether the subject resided in a "low return" block (see Table 1) (US Bureau of the Census 2000).

The natural estimate of the In-Census Out-A.C.E. cell in Table 1 is $\hat{\psi}^S_{k10} = z^S_{k1.} - y^S_{k11}$. This result is often negative; in addition, further assumptions are needed to estimate the Out-Census Out-A.C.E. cell. The most straightforward is to assume that 1) the events of capture and recapture are independent and 2) either the capture or the recapture probabilities are equal across individuals; then $\hat{\psi}^S_{k00} = \hat{\psi}^S_{k10}\hat{\psi}^S_{k01}/\hat{\psi}^S_{k11}$, and $\hat{\psi}^{S\ I}_{k..} = \hat{\psi}^S_{k11} + \hat{\psi}^S_{k10} + \hat{\psi}^S_{k01} + \hat{\psi}^S_{k00} = z^S_{k1.} y^S_{k.1}/y^S_{k11}$. The Census Bureau attempts to ensure 1) through independent enumeration of households and residents during the A.C.E.; stratification into the k poststrata (Sekar and Deming 1949) helps to ensure 2), reducing the effect of "correlation bias" or "heterogeneity bias" This is the standard undercount adjustment made by the Census and is referred to as "dual system estimation" (DSE).

Incorporating Data from Demographic Analysis

The plausibility of the assumption of zero correlation bias within post-strata can be assessed by comparing the sex ratios in subpopulations obtained under the DSE model with estimates from demographic analysis (DA). The evidence for the 2000 Census is that correlation bias exists in at least some subgroups: for example, in the 2000 Census, the DA estimate of the sex ratio among the non-institutionalized African-American population 30-49 was

Table 1: Observed data and associated underlying population parameters from Census and Accuracy and Coverage Evaluation (A.C.E.) Survey: y^S_{k11} and y^S_{k01} are estimated counts of individuals in and out of the Census on the basis of the A.C.E. interviewing and follow-up (estimated from the P-sample); $z^S_{k1.}$ is the Census count, minus imputations and an estimate of erroneous enumerations (estimated from the E-sample); ψ^S_{kij} is the population that would reside in the ijth cell in the kth poststratum for gender S if the A.C.E. had been a complete census.

		Observed Data			Underlying Parameters		
		A.C.E.			A.C.E.		
		In	Out		In	Out	
Census	In	y^S_{k11}		$z^S_{k1.}$	ψ^S_{k11}	ψ^S_{k10}	$\psi^S_{k1.}$
	Out	y^S_{k01}			ψ^S_{k01}	ψ^S_{k00}	$\psi^S_{k0.}$
		$y^S_{k.1}$			$\psi^S_{k.1}$	$\psi^S_{k.0}$	$\psi_{k..}$

0.89, compared with the unadjusted Census sex ratio estimate of 0.80 and the the DSE-adjusted estimate of 0.82. To correct for this apparent undercount of men relative to women, Wolter (1990) suggested attributing the low observed male-female ratio to an undercount of males. Specifically, he assumed that the ratio of the odds of enumeration in the Census for those included in the follow-up survey to the odds of enumeration in the Census for those not in the follow-up was 1 for all females and an unknown parameter $\theta^M \in (0, \infty)$ for all males. Under this model, the total population $\hat{\psi}^{DA}_{...}$ is estimated by $(1+\rho)\hat{\psi}^F_{...}$, where ρ the overall male-female ratio (assumed known from DA) and $\hat{\psi}^F_{...}$ is estimated under the independence model. Bell (1993) extended this idea to the stratified case by assuming that

$$\text{FOR: } \theta^M_k = \frac{\psi^M_{k11}/\psi^M_{k10}}{\psi^M_{k01}/\psi^M_{k00}} = \theta^M \text{ and } \theta^F_k = \theta^F = 1 \text{ for all } k. \tag{1}$$

