Market Structure and the Flow of Information in Repeated Auctions

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Abstract

There is an ongoing public policy debate regarding vertical integration and its concomitant information flows. Of particular concern is that the information derived by an auctioneer (such as a distributor) will be shared with its integrated bidder (such as a manufacturer), leading to a reduction in competition between the bidders. Similar competitive concerns arise regarding bid revelation policies, such as those used in public sector procurement. Modeling such situations as the repeated auctions introduced in Thomas [1996a], this paper examines the transmission of private information via the auction outcomes, and shows how that transmission is affected by the changes in market structure described above. These concerns are not present in the existing auction literature, because such information transmission is irrelevant both in series of independent auctions and in sequential auctions in which participants desire only a single item. However, when bidders desire multiple items, and when the values of those items to a bidder are correlated, the incentive to learn about opponents' values and to obscure one's own drive equilibria to depart systematically from those in standard models. I examine structural information transmission, created through various policies for conducting auctions, and its effect on strategic information transmission, which arises as an optimal response to given structural policies. I initially model information acquisition in situations where bidders do not see rival bids and learn only the identity of the winner. I then extend this model to show how the desire to conceal information about oneself affects behavior by examining repeated auctions with publicly announced bids. The auctioneers prefer a policy of revealing all bids to revealing only the winner's bid and to revealing no bid information. Finally, I show that a vertical merger between a buyer and a seller can be procompetitive due to both structural and strategic information transmission.

1 Introduction

There is an ongoing public policy debate regarding vertical integration and its concomitant information flows. Of particular concern is that the information derived by an auctioneer (such as a distributor) will be shared with its integrated bidder (such as a manufacturer), leading to a reduction in competition between the bidders. Similar competitive concerns arise regarding bid revelation policies, such as those used in public sector procurement. Modeling such situations as the repeated auctions introduced in Thomas [1996a], this paper examines the transmission of private information via the auction outcomes, and shows how that transmission is affected by the changes in market structure described above. These concerns are not present in the existing auction literature, because such information transmission is irrelevant both in series of independent auctions and in sequential auctions in which participants desire only a single item. However, when bidders desire multiple items, and when the values of those items to a bidder are correlated, the incentive to learn about opponents' values and to obscure one's own drive equilibria to depart systematically from those in standard models.

It is certainly true in auction markets that firms often face the same competitors at several points in time. For example, a few large firms compete for government defense contracts, while local construction firms frequently compete for jobs in both the public and private sector. Additionally, private firms submit bids to hire goods and services, as with logging companies bidding for timber from the U.S. Forest Service, and with manufacturers bidding for the services of distributors in the pharmaceutical industry.

In all of these situations there is a small population of bidders whose members face each other repeatedly over time. Each interaction provides information about privately known attributes of the participating firms, the form and precision of which depends upon the specifics of the auction framework used. For example, in municipal construction contracting, firms' bids are publicly revealed after the contract is awarded. In contrast, in many private sector auctions, bids are considered proprietary information.

The value of the information gained by repeated interaction depends on the serial persistence of firms' attributes. If the bidder's valuations for items are independent across auctions, then no information is gained from previous interaction. However, when a bidder's valuations are correlated across auctions, information is transmitted simply by winning or losing an auction. That is, bidders learn about one another even if no bid information is revealed. For example, suppose that two bidders with continuously distributed valuations for two identical objects bid on the first object in a first price auction using the bid function $b(v)$ shown in Figure 1. By losing, player 1 with valuation v_1 knows $v_2 \geq v_1$. Similarly, by winning, player 2 knows $v_2 \geq v_1$. Consequently, dynamic interaction and the resulting information transmission mechanism suggest that, if there is a second auction, then the bidding behavior in the first auction differs from that in a single item auction. The desire to learn about one's opponents and to conceal information about oneself provides strong incentives for strategic behavior.[1]

In this paper I investigate two policies which affect structural information transmission, the information revealed through the basic market structure. I concurrently examine strategic information transmission, which arises as an optimal response to given structural policies. Following the model introduced in Thomas [1996a], I describe a situation in which bidders do not see rival bids and learn only the identity of the winner, ex post. This will be the benchmark against which I compare the following two changes in market structure.

First, I examine the impact of bid revelation policies on bidder behavior and auction outcomes. Comparing a full revelation policy with the benchmark suggests that current policies in public sector procurement lead to stronger competition than a policy which reveals no bid information. This result may be used to counter arguments that current bid revelation policies stifle competition in the public sector procurement process.[2] Moreover, taking as given that some bid revelation policy is required, I show that revealing all bids leads to greater revenues for auctioneers than does revealing only the winner's bid. Finally, as is also true when bids are not revealed, I find that a single seller bound by bid revelation rules prefers to bundle items for sale. This suggests that long term contracts should be preferred by firms accepting bids from input suppliers, for example.

Second, in light of recent government interest regarding information flows between vertically related firms, I examine the impact on firm behavior and consumer welfare of a vertical merger between a buyer and a seller. This is relevant if manufacturers compete for the services of distributors, for example, and a manufacturer and a distributor consider merging. I find that the merged manufacturer and distributor are willing to commit to a policy which prevents the transfer of information about opponents' bids, a remedy which has been applied in recent decisions. However, I also show that if bid information is transmitted vertically, then expected payments by all distributors (the receivers of bids) are lower than if there were no merger. These lower bids translate into lower

[1]See Thomas [1996c] for a more general discussion of this phenomenon.

[2]Of course, other factors may determine the efficacy of bid revelation policies, such as concerns about collusive behavior.

consumer prices, and consequently the vertical merger is procompetitive.

The issues raised above are intriguing. The reason they have not been previously analyzed is that, despite the recent growth of interest in sequential auctions, no work[3] examines the simple model I propose here. Other papers dealing with repeated auctions do not examine interrelationships between auctions and how they drive the asymmetric nature of learning, with its consequent effect on behavior. These issues are critical to understanding how decision makers act in dynamic situations with incomplete information.

The rest of the paper is organized as follows. First, I briefly describe papers from the auction literature which are related to this topic. Section 2 introduces a repeated auction which is a simplification of the model in Thomas [1996a]. Section 3 examines the outcomes which arise under a public price announcement scheme. Section 4 discusses the implications of a merger between a buyer and a seller, relative to the pre-integration equilibrium, while Section 5 briefly concludes.

