On X-11 Seasonal Adjustment and estimation of its variance

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Introduction

Most official seasonal adjustments use the X-11 method and its extensions, available for instance in the Census Bureau's X-12-ARIMA software (Findley et al, 1998). An important problem with the use of this method is how to estimate the variances of the estimators of the seasonal effects and the other components that the method produces. Wolter and Monsour (1981) propose an approach to variance estimation that uses the linear approximation to X-11. The methods of Pfeffermann (1994) and Bell & Kramer (1999) build on and extend this approach. The three methods use different definitions of the variance.

In this paper we propose new definitions of the seasonal and trend components under which the X-11 estimators of the trend and the seasonal components are almost unbiased in the central part of the series. Next, we define the variance and Mean Square Error (MSE) of the X-11 estimators with respect to the newly defined trend and seasonal components and we show that under these definitions the variance estimators of Pfeffermann (1994) are unbiased. We investigate the behavior of the X-11 estimators of the newly defined trend and seasonal components at the two ends of the observed series where they are biased and suggest a bias correction procedure.

The results are illustrated by a small simulation study based on the "Education and Health Services employment" (EDHS) series, obtained as part of the Current Employment Statistics program. Finally we estimate the bias corrected MSE of the X-11 estimators for EDHS.

Bias, Variance and MSE of X-11 estimators and their estimation

We begin with the usual notion that an economic time series can be decomposed into a trend or trend-cycle component T_t, a seasonal component S_t, and an irregular term, I_t; $Y_t = T_t + S_t + I_t$. Here we consider for simplicity the additive decomposition but the results can be generalized to the multiplicative decomposition, $Y_t = T_t \times S_t \times I_t$, using similar considerations as in Pfeffermann *et al*. (1995). Typically, the data are obtained from a sample survey, such that the observed value, y_t, can be expressed as the population value, Y_t, plus a sampling error,

$$y_t = Y_t + \varepsilon_t = T_t + S_t + e_t,\ \ e_t = I_t + \varepsilon_t,\ \ E[e_t \mid (S_n, T_n; n = 1,..., N)] = 0;\quad t = 0,..., N. \tag{1}$$

The X-11 program applies a sequence of moving averages or linear filters to the observed data. Thus the X-11 estimators of the trend and the seasonal components can be approximated as,

$$\hat{S}_t = \sum_{k=-N+t}^{t-1} w_{kt}^S y_{t-k},\quad \hat{T}_t = \sum_{k=-N+t}^{t-1} w_{kt}^T y_{t-k}, \tag{2}$$

where the filters w_{kt}^S and w_{kt}^T are defined by the X-11 program options for the given time interval $t = 1,..., N$. Moreover, at the central part of the series the filters are time-invariant, $w_{kt}^S = w_k^S$, for $a_S \le t \le N - a_S$, $w_{kt}^T = w_k^T$, for $a_T \le t \le N - a_T$, where a_S, a_T are also defined by the X-11 program options. Note also that $w_{kt}^T = w_k^T = 0$ if $k \notin [-a_T, a_T]$ and $w_{kt}^S = w_k^S = 0$ if $k \notin [-a_S, a_S]$, and

$w_{kt}^{T} = w_{kt}^{S} = 0$ if $t-k \notin [1,...,N]$. To simplify summation indexes, we denote for a given series Z, $\sum_k w_{kt}^{C} Z_{t-k} = \sum_{k: w_{kt}^{C} \neq 0} w_{kt}^{C} Z_{t-k}$ and $\sum_k w_{k}^{C} Z_{t-k} = \sum_{k: w_{k}^{C} \neq 0} w_{kt}^{C} Z_{t-k}$, $C = S$ or T.

Remark 1. X-11 and its extensions, like X-12 ARIMA include also “non-linear” operations such as the identification and estimation of ARIMA models and the identification and gradual replacement of extreme observations. We assume that the time series under consideration is already corrected for outliers. The effects of the identification and non-linear estimation of ARIMA models are generally minor, see, e.g., Pfeffermann *et al*. (1995) and Pfeffermann *et al*. (2000).