We term this the fixed odds ratio (FOR) model. Under (1), Bell estimates the total population size within each poststratum by $\hat{\psi}^\theta_{k..} = \hat{\psi}^I_{k..} + (\hat{\theta}-1)\hat{\psi}^I_{k00} = \hat{\psi}^I_{k..} + (\hat{\psi}^{DA}_{...} - \hat{\psi}^I_{...})(\hat{\psi}^I_{k00}/\hat{\psi}^I_{.00})$. Wolter and Bell's approach can be applied to alternative models of behavior. We consider two others: the fixed relative-risk (FRR) model (Bell 1993) assumes a constant relative risk for enumeration in the Census and A.C.E. for males and independence for females:

$$\text{FRR: } \gamma^M_k = \frac{\psi^M_{k11}/\psi^M_{k1.}}{\psi^M_{k01}/\psi^M_{k0.}} = \gamma^M \text{ and } \gamma^F_k = 1 \text{ for all } k, \tag{2}$$

while the "two group" (TG) model (US Bureau of the Census 1999) assumes a constant proportion in the Census among those captured in the A.C.E. relative to the proportion in the Census among the entire population:

$$\text{TG: } \eta^M_k = \frac{\psi^M_{k11}/\psi^M_{k.1}}{\psi^M_{k1.}/\psi^M_{k..}} = \eta^M \text{ and } \eta^F_k = 1 \text{ for all } k. \tag{3}$$

Under the FRR and TG models, the total population $\hat{\psi}^{DA}_{...}$ is estimated under the same demographic constraints as the FOR model. The total population size within each poststratum is then given by $\hat{\psi}^\gamma_{k..} = \hat{\psi}^I_{k..} + (\hat{\psi}^{DA}_{...} - \hat{\psi}^I_{...})([\hat{\psi}_{k01} + \hat{\psi}^I_{k00}]/[(\hat{\psi}_{.01} + \hat{\psi}^I_{.00}])$ under the FRR model and $\hat{\psi}^\eta_{k..} = \frac{\hat{\psi}^{DA}_{...}}{\hat{\psi}^I_{...}}\hat{\psi}^I_{k..}$ under the TG model. Thus the TG model has the property that the discrepancy between the DA estimate and the independence estimate is allocated to the poststrata in proportion to the independence estimate.

Since ρ is estimated using demographic analysis within an age-race group, the models are applied separately to each of the six age-race domains: 18-29, 30-40, and 50 or older, by African-American versus non-African-American.

Bayesian Models for Census Evaluation

Because the cell counts are sums, we model the data from the kth postratum as $(y^S_{k11}\ y^S_{k01}\ z^S_{k1.})^T \sim N_3\left((\psi^S_{k11}\ \psi^S_{k01}\ \psi^S_{k1.})^T, \Sigma_k\right)$. The covariance Σ_k is estimated via a jackknife procedure and is treated as known.

Assuming the fixed odds ratio (FOR) model given by (1), we reparameterized the eight population counts in poststratum k, $\psi_k = \{\psi^S_{kij} : i = 0,1; j = 0,1; S = M,F\}$, as $\psi^*_k = (\psi_{k..}, \rho_k, \delta^M_k, \delta^F_k, \phi^M_k, \phi^F_k, \theta^M_k, \theta^F_k)$ where:

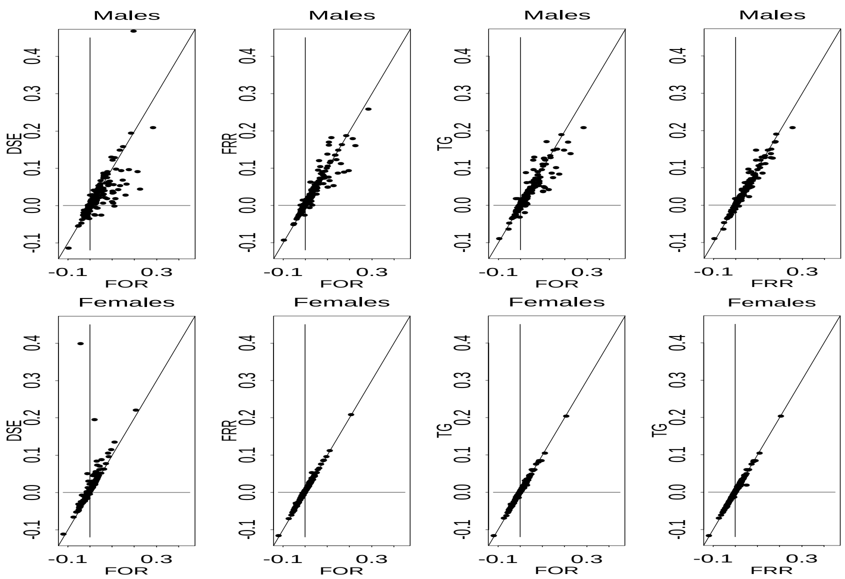

Figure 1: DSE estimates and posterior mean estimates of under/overcount in 384 poststrata under the FOR, FRR, and TG models: FOR vs. DSE, FRR, and TG; FRR vs. TG; separately for males and females.