Background Literature

The typical model of a multi-period auction has either a series of independent single item auctions in which participants receive new draws from the distribution of types each period or a series of single item auctions in which each participant desires only one of the items. Thomas [1996a] introduces the idea of repeated auctions, in which participants would like to acquire all of the items and in which a participant's values for all the items are identical. Other papers examine related topics in multi-period auctions, but do not discuss the serial linkage of auctions. Because no other work examines such a model, the question of vertical integration and bid revelation are largely irrelevant and are certainly different from the questions posed here. If the auctions are independent, then both bid revelation and vertical integration will not change behavior, beyond the vertical integration causing a change in behavior in the auction involving the merged entity. In a traditional sequential auction, vertical integration may have some effect, while Milgrom and Weber [1982b] show that revealing the winner's bid has no effect on behavior.[4] The papers by Hausch [1988] and Bikhchandani [1988] use bid revelation, but do not make comparisons of different revelation policies.

2 A Two Period Repeated Auction

This section presents a model in which two players[5] bid for two identical items. Each player has a valuation v for each object, which is constant regardless of the number of items acquired.[6] I call

[3]With the exception of this paper's predecessor, Thomas [1996a].

[4]Other issues regarding the repeated auction process in general are discussed more fully in Thomas [1996a].

[5]I will use the terms player, bidder, and firm interchangeably.

[6]I find this extreme to be more plausible in an industrial organization context than the assumption that the utility

this a "repeated auction" to contrast it with a "sequential auction" in which players desire only a single item. For simplicity I assume that players can be one of two types.

First, I characterize the equilibrium of a series of T independent single item asymmetric auctions. There can be no learning in such an environment, so it will be the benchmark showing how behavior changes once players have incentives to acquire information about their rivals. Second, I characterize the equilibrium of the two period repeated auction in which players learn only the identity of the winner. I show that bidding competition in auction one is softer than if there were no later auction. Also, bidding in auction two is softer than if there were no previous auction. Thus, the bidders earn higher payoffs in the repeated auction than if the two auctions were independent or were held simultaneously.

To avoid any confusion, recall that the analyses of buying auctions and selling auctions are conceptually identical. The model I use is of a buying auction in which bidders are buyers and auctioneers are sellers. However, some of the examples I use have bidders as sellers. When I offer insights about such situations I adjust my explanations accordingly.

A General Asymmetric Auction

Consider an asymmetric first price auction in which two players, A and B, can be one of two types, v_L and v_H, with $v_L < v_H$. For simplicity, assume $v_L = 0$.[7] Player A is type v_H with probability α, while player B is type v_H with probability β. Without loss of generality, assume that $\alpha \leq \beta$.[8] Suppose there are T items to be sold, and that a buyer has an identical valuation for each. If the items are sold simultaneously, then it is as if there is a series of T independent auctions. Therefore, the equilibrium is a T times repeated version of the equilibrium from a single item auction. A buyer may submit the same bid for each item, or may draw each bid from the mixed strategy distribution of bids. This indifference arises because the bidders are assumed to be risk neutral and both methods of bidding offer the same expected payoff.

Alternatively, suppose the T items are sold sequentially but are only stochastically equivalent to the bidders. That is, a bidder receives a new draw from the distribution of valuations for each item. In this case, each auction is equivalent to a single item auction. Information from prior

of a second object is zero, as would be the case in a standard sequential auction. Of course, more generally it would be nice to consider positive and negative synergies. My extreme assumption makes the analysis easier, while still adding the correlation of values across auctions.

[7]This assumption will not affect the qualitative nature of the results. Suppose $0 < v_L < v_H$. Type v_L players earn zero in equilibirum and bid v_L. Methods similar to those described following Proposition 1 may be used to correct for any problems associated with the possibility of ties.

[8]This model follows and extends example 2 in Maskin and Riley [1995]

auctions has no effect on beliefs and hence on behavior in the current auction.[9]

Let $F_A(b)$ and $F_B(b)$ be the equilibrium bid distributions used by type v_H players A and B in each period. Let $\pi^L(v \mid \alpha, \beta)$ and $\pi^H(v \mid \alpha, \beta)$ denote the equilibrium expected profits of player A and player B, respectively, whose type is v when the probabilities that A and B are type v_H are α and β. The L and H refer to A's more likely being a low valued player than B. Similarly, let $P^L(v \mid \alpha, \beta)$ and $P^H(v \mid \alpha, \beta)$ denote the equilibrium expected payments of player A and B, respectively, when their type is v and the probabilities that A and B are type v_H are α and β. Clearly $P^L(0 \mid \alpha, \beta) = P^H(0 \mid \alpha, \beta) = 0$. More interesting are the expected payments of the high valuation players. Finally, let $R(\alpha, \beta)$ denote the seller's expected revenue when player A and player B are type v_H with probability α and β.

Proposition 1 *The equilibrium in the asymmetric T item simultaneous auction is in mixed strategies and has the following properties:*

1) Type v_H players mix over $[0, \alpha v_H]$.
2) $F_A(b) = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{v_H}{v_H - b}\right) - \left(\frac{1-\alpha}{\alpha}\right)$
3) $F_B(b) = \left(\frac{1-\alpha}{\beta}\right)\left(\frac{v_H}{v_H - b}\right) - \left(\frac{1-\beta}{\beta}\right)$
4) $\pi^L(v_H \mid \alpha, \beta) = \pi^H(v_H \mid \alpha, \beta) = (1-\alpha) v_H T$
5) $P^L(v_H \mid \alpha, \beta) = \left(\frac{\alpha v_H}{2}\right) T$
6) $P^H(v_H \mid \alpha, \beta) = \left(\frac{\alpha^2 v_H}{2\beta}\right) T$
7) $R(\alpha, \beta) = (\alpha^2 v_H) T$

Proof: See the Appendix.