Assuming that S_t and T_t are well defined (although never observed) for $-\infty < t < \infty$, define, $S_t^{x11} = \sum_k w_k^{S} (T_{t-k} + S_{t-k})$, $T_t^{x11} = \sum_k w_k^{T} (T_{t-k} + S_{t-k})$, such that S_t^{x11} and T_t^{x11} are the outputs of applying the symmetric filters to the signal of the infinite series at each time point $t = 1,...,N$. Denote $\mathbf{S} = \{S_t, -\infty < t < \infty\}$ and $\mathbf{T} = \{T_t, -\infty < t < \infty\}$. Note from (1) that $S_t^{x11} = E[\hat{S}_t \mid \mathbf{S,T}]$ when $a_S \le t \le N - a_S$ and $T_t^{x11} = E[\hat{T}_t \mid \mathbf{S,T}]$ when $a_T \le t \le N - a_T$, which implies the following obvious result.

Result 1. Let $e_t^{x11} = y_t - T_t^{x11} - S_t^{x11}$. X-11 decomposes therefore the observed series into the ‘X-11-trend’ T_t^{x11}, the ‘X-11-seasonal component’ S_t^{x11}, and the ‘X-11 error’, e_t^{x11};

$$y_t = T_t^{x11} + S_t^{x11} + e_t^{x11}, \tag{3}$$

and at the center part of the series, $\max(a_S, a_T) \le t \le N - \max(a_S, a_T)$, the X-11 estimators of the trend and the seasonal components are almost unbiased with respect to the decomposition (3).

Remark 2. The decomposition defined by (3) into a seasonal component, a trend component and an error term is clearly not unique; see, for example, the discussion in Hilmer and Tiao (1982). Bell and Kramer (1999) use a similar decomposition: they define the “target” of the seasonal adjustment as the adjusted series that would be obtained if there was no sampling error and there are sufficient data before and after the time points of interest for the application of the symmetric filter (Bell and Kramer 1999, page 15). Thus, the Bell and Cramer seasonal and trend components are defined as, $S_t^{Bell,Kramer} = \sum_k w_k^{S} (T_{t-k} + S_{t-k} + I_{t-k})$ and $T_t^{Bell,Kramer} = \sum_k w_k^{T} (T_{t-k} + S_{t-k} + I_{t-k})$. The difference between (3) and the Bell and Kramer decomposition is therefore that the latter decomposition considers the irregular term as a part of the signal. As a result, the MSE of the X-11 estimators of the components defined by the decomposition (3) (see below), is generally higher than the MSE of the X-11 estimators of the components defined by the Bell and Kramer decomposition.

The bias, variance and MSE of the X-11 estimators with respect to decomposition (3), conditional on the true components **S, T** are obtained as follows:

$$Bias[\hat{S}_t \mid \mathbf{S,T}] = \sum_k (w_{kt}^{S} - w_k^{S})(T_{t-k} + S_{t-k}) \tag{4}$$

$$Var[\hat{S}_t \mid \mathbf{S,T}] = E\{[\sum_k w_{kt}^{S} y_{t-k} - E(\sum_k w_{kt}^{S} y_{t-k} \mid \mathbf{S,T})]^2 \mid \mathbf{S,T}\} =$$

$$E\{[\sum_k w_{kt}^{S} (y_{t-k} - S_{t-k} - T_{t-k})]^2 \mid \mathbf{S,T}\} = E[(\sum_k w_{kt}^{S} e_{t-k})^2 \mid \mathbf{S,T}], \tag{5}$$

$$MSE[\hat{S}_t] = E[(\hat{S}_t - S_t^{x11})^2 \mid \mathbf{S,T}] = Var[\hat{S}_t \mid \mathbf{S,T}] + Bias^2[\hat{S}_t \mid \mathbf{S,T}], \tag{6}$$

and similarly for the trend.