1. $\psi_{k..}$, the total population count in poststratum k, with prior $N(\hat{\psi}^I_{k..}, (\hat{\psi}^I_{k..})^4)$, a nearly flat prior corresponding to our lack of knowledge about the total population counts in each poststratum;
2. $(\psi^M_{k..})/(\psi^F_{k..}) = \rho_k$, the sex ratio ($\psi^M_{k..} + \psi^F_{k..} = \psi_{k..}$), with prior $N(\rho, 1)$ subject to the constraint that $\sum_k w_k \rho_k = \rho$ where $w_k = (\psi_{k..})/(\sum_k \psi_{k..})$ and ρ is the DA-estimated nationwide sex ratio;
3. $\delta^S_k = \Phi^{-1}\left(\psi^S_{k1.}/\psi^S_{k..}\right)$, where $\psi^S_{k1.}/\psi^S_{k..}$ is the proportion of the total population captured in the adjusted Census counts, and $\delta^S_k \sim N(x^S_k \beta^S_\delta, (\sigma^2)^S_\delta)$, with $\beta^S_\delta \sim N(0,\ \text{diag } 1000)$ and $\sigma^2_\delta \sim INV-GAMMA(.01, .01)$;
4. $\phi^S_k = \Phi^{-1}\left(\psi^S_{k11}/\psi^S_{k1.}\right)$, where $\psi^S_{k11}/\psi^S_{k1.}$ is the proportion of the Census cases enumerated in the A.C.E. ("match total"), and $\phi^S_k \sim N(x^S_k \beta^S_\phi, (\sigma^2)^S_\phi)$, with $\beta^S_\phi \sim N(0,\ \text{diag } 1000)$ and $\sigma^2_\phi \sim INV-GAMMA(.01, .01)$; and
5. $\frac{\psi^S_{k11}/\psi^S_{k10}}{\psi^S_{k01}/\psi^S_{k00}} = \theta^S_k$, the odds among sex S of being included in the A.C.E. among Census respondents relative to to odds of being included in the A.C.E. among Census non-respondents; we assume $\theta^M_k = \theta^M \sim GAMMA(\alpha, \beta)$ for all k; $\theta^F_k = 1$ for all k, where α and β to maximize the correlation bias prior variance under the constraint that that 95% of the prior distribution lies within 1.0 and 11.0.

The probit regression models include an intercept and dummy variables for owner versus renter and high versus low return-rate households in the African-American age-race domains, and an intercept and dummy variables for ownership, return rate, and ethnicity in the the non-African-American age-race domains.

The fixed relative risk (FRR) model (2) replaces θ^S_k with $\gamma^S_k = \frac{\psi^S_{k11}/\psi^S_{k1.}}{\psi^S_{k01}/\psi^S_{k0.}}$, the proportion in the A.C.E. among Census respondents relative to the proportion in the A.C.E. among Census non-respondents for sex S; the two group (TG) model (3) replaces θ^S_k with $\eta^S_k = \frac{\psi^S_{k11}/\psi^S_{k.1}}{\psi^S_{k1.}/\psi^S_{k..}}$, the proportion in the Census among A.C.E. respondents relative to the proportion in the Census among the entire population.

We used Gibbs sampling to draw estimates of the population parameters from their joint posterior distribution. To determine that convergence of the Gibbs sampler had been achieved, five chains of 1000 draws with different startpoints were run, and the between- and within-sequence posterior variances and for each (scalar) parameter was calculated after dropping the first 100 draws as "burn-in".

Domain	Bayes Factor FRR vs. TG	FRR vs. FOR
African-American 18-29	16.0	.43
African-American 30-49	22.4	5.44×10^{-3}
African-American 50+	2.39	.052
Non-African-American 18-29	2.20×10^{13}	7.85
Non-African-American 30-49	2.22×10^{5}	1.06
Non-African-American 50+	8.83×10^{11}	1.1×10^{6}

Table 2: Bayes factors (posterior odds that the model is correct) comparing fixed relative risk model (FRR), two group model (TG), and fixed odds ratio model (FOR) within each age-sex domain.