From part 3, one sees that if B is more likely to be v_H than is A, then B will bid zero (v_L) with positive probability. The careful reader will notice that if $F_B(0)$ is positive, then B faces the risk of tying with a type v_L player A. This implies that B should bid slightly above zero, thus unraveling the purported equilibrium. This problem is minor and may be resolved in two ways. First, one may assume that in equilibrium type v_L players do not submit a bid. Second, one may assume the existence of an infinitesimal bid $\epsilon > 0$ on which the probability mass actually rests. These methods are employed in Maskin and Riley [1994], and they are typically equivalent. However, in Sections 3 and 4 I examine situations in which bids are revealed, and thus firms may truly wish to bid zero and hide the fact that they are type v_H. For this reason I employ the first method, which still

[9] Concerns about collusion are trivially dismissed in the simultaneous sale. In the sequential sale, collusion unravels via backward induction arguments.

yields Nash equilibrium outcomes.[10] Further, I assume that the seller reveals a "no bid" as a zero bid when auction policy requires bid revelation. This causes bidders to assess positive probability that their opponent is type v_H, which induces higher bidding than if bidders knew they faced a type v_L opponent.

Both type v_H players have the same expected profit in equilibrium. This is so because each type v_H player has the same high bid in equilibrium. The high bid wins with probability one and yields payoff $v_H - \bar{b}$. Because each bid in a mixed strategy equilibrium gives the same expected payoff both A and B have the same expected payoff.

In the asymmetric case, a type v_H player A's expected payment is higher than a type v_H player B's. This appears trivial, because parts 2 and 3 show that A's equilibrium bid distribution stochastically dominates B's. It also appears strange because both players have the same expected payoff. However, when determining expected payments one must take into account not simply the expected bid made, but also the probability that that bid is successful. Thus, a given bid by player A may be less likely to win than the same bid by player B, because A more likely faces a competitor bidding a positive amount than does B.

Proposition 1 allows easy computation of continuation values for auctions with asymmetric beliefs. In the repeated auctions I analyze, players begin with symmetric beliefs, but their actions today determine their updated beliefs about their opponents tomorrow. By using the template above, bidders can calculate the payoff they expect to receive in the future for given actions of theirs in the current auction. To do so, they simply define α and β by the appropriately updated beliefs based on their current actions.

A second reason for constructing this benchmark is that the repeated auction involves bidding behavior that is used to learn about the valuations of one's competitors. This benchmark, by construction, does not exhibit such behavior. By comparing the two, it is possible to show how the desire to elicit information about one's rivals leads to changes in bidding behavior and auction outcomes. This comparison also shows whether a seller of T identical items should sell them simultaneously or sequentially.[11]

The Two Period Bidding Model

I now solve for the symmetric equilibrium of the two period repeated auction to see how bidding

[10] Although I informally examine what happens when this method is not used.

[11] This last part is equivalent to having a buyer's option, in that one should see the single shot bids. Bidders will not post the low bids that arise in the repeated setting because once a bidder has won he will buy all of the items at the current bid. This induces bidders to bid more aggressively than without the buyer's option.

in the two periods differs. I also compare bidding in the repeated auction to that in a series of independent auctions. As is usual in this sort of dynamic game, I begin in auction two, taking as given some bid distribution from auction one.[12]

Auction Two

Suppose that type v_H players A and B used the bid distribution $F_1(b)$ on $[0, \bar{b}_1]$ in auction one, and that player B won with a bid b_{B1}, while player A lost with bid b_{A1}. Initial beliefs put probability $\frac{1}{2}$ that either player is type v_H. Following auction one, it is now common knowledge that B is type v_H, while player B's updated belief that player A is type v_H is $\frac{F_1(b_{B1})}{1+F_1(b_{B1})}$. Notice that, conditional on winning, B learns more about A's type the lower was B's winning bid. Similarly, a type v_H player A learns more about B's first period bid the higher was A's losing bid.

Though the type v_H players used mixed strategies in auction one, I can characterize an equilibrium in auction two in which the loser of auction one uses a pure strategy and the winner of auction one uses a mixed strategy. In this equilibrium, players condition their bid or bid distribution in auction two on their bid in auction one and whether the bid was successful or not.

Firm A's Problem

Suppose that for a given successful bid b_{B1}, B bids according to

$$F_B(b|b_{B1}) = \frac{1}{1 + F_1(b_{B1})} \left(\frac{v_H}{v_H - b} \right).$$

This is the mixed strategy distribution B would use if B were known to be type v_H and A were type v_H with probability $\frac{1}{1+F_1(b_{B1})}$. A maximizes its expected profits for a bid a, given b_{A1}:

$$\pi_2^A(a \mid b_{A1}) = \frac{\int_{b_{A1}}^{\bar{b}_1} F_B(a|s)F_1'(s)ds}{1 - F_1(b_{A1})}[v_H - a].$$

One concern with this formulation of A's expected payoff is that there may exist pairs of (a, s) such that $F_B(a|s) > 1$, when for those pairs it should be noted $F_B(a|s) = 1$. However, if A is restricted to bids $a \leq \frac{v_H F_1(b_{A1})}{1+F_1(b_{A1})}$, the maximum bid if $\alpha = \frac{F_1(b_{A1})}{1+F_1(b_{A1})}$, then $F_B(a|s) \leq 1$ for all $s \in [b_{A1}, \bar{b}_1]$. Thus, A's expected payoff may be re-written as

$$\pi_2^A(a \mid b_{A1}) = \frac{v_H}{1 - F_1(b_{A1})} \int_{b_{A1}}^{\bar{b}_1} \frac{F_1'(s)}{1 + F_1(s)} ds = \frac{v_H}{1 - F_1(b_{A1})} \left[\ln(2) - \ln[1 + F_1(b_{A1})]\right],$$

[12] Note that there will be no out of equilibrium beliefs, because players will not see the action of their opponents, only auction outcomes.

which is independent of A's bid $a \leq \frac{v_H F_1(b_{A1})}{1+F_1(b_{A1})}$.

Firm B's Problem

Suppose that for a given unsuccessful bid b_{A1} in auction one, A bids according to the strictly increasing function $L(b_{A1}) = \frac{v_H F_1(b_{A1})}{1+F_1(b_{A1})}$ in auction two. A is willing to make this bid according to the preceding analysis. B wins with bid b if and only if

$$b > \frac{v_H F_1(b_{A1})}{1+F_1(b_{A1})} \Leftrightarrow F_1^{-1}\left(\frac{b}{v_H - b}\right) > b_{A1}.$$

Thus, B's expected payoff is

$$\pi_2^B(b \mid b_{B1}) = \left[\frac{1}{1+F_1(b_{B1})} + \left(\frac{F_1(b_{B1})}{1+F_1(b_{B1})}\right)\left(\frac{F_1\left(F_1^{-1}\left(\frac{b}{v_H-b}\right)\right)}{F_1(b_{B1})}\right)\right][v_H - b] = \frac{v_H}{1+F_1(b_{B1})},$$

which is independent of b. Thus, B is willing to bid according to $F_B(b|b_{B1}) = \frac{1}{1+F_1(b_{B1})}\left(\frac{v_H}{v_H-b}\right)$.