By Eq. 5, the variance of the X-11 estimator of the seasonal component is a linear combination of the covariances, $Cov(e_t, e_k)$, $t, k = 1,...,N$. Following Pfeffermann (1994), let

$$R_t = y_t - \hat{S}_t - \hat{T}_t = \sum_k a_{kt} y_{t-k}\ ,\ a_{0t} = 1 - w_{0t}^S - w_{0t}^T,\ a_{kt} = -w_{kt}^S - w_{kt}^T,\ k \neq 0\ ,$$

define the linear filter approximation of the X-11 residual term. Then,

$$Var(R_t \mid \mathbf{T,S}) = E\{[\sum_k a_{kt}(y_{t-k} - E(y_{t-k} \mid \mathbf{T,S})]^2 \mid \mathbf{T,S}\} = Var(\sum_k a_{kt} e_{t-k} \mid \mathbf{T,S})$$
$$Cov(R_t, R_m \mid \mathbf{T,S}) = Cov(\sum_k a_{kt} e_{t-k}, \sum_k a_{km} e_{m-k} \mid \mathbf{T,S}) = \sum_k \sum_l a_{kt} a_{lm} Cov(e_{t-k}, e_{m-l} \mid \mathbf{T,S})\ . \qquad (7)$$

It follows from (7) that $v_{tm} = Cov(e_t, e_m \mid \mathbf{S}, \mathbf{T})$, $t, m = 1,..., N$, and $u_{tm} = Cov(R_t, R_m \mid \mathbf{S}, \mathbf{T})$, $t, m = 1,..., N$, are related by the system of linear equations,

$$\mathbf{U} = D\mathbf{V}\ , \qquad (8)$$

where the matrix D is defined by the weights $a_{kt}, t, k = 1,..., N$ through (7). Since the X-11 residuals, R_t are observed for $t = 1,..., N$, (and assuming e_t is independent of the true trend and the seasonal components), $Cov(R_t, R_k \mid \mathbf{S}, \mathbf{T})$ can be estimated from the observed series at least at the central part of the series, $t = t^*,..., N - t^*$ for some $t^* > 0$. However, the number of equations in (8) for $t = t^*,..., N - t^*$ is smaller than the number of unknown covariates $v_{tm} = Cov(e_t, e_m \mid \mathbf{S}, \mathbf{T})$, and therefore (8) can not be solved directly and the solution is very unstable. A possible way to overcome this problem is by assuming that the covariances v_{tk} are negligible (and hence set to zero) for $|t - k| > C$ for some constant *C*, which allows then to solve the reduced set of equations obtained from (8). See Pfeffermann (1994), Pfeffermann and Scott (1997) and Chen *et al*. (2003), for different approaches to the estimation of $\mathbf{U}$ and $\mathbf{V}$.

Remark 3. Pfeffermann (1994) developed his variance estimators under the Postulate: $\sum_k a_{kt}(S_{t-k} + T_{t-k}) \cong 0$ at the center of the series. Although this assumption seems to hold approximately in practice, it is essentially impossible to test it. Note that this Postulate implies that $(\mathbf{T,S}) = (\mathbf{T}^{x11}, \mathbf{S}^{x11})$ at the center of the series, which is not generally true, see the results of the simulation study below. On the other hand, as shown above, Pfeffermann's (1994) method produces consistent estimators for the variance defined by (5).

Estimation of the MSE of the X-11 estimators is complicated. The error term, e_t , can be usually assumed to be independent of the true trend and the seasonal components, and therefore the variance in (5) does not depend on the signal. On the other hand, by (4), the bias of the estimator is a function of $\mathbf{S,T}$ and its value depends on the particular realization of the signal and therefore estimating of the bias requires strict model assumptions that could be hard to validate. Instead of estimating the MSE given the trend and the seasonal components, we propose therefore to estimate instead the expected MSE,

$$E\{MSE[\hat{S}_t]\} = E\{E[(\hat{S}_t - S_t^{x11})^2 \mid \mathbf{S,T}]\} = E\{Var[\hat{S}_t \mid \mathbf{S,T}]\} + E\{Bias^2[\hat{S}_t \mid \mathbf{S,T}]\}\ . \qquad (9)$$

Note that $E\{MSE[\hat{S}_t]\}$ can be considered as the best predictor of $MSE[\hat{S}_t]$ under a square loss function. Assuming that the error term, e_t is independent of the true trend and the seasonal components, the first term in (9) does not depend on the signal and therefore it can be estimated by use of Pfeffermann (1994) method. The second term can be estimated by the following parametric bootstrap procedure, which is illustrated in the simulation study:

(a) Fit a parametric model and estimate the parameters of the separate models identified for the trend, the seasonal component, the irregular term and the sampling errors. See Steps 1–3 of the simulation study where we use the models identified by SEATS, accounting for the sampling error information.