Results

Total population estimates

Figure 1 plots the posterior means of the undercount estimates under the FOR model against a) the DSE undercount estimates, b) the posterior means of the undercount estimates under the FRR model, c) the posterior means of the undercount estimates under the TG model, and d) the undercount estimates under the FRR model against the undercount estimates under the TG model. Examining these plots, together with the specific poststrata associated with each point, indicates that undercount tends to be greatest among young and middle-aged male minorities living in rental housing units. Here the FOR, FRR and TG models are estimating positive correlation bias, resulting in a higher estimated undercount than that under the DSE model. The female estimates of undercount are of course more similar under the four models due to the common assumption of independence, although three poorly-estimated poststrata (female 50+, Native American and Hispanic in rental households; male 18-29 Native American in owner-occupied households) with high DSE undercount estimates are pulled toward 0. A greater degree of undercount is suggested under the FOR than the FRR or TG models in minority strata. The FRR and TG models tend to give more similar estimates.

Effect of Poststratum Characteristics on Undercount

Examining the probit regression parameters, it appears that older persons, Caucasians, males, and those in in owner-occupied dwelling units tended to have higher capture and recapture rates. Also, recapture rates tended to be higher than capture rates for a given set poststratum predictors. Interestingly, high postal return rates were associated with only slight increases in capture and recapture rates after adjusting for race/ethnicity, sex, and ownership status.

Model Checking/Model Selection Results

Posterior Predictive Distributions. We obtain the posterior predictive distribution of the $3K$ cell counts in each age-race domain by using the Gibbs sampling draws as ${\psi^*}_k^{rep}$, backtransforming to ψ_k^{rep}, and then generating $\left((y^S_{k11})^{pred}\,(y^S_{k01}))^{pred}\,(z^S_{k1.}))^{pred}\right)^T$. By comparing $(z^S_{k1.})^{obs} - (y^S_{k11})^{obs}$ with $(z^S_{k1.})^{rep} - (y^S_{k11})^{rep}$, we can determine whether some of the negative In-Census-Out-A.C.E. cells may be due to systematic differences in the Census and A.C.E. methods of enumeration, rather than simply sampling variability. We found that 9 of the 384 poststrata have PPD p-values of .1 or less under one or more of the models. Four of these poststrata are Caucasian 39-49 and 50 and older in owner-occupied dwellings units in small and non-MSA region in the Northeast with low return rates (PS 212-217); since all the age-sex categories share the same set of geographic blocks, there is some evidence to suggest that overestimation of the number of matches from the A.C.E. or overestimation of the rate of erroneous enumerations from the Census may be present in these blocks.

Model Selection. Following the method of Chib (1995), we determine the Bayes factors comparing the FOR, FRR, and TG models within each of the six age-race domains (see Table 2). The FOR model is strongly favored among African-Americans 30-49 and 50+. The FRR model is strongly favored among non-African-Americans 50+ and moderately favored among non-African-Americans 18-29. The FOR and FRR models have approximate parity among African-American 18-29 and non-African-Americans 30-49. In none of the six age-race domains does the TG model have substantial posterior probability.

Model Averaging. Tables 3 and 4 give the DSE estimates and approximate 95% confidence intervals of