By examining the maximization problems of firms A and B, it is clear that for a given mixed strategy distribution $F_1(b)$ in auction one, the equilibrium in auction two is given by the pure strategy $L(b) = \frac{v_H F_1(b)}{1+F_1(b)}$ for the loser of auction one and the mixed strategy distribution $F_B(b|b_1) = \frac{1}{1+F_1(b_1)}\left(\frac{v_H}{v_H-b}\right)$ for the winner of auction one.

Characterization of Equilibrium

Let $\pi_W^*(b)$ denote the equilibrium expected payoff in auction two of a type v_H player who won auction one with bid b. Define $\pi_L^*(b)$ analogously for a type v_H loser of auction one. Let $\hat{\pi}$ be the expected profit for a type v_H player in a symmetric single shot auction. From Proposition 1, with $\alpha = \beta = \frac{1}{2}$, $\hat{\pi} = \frac{v_H}{2}$.

Theorem 1 *For all first period bids $b \in [0, \bar{b}_1)$, $\pi_W^*(b) > \hat{\pi}$ and $\pi_L^*(b) > \hat{\pi}$.*

Proof: From the derivation above, for all first period bids $b \in [0, \bar{b}_1)$,

$$\pi_W^*(b) = \frac{v_H}{1+F_1(b)} > \frac{v_H}{2} = \hat{\pi}$$

and

$$\pi_L^*(b) = \frac{v_H}{1-F_1(b)}\left[\ln(2) - \ln[1+F_1(b)]\right] > \frac{v_H}{2} = \hat{\pi}.$$

This is the desired result. □

Given the characterization of bidding in the two possible contingencies of auction two, having either won or lost auction one, it is possible to determine optimal bidding behavior in the first auction.

Auction One

The expected payoff to player B from bidding b in auction one, given optimal play in auction two and assuming that a type v_H player A is using $F_1(b)$, is

$$\begin{aligned}\pi^B(b \mid v_H) &= \frac{1}{2}\left[1+F_1(b)\right]\left[(v_H-b)+\pi_W^*(b)\right]+\frac{1}{2}\left[1-F_1(b)\right]\pi_L^*(b)\\ &= \frac{1}{2}\left[1+F_1(b)\right](v_H-b)+\frac{v_H}{2}+\frac{v_H}{2}\left[\ln(2)-\ln\left(1+F_1(b)\right)\right]\end{aligned}$$

Proposition 2 *The equilibrium bid distribution in auction one, $F_1(b)$, is the solution to*

$$F_1(b)=\left(\frac{b}{v_H-b}\right)+\left(\frac{v_H}{v_H-b}\right)\ln\left(1+F_1(b)\right).$$

Proof: Because bidders use a mixed strategy in auction one, each bid in the bid distribution yields the same total expected payoff over the entire auction. Thus, $\pi^B(b \mid v_H)=\pi^B(0 \mid v_H)$, so

$$\frac{1}{2}\left[1+F_1(b)\right](v_H-b)+\frac{v_H}{2}+\frac{v_H}{2}\left[\ln(2)-\ln\left(1+F_1(b)\right)\right]=v_H+\frac{v_H}{2}\ln(2).$$

Minor algebra yields

$$F_1(b)=\left(\frac{b}{v_H-b}\right)+\left(\frac{v_H}{v_H-b}\right)\ln\left(1+F_1(b)\right),$$

which is the desired result. □

It is easy to show that equilibrium bids in auction one are lower on average if there is a later auction. That is, bidders bid less aggressively if there exists a later auction.

Proposition 3 *The equilibrium bid distribution from Proposition 1 with $\alpha=\beta=\frac{1}{2}$ first order stochastically dominates $F_1(b)$.*

Proof: Follows from a simple comparison of the two distributions. □

It is straightforward to rank the profits from the repeated auction and the series of independent auctions.

Theorem 2 *Each bidder's expected payoff is higher in the two period repeated auction than in a series of two independent auctions.*

Proof: Because bidders use a mixed strategy in auction one, each bid in the bid distribution yields the same total expected payoff over the entire auction. B's expected payoff from bidding zero in auction one is

$$\pi^B(0 \mid v_H) = \frac{v_H}{2} + \frac{v_H}{2} + \frac{v_H}{2}\ln(2) > v_H.$$

This shows that the repeated auction offers the bidders higher expected payoffs than does a sequence of two independent auctions, which yields an expected payoff to a type v_H bidder of v_H. $\square$

There are two main implications of this softening of bidding behavior relative to a series of independent auctions. First, if there is only one seller (that is, if seller 1 and seller 2 are actually the same firm), then selling the items simultaneously is strictly preferred to selling them sequentially. For example, a company hiring services through a procurement auction should prefer signing long term contracts to short term contracts. This preference is strictly due to information transmission, and it ignores other reasons for wanting long term contracts, such as negotiation costs and fears about collusion.

Second, if the sellers are different, then they would like to differentiate their products. This preference for differentiation is not because bidders will substitute one seller's product for the other's; bidders want to acquire both items. The sellers wish to differentiate their products because information acquisition leads to softer bidding, something absent from the series of independent auctions in Proposition 1.

This analysis is complicated by the fact that bids are not revealed, so the updated beliefs about one's rivals are unknown. A simpler method is presented in the next section, which examines how changing the auction framework to require bid revelation changes bidder strategies and auction outcomes.

3 The Effect of Price Announcements

Most public sector auctions, such as those for construction contracts, reveal the bids submitted by all participants. This ensures honesty by preventing transfers from bidders to the auctioneer, a government representative, from altering auction outcomes.

While this concern has merit, it is well known in the industrial organization literature that revealing private information or reducing the noise of unobservables enhances collusion. Stigler [1964] notes that "collusion will always be more effective against buyers who report correctly and fully the prices tendered to them." Moreover, Green and Porter [1984] show that the ability of firms to collude decreases and the incidence of price wars increases because firms cannot observe their opponents' output and are uncertain about demand. If firms could monitor each other's output choices, then they could collude and avoid price wars, even with low demand realizations.