(b) Generate *B* series, $\mathbf{y}^b, b = 1,..., B$, each of sufficient length for applying the symmetric filters to the central N months of the generated series, by independently generating the four component series, and store the trend and the seasonal components. For each generated series compute the bias, $B_t^b = \sum_k (w_{kt}^S - w_k^S)(T_{t-k}^b + S_{t-k}^b)$, see Steps 4-6 of the simulation study.

(c) Estimate $E\{Bias^2[\hat{S}_t \mid \mathbf{S},\mathbf{T}]\}$ by averaging $(B_t^b)^2$ over $b = 1,...,B$., see Eq. 10 .

Remark 4. Bell and Kramer (1999) estimate the unconditional MSE of the X-11 estimator with respect to $\mathbf{S}^{Bell,Kramer}, \mathbf{T}^{Bell,Kramer}$, that is, they estimate $E[(\hat{S}_t - S_t^{Bell,Kramer})^2]$, instead of estimating $E[(\hat{S}_t - S_t^{Bell,Kramer})^2 \mid \mathbf{S},\mathbf{T}]$, which is similar to the use of (9).

Simulation study

We illustrate the implications of the use of the new definitions of the trend and the seasonal component in (3), and their estimation, and compare it to the estimation of the corresponding 'true' components by use of simulations. The simulations use the models fitted to the series "Education and Health Services employment" (EDHS), with observations from January, 1996 through December, 2005, (*N*=120).
We also estimate the MSEs of the X-11 trend and seasonally adjusted estimators of the true series.

Our interest in this series is in the month-to-month change in employment. As explained in Scott, *et al*. (2004), we consider the log ratios of the EDHS series, corrected for outliers as the original series (see Remark 1).

As the main objectives of the study we consider the estimation of the trend and the seasonally adjusted (SA) series. Correspondingly to decomposition (3), the definition of "X-11 SA series" is, $A_t^{x11} = Y_t - S_t^{x11}$. X-11 estimate of the SA series (SAE) is defined as $\hat{A}_t = y_t - \hat{S}_t = \sum_k w_{kt}^A y_{t-k}$ where $w_{0t}^A = 1 - w_{0t}^S, \quad w_{kt}^A = -w_{kt}^S,\ k \neq 0$.

The study consists of the following steps:

Step 1. Fit an ARIMA model to the observed series using X-12-ARIMA..

Step 2. Re-estimate the parameters of the model by use of REGCOMNT program (Bell, 2003), accounting for the presence of the sampling error component. (We model this component using the autocovariance estimates of the sampling errors as computed by the Bureau of Labor Statistics.)

Step 3. Identify the trend, seasonal, and irregular component models by use of signal extraction, available in the experimental software X-12-SEATS developed by the Census Bureau. The models and parameter estimates used in the simulation study are as follows:

Trend- T_t; ARIMA(1,1,2) with parameters -.90, .06, -.94 and disturbance variance 0.5;
Seasonal component- S_t; ARIMA(11,0,11) model with AR-coefficients equal to 1, MA-coefficients equal to, .70, .42, .17, -.04, -.20, -.30, -.37, -.39, -.38, -.34, -.28, and disturbance variance 4.5;
Irregular component- I_t; white noise with disturbance variance 18.0;
Sampling error- ε_t; MA(1) with MA-coefficient -.15 and disturbance variance 58.68.

Step 4. Generate independently 3,000 series from the component models developed in Step 3 and add them up to form new original series y_t^b. Each generated series has length *N*+96, $N = 120$, so that the application of X-11 to the full series gives values approximately equal to the "final" X-11 values for the central *N* points. Store the series y_t^b and their components, $b = 1,...,3{,}000$.

Step 5. Fix the form of the X-11-ARIMA model for the original series. For each series generated in Step 4 estimate the parameters of the ARIMA models and compute the filter weights w_{kt}^{A} and w_{kt}^{T}, reflecting extrapolation with forecasts based on the model identified for the series.