Male	DSE	FOR	FRR	TG	Ave.
African-American 18-29	$4.5_{(2.6,6.2)}$	$12.2_{(10.4,13.9)}$	$12.1_{(10.5,13.8)}$	$12.1_{(10.4,13.8)}$	$12.1_{(10.5,13.8)}$
African-American 30-49	$2.8_{(1.7,4.0)}$	$12.2_{(11.2,13.2)}$	$12.2_{(11.3,13.2)}$	$12.2_{(11.2,13.2)}$	$12.2_{(11.2,13.2)}$
African-American 50+	$-.7_{(-2.0,0.5)}$	$4.0_{(2.9,5.0)}$	$4.0_{(3.0,5.1)}$	$4.1_{(3.0,5.1)}$	$4.0_{(2.9,5.0)}$
Caucasian 18-29	$2.4_{(1.7,3.1)}$	$1.8_{(1.3,2.4)}$	$1.8_{(1.2,2.4)}$	$1.4_{(0.7,2.2)}$	$1.8_{(1.2,2.4)}$
Caucasian 30-49	$1.3_{(1.0,1.7)}$	$1.7_{(1.3,2.1)}$	$2.0_{(1.6,2.3)}$	$1.9_{(1.5,2.3)}$	$1.8_{(1.4,2.3)}$
Caucasian 50+	$-0.3_{(-0.7,0.0)}$	$0.1_{(-0.2,0.4)}$	$0.3_{(0.0,0.6)}$	$0.2_{(-0.2,0.5)}$	$0.3_{(0.0,0.6)}$
Hispanic 18-29	$7.9_{(6.5,9.4)}$	$7.8_{(6.1,9.7)}$	$7.8_{(6.2,9.3)}$	$7.6_{(6.0,9.1)}$	$7.8_{(6.2,9.4)}$
Hispanic 30-49	$3.9_{(2.8,4.9)}$	$6.3_{(4.5,8.4)}$	$5.4_{(4.2,6.6)}$	$4.6_{(3.6,5.7)}$	$5.9_{(4.3,8.1)}$
Hispanic 50+	$0.3_{(-1.0,1.7)}$	$1.9_{(-0.1,4.6)}$	$1.5_{(-0.1,3.1)}$	$0.7_{(-0.6,2.1)}$	$1.5_{(-0.1,3.1)}$
Asian 18-29	$2.2_{(-0.6,4.9)}$	$1.6_{(-1.3,4.3)}$	$1.6_{(-1.3,4.4)}$	$1.5_{(-1.4,4.4)}$	$1.6_{(-1.3,4.4)}$
Asian 30-49	$2.4_{(0.5,4.3)}$	$2.8_{(0.6,5.2)}$	$3.3_{(1.2,5.5)}$	$3.1_{(1.2,4.9)}$	$3.1_{(0.9,5.4)}$
Asian 50+	$0.9_{(-1.0,2.9)}$	$2.5_{(-0.5,6.4)}$	$1.9_{(-0.2,4.2)}$	$1.3_{(-0.6,3.2)}$	$1.9_{(-0.2,4.2)}$

Table 3: Percent undercount estimates by age, race, and gender in 2000 Census: DSE estimate; FOR, FRR, and TG posterior means and posterior mean of average of FOR, FRR, and TG models. 95% credible interval in parentheses. Negative values indicate Census overcount.

undercount/overcount, together with the posterior mean and 95% posterior predictive estimates under the FOR, FRR, and TG models, for the nationwide age-race/ethnicity-gender groupings. The estimates of undercount among African-Americans are similar for the three models. The TG model tended to give smaller estimates of undercount (larger estimates of overcount) among Caucasians 18-29 and Asians, Hispanics, and Native Americans; the FOR model tended to give larger estimates of undercount (smaller estimates of overcount) among Asians, Hispanics, and Native Americans males of all ages. The last column of Tables 3 and 4 give the posterior means and 95% posterior predictive intervals of the overcount/undercount from the combination of the three models, obtained by averaging the posterior distribution of the poststrata total under a given model by the posterior probability of the D_jth model being correct, assuming that all three models are equally probable *a priori*.

Discussion

Our Bayesian approach to incorporating information from the follow-up Accuracy and Coverage Evaluation survey and demographic analysis addresses the problem of "negative cell counts" in In-Census Out-ACE cells and provides inference about population total estimates that account for all modeled sources of uncertainty. By supplying proper priors and assuming exchangeability, the proposed model shrinks outlying estimates of Census undercount toward overall means.

There does not appear to be strong evidence of bias in match rate or erroneous enumeration estimates that would lead to excessive estimates of negative cell counts. An exception is the Caucasian, owner-occupied, small and non-MSA region with low return rates, where large negative cells appear to be due to factors other than sampling variability. Review of these blocks that make up this poststratum may be warranted.

It appears that, of the three models considered, the two group model never fits as well as the fixed odds ratio and fixed relative risk models. The two group model allocates the DA-estimated undercount in proportion to the DSE population estimates, an unattractive feature since failure of the independence assumption underlying the DSE is what induces the correlation bias and consequently the undercount. The FRR model tends to fit best when correlation bias is low to moderate (as in the non-African-American domains), while the FOR model tends to fit best when correlation bias is high (as in the African-American domains). While this is in part influenced by our formulation of the priors, it is also consistent with the FRR model tending to give undercount estimates that are intermediate between the larger undercount estimates of fixed odds ratio model and the small undercount estimates of the two group model.