A related yet often overlooked point when discussing collusion in auctions is that firms' ability to collude is reduced when they are uncertain about one another's production costs.[13] This gives another strong motive not to reveal bids in repeated auctions, because otherwise firms will form better estimates about the costs of their rivals. From the point of view of a government department soliciting bids from contractors, these two points suggest that bid revelation leads to higher expected payments.

One issue not previously considered in the procurement context is the informational effect on non-collusive bidding behavior which arises from bid revelation. In a single item auction, or equivalently, when firms redraw their types for each auction in a repeated setting, the information gained through seeing rivals' bids is irrelevant.[14] However, once firms face each other repeatedly and with unchanging valuations, such information can be crucial. If the objects sought have a common value aspect, then the bids of other players reveal information about the common state variable. Similarly, if the objects have private value aspects, then the bids of other players reveal information about their type. Given the properties of equilibrium in Section 2, with no bid announcements, a natural and compelling question to ask is how the price announcement affects bidding behavior and expected revenues.

I use the model developed in Section 2 to examine the effect of bid revelation.[15] In particular, I study two different bid announcement policies seen in practice. The first entails revealing all bids, while the second reveals only the bid of the winner. Because generating public information about firms' costs leads to more vigorous competition when firms set prices, revealing all bids leads to greater revenues for the auctioneer than revealing only the winner's bid, which helps counter arguments that current policies in public sector procurement stifle competitive behavior. Surprisingly, revealing no bids leads to the identical revenues for the auctioneer as revealing only the winner's bid.

Revealing All Bids

Suppose A and B participate in a two item repeated auction, but both players' first period bids are

[13] Refer to the information sharing literature, such as Vives [1984], Gal-Or [1986], and Shapiro [1986].

[14] Unless a firm's distribution of types is unknown. Then knowing the draws a firm has had will afffect beliefs about the true distribution of that firm's type.

[15] Hausch [1988] presents a repeated auction model with bid revelation, but his model is inapplicable to the independent private values framework I use. However, he does explain behavior in the context of the common and the strictly mixed common and private value frameworks I mentioned above.

revealed after auction one. As in Section 2, I assume that type v_L players submit no bids in either auction, so the only interest is in the behavior of the type v_H players. It is easy to show that the incentive for type v_H players to deviate from either a pure pooling equilibrium or a pure separating equilibrium is too great, and that the only equilibrium is semi-pooling.

Pooling equilibria entail bids of zero in auction one by both types of players. More precisely, type v_L bidders do not bid, while type v_H bidders bid zero, with the seller revealing both as zero bids.[16] The winner, if there is one,[17] is revealed to be type v_H, but no information is revealed about the loser. Thus, the second auction will have the same expected outcome as the single item symmetric auction in Proposition 1. The following proposition shows that type v_H bidders have an incentive to deviate from a pooling strategy. By bidding $\epsilon > 0$ a type v_H bidder can win auction one for sure without affecting his payoff in auction two.

Proposition 4 *There does not exist a pooling equilibrium in this game.*

Proof: See the Appendix.

In a pure separating equilibrium types v_L and v_H bid different amounts with probability one. A type v_H player has an incentive to bid zero and mimic a type v_L player, however. The bidder will still beat a type v_L rival and will convince a type v_H rival that a bid of zero is sufficient to win auction two. Thus, the bidder "lying in the grass" can obtain the second object for almost nothing.

Proposition 5 *There does not exist a pure separating equilibrium in this game.*

Proof: See the Appendix.

In a semi-pooling equilibrium, type v_H players bid zero in auction one with positive probability. Thus, seeing a zero bid by one's opponent is not proof that he is type v_L. However, positive bids will still reveal that one is type v_H. Proposition 6 describes bidding behavior in the first auction of a semi-pooling equilibrium.

Proposition 6 *The symmetric bid distribution in auction one with all bids announced,* $F_1^{AA}(b)$, *is* $F_1^{AA}(b) = \frac{b}{v_H - b} + \frac{v_H(\sqrt{17}-3)}{2(v_H - b)}$.

Proof: See the Appendix.

[16] The seller has an incentive to do this. If "no bids" were revealed as such, then the seller will earn zero in the second auction.

[17] Note that both players may be type v_L, in which case there will be no winner.

A winning player is revealed as type v_H no matter what bid revelation scheme is used. The reason for mimicking type v_L players is to hide one's type in case of a loss in auction one. This response intuitively contrasts with the results in both the single item symmetric auction and the two period auction without price announcements, both of which have no pooling of bids at zero. Because of the mimicry by the type v_H bidders in auction one, seller 1 does worse than if there were no later auction. Also, because of the likelihood of the auction one winner thinking the loser is type v_L, seller 2 does worse than if there were no previous auction.

Proposition 7 *In the semi-pooling equilibrium with all bids announced, the ex ante expected profits of seller 1 are*

$$R_{S1}\left(\frac{1}{2},\frac{1}{2}\right) = \left(\frac{5-\sqrt{17}}{4}\right)^2 v_H.$$

The ex ante expected profits of seller 2 are

$$R_{S2}\left(\frac{1}{2},\frac{1}{2}\right) = \frac{49-11\sqrt{17}}{4(\sqrt{17}-1)^2} v_H.$$

Proof: See the Appendix.

Because both sellers do worse in this dynamic environment relative to the static one, if there is a single seller who is legally required to release bid information, then she prefers selling them simultaneously. A preference for simultaneous sales arises in Froeb and McAfee [1988], which suggests making sales more lumpy to increase firms' incentive to cheat on collusive agreements. In the repeated auction model, the preference for simultaneous sales is because their use reduces the flow of information to bidders.

Another interpretation of the result that sellers do better in static auctions, as with the result without price announcements, is that the sellers do better when the auctions are independent. In a sense, the sellers would like to differentiate their products. They are harmed by the fact that they are perfect substitutes, though not because bidders are choosing one item instead of the other; bidders wish to acquire both items. The sellers are harmed for two reasons. First, the bidders use the similarity of the items to learn about each other's valuation and thus reduce their bids in auction two. Second, the learning is accomplished by bidding low in the first auction.