Step 6. For each series generated in Step 4 and the weights obtained in Step 5 define,

$$\hat{A}_t^b = \sum_k w_{kt}^A y_{t-k}^b , \qquad \hat{T}_t^b = \sum_k w_{kt}^T y_{t-k}^b ,$$

$$\tilde{A}_t^b = E(\hat{A}_t^b \mid \mathbf{T^b}, \mathbf{S^b}) = \sum_k w_{kt}^A (T_{t-k}^b + S_{t-k}^b) , \qquad \tilde{T}_t^b = E(\hat{T}_t^b \mid \mathbf{T^b}, \mathbf{S^b}) = \sum_k w_{kt}^T (T_{t-k}^b + S_{t-k}^b) ,$$

$$A_t^{x11,b} = \sum_k w_k^A (T_{t-k}^b + S_{t-k}^b) , \qquad T_t^{x11,b} = \sum_k w_k^T (T_{t-k}^b + S_{t-k}^b) ,$$

$$B_t^{A,b} = Bias[\hat{A}_t^b \mid \mathbf{S^b}, \mathbf{T^b}] = \sum_k (w_{kt}^A - w_k^A)(T_{t-k}^b + S_{t-k}^b) , \qquad B_t^{T,b} = Bias[\hat{T}_t^b \mid \mathbf{S^b}, \mathbf{T^b}] = \sum_k (w_{kt}^T - w_k^T)(T_{t-k}^b + S_{t-k}^b) ,$$

$$t = 48, \ldots, N+48 \, ; \; b = 1, \ldots, 3{,}000 .$$

Step 7. Compute the Empirical Root MSE of the X-11 estimators with respect to the 'true components' $\mathbf{A}^b, \mathbf{T}^b$ and with respect to the 'X-11 components' $\mathbf{A}^{x11,b}, \mathbf{T}^{x11,b}$, and the Empirical Standard Deviation (SD) of the X-11 estimators

$$RMSE_{\mathbf{T,S}}(\hat{A}_t^b) = \sqrt{\frac{1}{3{,}000} \sum_{b=1}^{3{,}000} (\hat{A}_t^b - A_t^b)^2} , \qquad RMSE_{\mathbf{T,S}}(\hat{T}_t^b) = \sqrt{\frac{1}{3{,}000} \sum_{b=1}^{3{,}000} (\hat{T}_t^b - T_t^b)^2} ;$$

$$RMSE_{x11}(\hat{A}_t^b) = \sqrt{\frac{1}{3{,}000} \sum_{b=1}^{3{,}000} (\hat{A}_t^b - A_t^{x11,b})^2} , \qquad RMSE_{x11}(\hat{T}_t^b) = \sqrt{\frac{1}{3{,}000} \sum_{b=1}^{3{,}000} (\hat{T}_t^b - T_t^{x11,b})^2} ;$$

$$SD(\hat{A}_t^b) = \sqrt{\frac{1}{3{,}000} \sum_{b=1}^{3{,}000} (\hat{A}_t^b - \tilde{A}_t^b)^2} , \qquad SD(\hat{T}_t^b) = \sqrt{\frac{1}{3{,}000} \sum_{b=1}^{3{,}000} (\hat{T}_t^b - \tilde{T}_t^b)^2} ,$$

Step 8. For each series generated in Step 4 estimate the variances of the X-11 estimators by the method developed in Pfeffermann (1994); see Pfeffermann (1994) and Pfeffermann and Scott (1997) for details. Denote the estimates by $\hat{V}(\hat{A}_t^b)$ and $\hat{V}(\hat{T}_t^b)$, and define,

$$\hat{SD}(\hat{A}_t^b) = \sqrt{\frac{1}{3{,}000} \sum_{b=1}^{3{,}000} \hat{V}(\hat{A}_t^b)} , \qquad \hat{SD}(\hat{T}_t^b) = \sqrt{\frac{1}{3{,}000} \sum_{b=1}^{3{,}000} \hat{V}(\hat{T}_t^b)} .$$

Estimation of MSE of X-11 Seasonally Adjusted and Trend Estimators for EDSH series