Finally, an issue specific to the 2000 Census is that a relatively large number of duplicate address may have appeared in the Master Address File (MAF) when local governments were allowed to review their portion of the MAF and add addresses that they believed were missing. An evaluation by Fay (2002) indicated that as much as 1.1% of the E-sample may consist of unflagged erroneous enumerations (Fay 2002). Both the Bayesian or non-Bayesian methods discussed here will suffer from upward bias in estimating undercount (or downward bias in

Female	DSE	FOR	FRR	TG	Ave.
African-American 18-29	$3.9_{(2.4,5.4)}$	$3.8_{(2.1,5.3)}$	$3.7_{(2.2,5.3)}$	$3.6_{(2.1,5.2)}$	$3.7_{(2.2,5.2)}$
African-American 30-49	$1.3_{(0.5,2.2)}$	$1.2_{(0.3,2.1)}$	$1.2_{(0.4,2.1)}$	$1.2_{(0.3,2.1)}$	$1.2_{(0.3,2.1)}$
African-American 50+	$-1.0_{(-2.0,0.1)}$	$-1.3_{(-2.4,-0.4)}$	$-1.3_{(-2.3,-0.3)}$	$-1.2_{(-2.3,-0.2)}$	$-1.3_{(-2.4,-0.3)}$
Caucasian 18-29	$1.3_{(0.7,1.9)}$	$1.1_{(0.5,1.7)}$	$1.0_{(0.4,1.6)}$	$0.8_{(0.1,1.4)}$	$1.0_{(0.4,1.6)}$
Caucasian 30-49	$0.8_{(0.5,1.2)}$	$0.6_{(0.3,1.0)}$	$0.7_{(0.4,1.0)}$	$0.5_{(0.2,0.9)}$	$0.7_{(0.3,1.0)}$
Caucasian 50+	$-1.0_{(-1.3,-0.6)}$	$-1.1_{(-1.4,-0.8)}$	$-1.0_{(-1.4,-0.7)}$	$-1.2_{(-1.6,-0.9)}$	$-1.0_{(-1.4,-0.7)}$
Hispanic 18-29	$4.1_{(2.9,5.3)}$	$4.2_{(2.9,5.6)}$	$4.2_{(2.9,5.4)}$	$3.8_{(2.5,5.1)}$	$4.2_{(2.9,5.4)}$
Hispanic 30-49	$1.3_{(0.4,2.2)}$	$1.1_{(0.2,1.9)}$	$1.1_{(0.2,2.1)}$	$1.0_{(0.1,1.9)}$	$1.1_{(0.2,2.0)}$
Hispanic 50+	$0.2_{(-1.0,1.4)}$	$-0.3_{(-1.6,0.8)}$	$-0.2_{(-1.4,0.9)}$	$-0.4_{(-1.6,0.8)}$	$-0.2_{(-1.4,0.9)}$
Asian 18-29	$1.5_{(-1.5,4.3)}$	$1.4_{(-1.5,4.4)}$	$1.2_{(-1.7,4.0)}$	$0.9_{(-2.0,3.8)}$	$1.2_{(-1.7,4.2)}$
Asian 30-49	$0.8_{(-0.7,2.3)}$	$0.7_{(-0.8,2.1)}$	$0.7_{(-0.7,2.1)}$	$0.5_{(-1.0,2.0)}$	$0.7_{(-0.8,2.1)}$
Asian 50+	$0.9_{(-0.9,2.6)}$	$0.6_{(-1.3,2.4)}$	$0.7_{(-1.1,2.5)}$	$0.5_{(-1.2,2.2)}$	$0.7_{(-1.1,2.5)}$

Table 4: Percent undercount estimates by age, race, and gender in 2000 Census: DSE estimate; FOR, FRR, and TG posterior means and posterior mean of average of FOR, FRR, and TG models. 95% credible interval in parentheses. Negative values indicate Census overcount.

estimating overcount) if undetected duplicates remain, since they will be treated as matched cases and left in the adjusted Census totals. However, because Bayesian estimates borrow strength from other poststrata, they may protect against a undetected duplicates in outlying poststrata being "multiplied," thus generating unduly large estimates of undercount.

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