Revealing the Winning Bid

In some auctions only the winner's bid is announced, a policy which should seemingly be preferred by the sellers. It reveals fewer bids, which eases fears about collusion, and yet it prevents corruption in the award process. Bid revelation greatly simplifies the analysis compared to that of the two period auction in Section 2. The difficulty there was that the winning firm's belief about the loser's type was not common knowledge. Bid revelation makes the winner's beliefs about the loser's type common knowledge, which permits the use of the asymmetric auction template developed in Section 2.

Proposition 8 *The symmetric equilibrium bid distribution in auction one with only the winner's bid revealed, $F_1^{WA}(b)$, is the solution to*

$$F_1^{WA}(b) = \frac{b}{v_H - b} + \left(\frac{v_H}{v_H - b}\right) \ln[1 + F_1^{WA}(b)].$$

Proof: See the Appendix.

Type v_H bidders do not pool at zero when only the winner's bid is revealed. The reason players submit zero bids when all bids are announced is to disguise their type when they lose. With only the winner's bid revealed there is no need to worry about disguising one's type after losing, so there is no incentive to pool at zero. This also explains why there are no pooling or semi-pooling equilibria.

Although revealing all bids leads to some pooling at zero, it is straightforward to show that the auctioneers do better when all bids are announced.

Theorem 3 *The sum of the sellers' expected profits are higher when all bids are announced than when only the winner's bid is announced and when no bids are announced.*

Proof: See the Appendix.

The intuition for this result is fairly simple. Although there is some pooling at zero when all bids are revealed, this is mitigated somewhat because the bidders' mixed strategy distribution includes bids strictly higher than those employed in the distribution when only the winner's bid is announced. Bidders potentially bid much higher because, if they are going to risk revealing they are type v_H by bidding a positive amount, then they have a strong incentive to win the first item. Thus, the net effect on revenues for the first seller are unclear. However, the environment following auction one tends to be more competitive when all auction one bids are announced. This is so because of the likelihood that, if both types are revealed to be v_H, then in auction two they will completely bid away any surplus they might hope to obtain ex ante.

This section illustrates how bidding behavior changes when the auction framework changes by revealing bids. Most importantly, it shows how a single auctioneer restricted to short term contracts is better off revealing all bids rather than only the winner's bid or revealing no bid information. This helps counter arguments that current policies in public sector procurement stifle competitive behavior.

4 Vertical Mergers

Understanding vertical integration requires examination of several difficult and often conflicting concepts. Beyond the typical efficiency enhancing and competition dampening claims about vertical mergers, there are issues concerning the flow of information between vertically related entities. If upstream horizontally related firms compete repeatedly for the services of downstream firms, and if the resulting vertical relationships entail the transfer of proprietary information, then upstream firms outside a particular merger may fear that information they transmit will flow directly to their horizontal competitor via the competitor's link to the downstream firm.[18] Not only might this transfer of information be anticompetitive by making it easier for the upstream firms to monitor its rivals and hence coordinate their actions, these mergers may induce upstream outsiders not to submit bids to integrated downstream firms, a consequence which could adversely affect social welfare.

One proposed remedy in these situations is the imposition of a firewall between the merging parties, a device which prohibits information gained by one party from being shared with the other. Such a device, if it can be successfully implemented, should alleviate concerns about allegedly harmful information flows. However, a larger and unanswered question is whether these information flows actually harm competition.

The literature on information sharing in oligopoly examines this question in a related context by modeling the incentive to share private cost and demand information by both price setting and quantity setting firms. Vives [1984] shows that quantity setting firms with private information about a common linear demand curve do not wish to share their private information, though price setting firms do. Shapiro [1986] shows that quantity setting firms with privately known linear marginal costs do want to share their private information. Most similar to this repeated auction context is Gal-Or [1986], which shows that price setting firms do not wish to share private cost information. The main difference is that the Gal-Or model is static while my formulation is

[18]Such concerns have arisen in the satellite (Martin Marietta/General Dynamics merger), pharmaceutical (Lilly/PCS merger), and data processing (First Data/Western Union merger) industries, among others.

dynamic. Perhaps surprisingly, however, the results are similar. The bidders know the information revelation is harmful but cannot avoid it.

To put more structure on this discussion, suppose that, following the analysis in Section 2, firm A merges with seller 1. The structural change induced by the merger clearly affects the bidding behavior of both A and B. Assuming that the merged firm wishes to maximize its joint profits, firm A wants firm B to win the object whenever B's bid is greater than v_A. As a result, A simply bids v_A in auction one, which has two effects. First, it causes B to change its behavior in auction one. Second, it causes both firms to change their bidding behavior in auction two.

Vertical information flows may cause additional effects, and the behavioral changes described above may be more pronounced if firm A (either secretly or publicly) sees B's bid from the first auction. The next two sections examine behavior in the presence or absence of an information firewall within the vertically integrated firm.[19]

Integration with a Firewall

Suppose A can credibly commit not to receive B's bid. It could do this by having the bids in the first auction evaluated by a third party or through fear of the government's reaction if the violation is detected. I continue to assume that type v_L players submit no bid in either auction. Furthermore, a type v_H player B will not submit a positive bid in auction one, because B cannot get the item if player A is type v_H. This framework and its consequent behavior generate a lot of information about the participants. If B wins auction one, then A knows that B is type v_H and B knows that A is type v_L, and this is common knowledge. B then bids zero in auction two.[20] If A wins auction one, then B knows that A is type v_H, and this is also common knowledge. However, in this case A does not know B's type, and in fact has learned nothing about B's type. B's payoff in this situation is therefore

$$\pi^B(0 \mid v_H) = \underbrace{\frac{1}{2}\left[v_H + \pi^H(v_H \mid 0, 1)\right]}_{\text{Against } v_L} + \underbrace{\frac{1}{2}\left[0 + \pi^L\left(v_H \mid \frac{1}{2}, 1\right)\right]}_{\text{Against } v_H}$$

[19]I am of course begging the question of which types of buyers merge. Clearly the type v_L bidders will not merge if there is a cost to doing so, and if the information gain is the only reason for the merger. However, if this information acquisition aspect is only a byproduct of the merger, then for now I can ignore the reasons for merging. Of course, the real reason for the merger, be it efficiency gains or increasing market power, must be considered when evaluating the results of this model.