For the original EDSH series estimate the variances of the X-11 estimators by the method developed in Pfeffermann (1994). Denote the estimates by $\hat{V}(\hat{A}_t)$ and $\hat{V}(\hat{T}_t)$. Using the estimates $B_t^{A,b}$ and $B_t^{T,b}$ obtained in Step 6 of the simulation study compute the estimates of the squared bias when estimating the 'X-11 components',

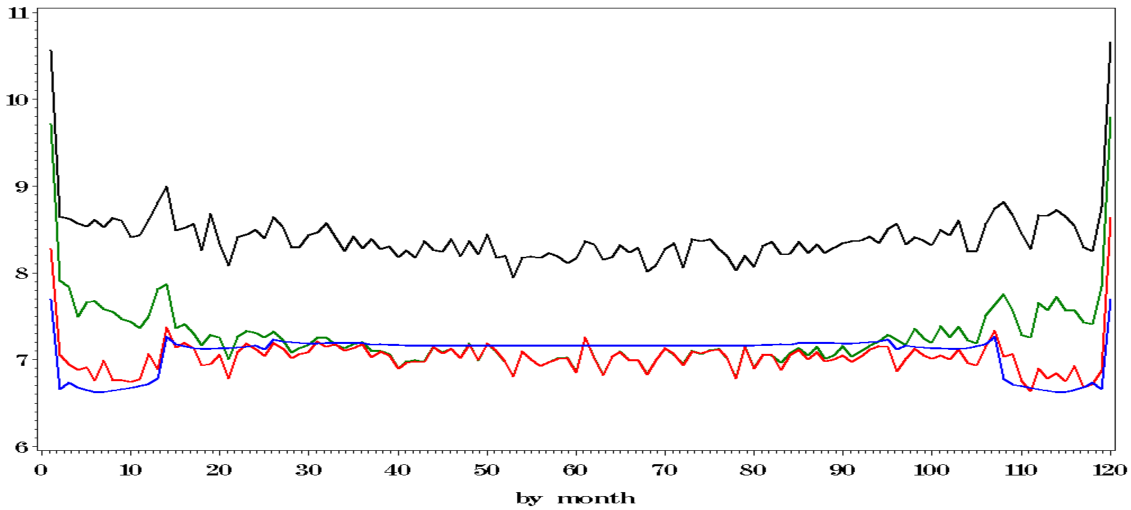
$$\hat{E}\{Bias^2[\hat{A}_t \mid \mathbf{S,T}]\} = \frac{1}{3{,}000} \sum_{b=1}^{3{,}000} (B_t^{A,b})^2 , \qquad \hat{E}\{Bias^2[\hat{T}_t \mid \mathbf{S,T}]\} = \frac{1}{3{,}000} \sum_{b=1}^{3{,}000} (B_t^{T,b})^2 , \tag{10}$$

and define the estimate of (9) for the original series as,

$$R\hat{M}SE(\hat{A}_t) = \sqrt{\hat{V}(\hat{A}_t) + \hat{E}\{Bias^2[\hat{A}_t \mid \mathbf{S,T}]\}}, \qquad R\hat{M}SE(\hat{T}_t) = \sqrt{\hat{V}(\hat{T}_t) + \hat{E}\{Bias^2[\hat{T}_t \mid \mathbf{S,T}]\}}. \tag{11}$$

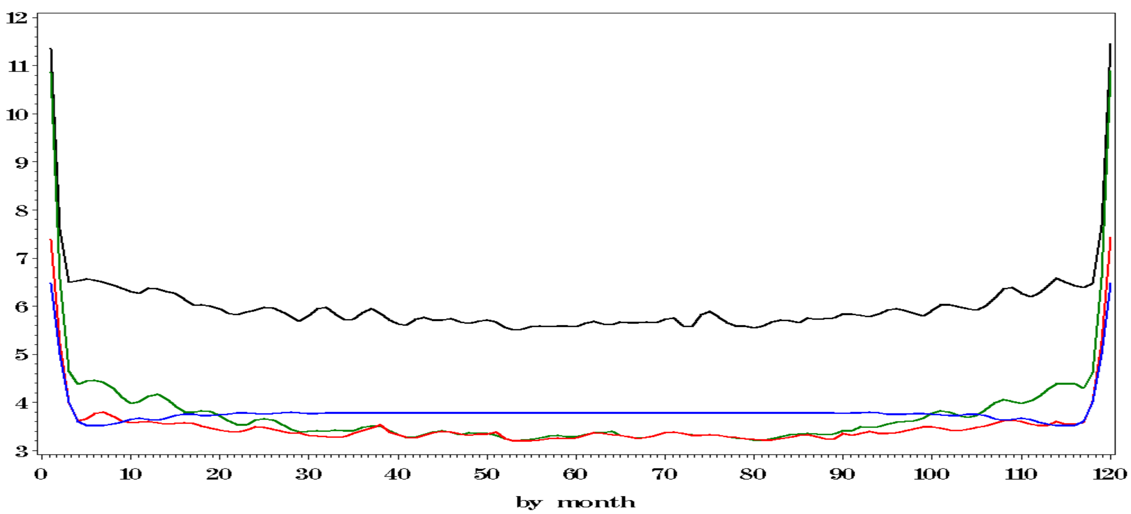
The results of the study are summarized in Figures 1–4.

Figure 1. Empirical SD and Root MSE of X-11 SA Estimators when estimating the true SA series, $\mathbf{A}^b = \mathbf{Y}^b - \mathbf{S}^b$ and when estimating the X-11 SA series $\mathbf{A}^{x11,b} = \mathbf{Y}^b - \mathbf{S}^{x11,b}$.



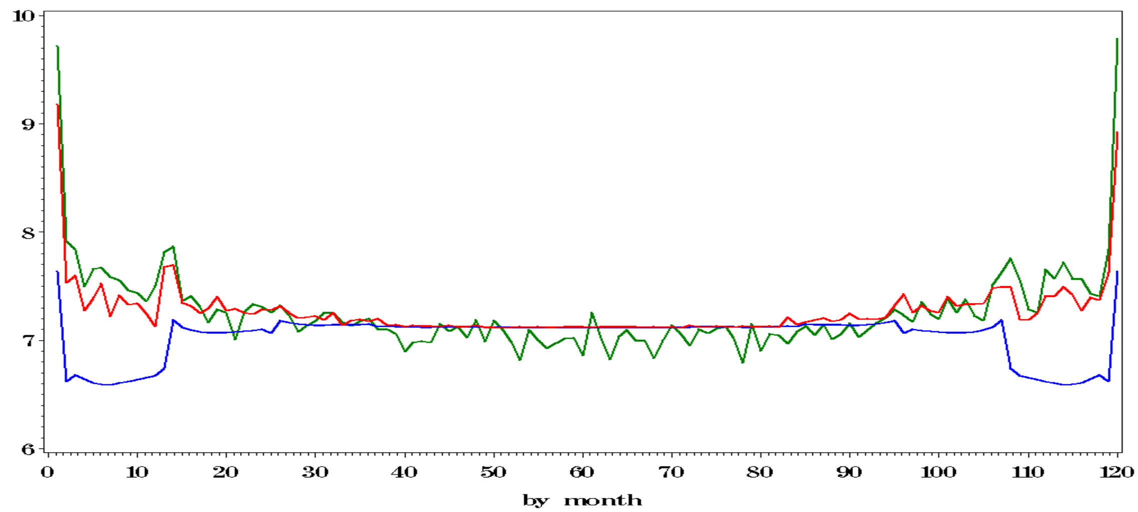
$SD(\hat{A}_t^b)$ is drawn in red, $\hat{SD}(\hat{A}_t^b)$ in blue, $RMSE_{\mathbf{T,S}}(\hat{A}_t^b)$ in black and $RMSE_{x11}(\hat{A}_t^b)$ in green.

Figure 2. Empirical SD and Root MSE of X-11 Trend Estimators when estimating the true Trend, $\mathbf{T}^b$ and when estimating the X-11 Trend $\mathbf{T}^{x11,b}$.