[20]If a type v_L player A should happen to deviate and bid zero in auction one, the only effect is that A may win and cause B to believe A is type v_H. This will cause B to bid higher in auction two, and will not affect A's payoff in any way. Thus, assuming a type v_L player A submits no bid is still a Nash equilibrium.

$$= \frac{1}{2}[v_H + v_H] + \frac{1}{2}\left(\frac{v_H}{2}\right) = \frac{5v_H}{4}.$$

B may choose to submit no bid in auction one, but then auction two is exactly the same as the single item symmetric auction. B's payoff over the two auctions is

$$\begin{aligned}\pi^B(\text{“no bid”} \mid v_H) &= \underbrace{\frac{1}{2}\left[0 + \pi^H\left(v_H \mid \frac{1}{2}, \frac{1}{2}\right)\right]}_{\text{Against } v_L} + \underbrace{\frac{1}{2}\left[0 + \pi^L\left(v_H \mid \frac{1}{2}, \frac{1}{2}\right)\right]}_{\text{Against } v_H} \\ &= \frac{v_H}{2}.\end{aligned}$$

B does better submitting a bid in auction one than not, because of the chance that player A is type v_L, which would allow B the opportunity of both acquiring the first object and getting the second for a low price. Therefore, B participates in auction one.

The payoff for a type v_H player A in auction one is $\frac{v_H}{2}$.[21] Because A learns nothing about B's type yet is known to be type v_H, A's payoff in auction two is

$$\pi_2^A(\cdot|v_H) = \pi^H\left(v_H \mid \frac{1}{2}, 1\right) = \frac{v_H}{2}.$$

The following proposition summarizes the sellers' payoffs.

Proposition 9 *Ex ante expected profits of seller 1 and seller 2 when there is no bid transmission between the upstream and downstream players are*

$$R_{S1}\left(\frac{1}{2}, \frac{1}{2}\right) = \frac{v_H}{4}.$$

and

$$R_{S2}\left(\frac{1}{2}, \frac{1}{2}\right) = \frac{v_H}{8}.$$

Proof: See the Appendix.

The bidding strategies of A and B imply that seller 1 gets $\frac{v_H}{2}$ if A is type v_H and zero otherwise. If player A is type v_L (which occurs with probability $\frac{1}{2}$), then seller 2 gets nothing; either B is also type v_L or B is type v_H yet knows that A is type v_L. If player A is type v_H, then seller 2 gets either the payoff from two type v_H players (if B is also type v_H) or only one type v_H player.

[21]For comparison purposes, I assume that the two merged firms split the rents.

Integration without a Firewall

Suppose A cannot credibly commit not to see B's bid, even seeing a "no bid" as such. As is the case when there is an internal wall, B will not bid a positive amount in auction one. Again, if B wins auction one, then A knows B is type v_H and B knows A is type v_L, and this is common knowledge. Also, if B loses auction one, then B knows A is type v_H. The difference that arises when A sees B's bid is that if B loses and has submitted a bid, then A knows that B is type v_H. B's payoff from bidding zero in auction one is

$$\begin{aligned}\pi^B(0 \mid v_H) &= \underbrace{\frac{1}{2}[v_H + \pi^H(v_H \mid 0,1)]}_{\text{Against } v_L} + \underbrace{\frac{1}{2}[0 + \pi^L(v_H \mid 1,1)]}_{\text{Against } v_H} \\ &= \frac{1}{2}[v_H + v_H] = v_H.\end{aligned}$$

If B submits no bid in auction one, the payoff is

$$\begin{aligned}\pi^B(\text{"no bid"} \mid v_H) &= \underbrace{\frac{1}{2}\left[0 + \pi^L\left(v_H \mid \frac{1}{2}, \frac{1}{2}\right)\right]}_{\text{Against } v_L} + \underbrace{\frac{1}{2}\left[0 + \pi^L\left(v_H \mid \frac{1}{2}, \frac{1}{2}\right)\right]}_{\text{Against } v_H} \\ &= \frac{v_H}{2}.\end{aligned}$$

Again, B participates in auction one. This is because there is the chance that player A is type v_L, and B does not wish to lose the opportunity of both acquiring the first object and getting the second for a low price. Regardless of the firewall, B does equally well in auction one. Examining the payoffs in the second auction, it is clear that B is harmed in auction two by the revelation of bid information. Thus, B definitely prefers there be a wall within the merged firm; B gets $\frac{5v_H}{4}$ versus v_H. This result corresponds to the potential concerns of outside firms.

A's payoff is not changed by the transmission of bid information. The payoff for a type v_H player A in auction one is $\frac{v_H}{2}$. Because A learns B's type following auction one, A receives a positive payoff in auction two if and only if B's type is v_L. Thus, A's payoff in auction two is

$$\pi_2^A(\cdot \mid v_H) = \frac{1}{2}\pi^H(0,1) = \frac{v_H}{2}.$$

Surprisingly, A is indifferent about seeing B's bid; A's total payoff over the two auctions is v_H in either case. This corresponds to the acquiescence of firms to having a firewall imposed; it either costs them nothing or is at least cheaper than the litigation costs. Moreover, because B bids the same way with or without the internal wall, A has no incentive to secretly learn B's bid. That is, the firewall is trivially enforced.

Proposition 10 *Ex ante expected profits of seller 1 and seller 2 when there is bid transmission between the upstream and downstream players are*

$$R_{S1}\left(\frac{1}{2},\frac{1}{2}\right) = \frac{v_H}{4}.$$

and

$$R_{S2}\left(\frac{1}{2},\frac{1}{2}\right) = \frac{v_H}{4}.$$

Proof: See the Appendix.

Seller 1's payoff is computed exactly as it was when there was a firewall. Seller 2's payoff changes, however. Seller 2 still gets zero if A is type v_L. However, seller 2 now receives zero if A is type v_H and B is type v_L, and receives v_H if both A and B are type v_H. Thus, the knowledge that information will be transmitted leads seller 2 to conclude ex ante that competition in auction two will be more vigorous than if no information were transmitted.

These analyses of expected seller revenues can be combined for a striking conclusion.