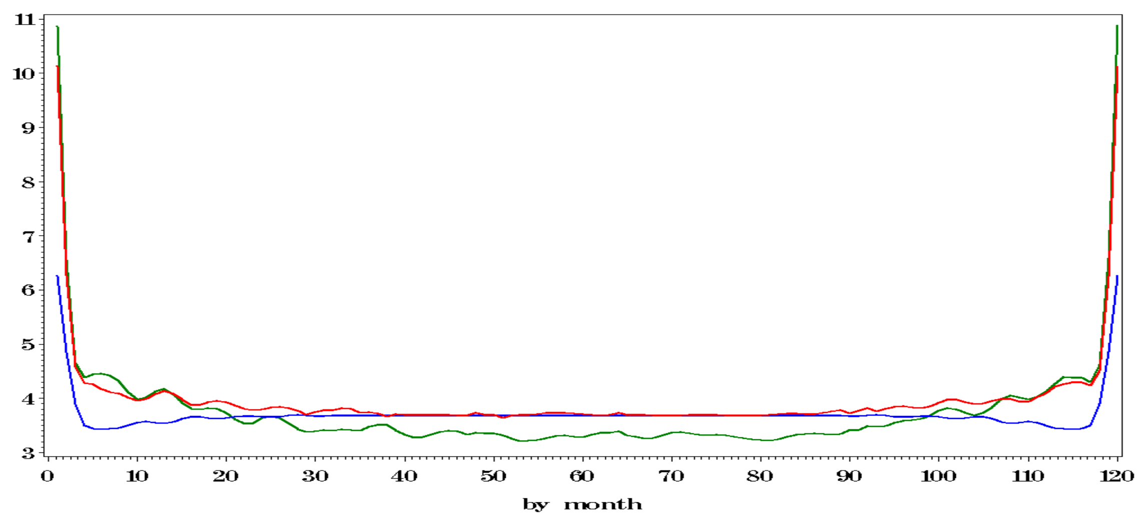
$SD(\hat{T}_t^b)$ is drawn in red, $\hat{SD}(\hat{T}_t^b)$ in blue, $RMSE_{\mathbf{T,S}}(\hat{T}_t^b)$ in black and $RMSE_{x11}(\hat{T}_t^b)$ in green.

Figure 3. Empirical Root MSE of X-11 SAE when estimating the X-11 SA series $\mathbf{A}^{x11,b} = \mathbf{Y}^b - \mathbf{S}^{x11,b}$, and estimates of the SD and Root MSE of X-11 SAE when estimating the X-11 SA series for the original EDHS series (see Eq. 11).



$RMSE_{x11}(\hat{A}_t^b)$ *is drawn in green,* $\hat{RMSE}(\hat{A}_t)$ *in red and* $\hat{SD}(\hat{A}_t)$ *in blue.*

Figure 4. Empirical Root MSE of the X-11 trend estimator when estimating the X-11 trend component $\mathbf{T}^{x11,b}$, and estimates of the Root MSE and Standard Deviations of the X-11 trend estimator when estimating the X-11 Trend component for the original EDHS series (see Eq. 11).



$RMSE_{x11}(\hat{T}_t^b)$ *is drawn in green,* $\hat{RMSE}(\hat{T}_t)$ *in red and* $\hat{SD}(\hat{T}_t)$ *in blue.*

Conclusions.

1) In this study the Empirical RMSEs of the X-11 estimators when estimating the true components are higher than the Empirical RMSEs when estimating the X-11 components defined by (3), which illustrates that the X-11 decomposition (3) is different from the 'true' decomposition used in the simulations**.**

2) For the 5 years in the center of the series the X-11 estimators are almost unbiased when estimating the newly defined X-11 components but at the beginning and at the end of the series there are nonnegligible biases.

3) Pfeffermann (1994) estimate of the variance approximates closely the empirical variance of the X-11 SAE when estimating the X-11 SA component, and overestimates slightly the empirical variance of the X-11 Trend estimator when estimating the X-11 trend.

4) Figures 3 and 4 illustrate that the proposed RMSE estimator (Eq. 10) corrects the bias incurred by the use of Pfeffermann (1994) variance estimators at the ends of the series when estimating the newly defined X-11 components. More extensive simulations are needed in order to test that the bias corrections are of the right magnitude.

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