Theorem 4 *Seller 1 prefers merging with bidder A to not merging, regardless of the existence of a firewall. Seller 2 prefers having seller 1 and bidder A merge and transmit information both to having no merger take place and to having a firewall imposed as a condition for merger.*

Proof: If seller 1 and bidder A merge, seller 1 gets $\frac{v_H}{4}$. In the pre-merger situation in Section 2, seller 1 gets strictly less than $\frac{v_H}{4}$. A similar argument holds for seller 2. □

Both sellers do better with the merger and information transmission than in the pre-merger situation. The driving force behind this is B's knowledge that A knows B's type if there is no firewall. This leads to more aggressive bidding in auction two when both players are type v_H. In the context of a manufacturer/distributor relationship, this result implies that distributors (the receivers of bids) expect to pay less. Lower bids to distributors translate into lower prices for final consumers, suggesting that some of the concern about vertical integration involving proprietary information issues is misplaced.

The above analysis assumes that firm A merges with seller 1. If the welfare predictions change if A merges with seller 2, for example, then care must be taken when using this work to evaluate real world mergers. If A merges with seller 2 before the repeated auction process begin, then in auction two behavior is completely determined: a type v_H player A bids v_H, while a type v_H player B bids zero. Because first period behavior does not affect second period behavior, bidding in auction one follows the predictions in Proposition 1. Thus, the sellers still prefer the vertical merger take place if A merges with seller 2 rather than seller 1, though now the firewall has no effect on the sellers' payoffs.[22]

There are three realistic extensions of this merger question which are beyond the scope of this paper. First, there might be $T > 2$ auctioneers. If A merges with an auctioneer besides the first or the last (the two cases just analyzed), then there may be consequences for bidder behavior in early auctions. Second, the vertical merger might take place after some auctions have taken place. In this case, potential downstream merger partners may use the outcomes from early rounds to decide with which firm to merge. This follows because the sellers in this model prefer merging with type v_H players rather than type v_L. Third, there is the distinct possibility that the bidders' characteristics are not fixed from period to period but simply have some systematic relationship. This paper presents a first step in considering that question.

6 Conclusion

This paper models a small population of bidders whose members face each other repeatedly over time. By assuming bidders desire multiple objects, and that the values of those items to a bidder are correlated, I address strategic problems that do not arise either in a sequential auction in which bidders desire only a single item, or in a series of independent single item auctions. With the discrete model I use, I discover the following about three structural information policies and the resulting strategic information transmission.

Without price announcements, bidding competition is softer in both the first and second auctions than in a series of two independent auctions. Competition is softer in auction one as firms try to acquire information about their rivals. It is softer in auction two because the winner of auction one now more firmly believes he faces an inefficient opponent. The equilibrium outcome implies that a single seller of the two items prefers a simultaneous sale. If the auctioneer is a firm acquiring inputs, then this suggests that long term contracts are preferred to short term contracts to eliminate information acquisition and soft bidding by input suppliers. Also, if the sellers are different, then they prefer the auctions to be independent. In a sense, the sellers prefer to differentiate their

[22]More precisely, both sellers have expected revenues of $\frac{v_H}{4}$, which is better than their payoffs premerger and is exactly the same as if player A and seller 1 merge with no firewall.

products.

With price announcements, the sellers do better having all bids announced rather than having only the winner's bid announced. This follows because revealing all bids generates a great deal of public information which leads to vigorous competition. For similar reasons, the sellers do better having all bids announced rather than revealing no bid information. These two results may be used to counter arguments that current bid revelation policies stifle competition in public sector procurement. A single seller, such as the government, prefers simultaneous sales rather than sequential sales, for the same reasons as in the repeated auction without price announcements. In Froeb and McAfee [1988], simultaneous sales are preferred because they reduce the possibility of collusion by making orders more lumpy. Here the preference arises because simultaneous sales prevent firms from gaining information about their rivals and eliminate the resulting soft competition.

With mergers, I find that bidders outside the merger would like to know that there is no transmission of bid data between the upstream and downstream members of the merged entity. Surprisingly, I find that the inside bidder is indifferent between knowing and not knowing the bids of rival bidders. Also, because the outsider's action in the first auction is unchanged by the presence of an internal wall, the insider has no incentive to secretly see the bid of the outsider. Finally, the sellers do better with the merger and no firewall than in the pre-merger equilibrium. In a manufacturer/distributor relationship, this result implies that distributors pay less on average, which leads to lower prices for final consumers and to the conclusion that the vertical merger is procompetitive. Therefore, some of the concern about informationally related anticompetitive effects of vertical mergers may be misplaced.

Though these results are from a two period, two bidder model, I believe the character of the results remains as the number of auctions or bidders increases. For example, in the benchmark with no bid revelation, bids should start out lower than usual, then go up over time, albeit more slowly than in the two period case. The reason they will rise more slowly is that, say in the second of three auctions, while the environment has grown more competitive following the first auction, there is still reason to try to elicit information for use in the third auction. Of perhaps greater importance is the robustness of these results to a change in the distribution of types. In particular, using a continuum of types may generate, following bid revelation or vertical integration, the complex pooling that arises in models exhibiting the ratchet effect.[23]

Appendix

[23] See Laffont and Tirole [1993].

Proof of Proposition 1: 1) Suppose player A bids a when his type is v_H, while Player B bids b. If $b > a$, then B prefers bidding $\frac{a+b}{2}$, which still wins with probability one but involves a lower payment. The reverse holds when $a > b$. If $a = b$, then B's payoff is

$$\pi^B(b \mid v_H) = \alpha(v_H - b) + (1-\alpha)\left(\frac{1}{2}\right)(v_H - b) = (v_H - b)\left(\frac{1+\alpha}{2}\right).$$

B's payoff from bidding $b + \epsilon$ is

$$\pi^B(b+\epsilon \mid v_H) = v_H - b - \epsilon.$$

For small ϵ, $\pi^B(b+\epsilon \mid v_H) > \pi^B(b \mid v_H)$, so B cannot bid b in equilibrium. Thus, there is no pure strategy equilibrium.

If both players do not mix over the same set of bids, then one player has an incentive not to use the higher bids. Suppose the range of bids is not a connected interval. For example, suppose bids are over $(\underline{b}_1, \bar{b}_1)$ and $(\underline{b}_2, \bar{b}_2)$ with $\bar{b}_1 < \underline{b}_2$. Then a player bidding very near $\underline{b}_2$ could lower his bid to $\frac{\bar{b}_1+\underline{b}_2}{2}$, decreasing his probability of winning by an arbitrarily small amount yet lowering his expected payment when he wins. Thus, both players bid over an interval $[\underline{b}, \bar{b}]$.

Suppose player A bids bid $b \in (\underline{b}, \bar{b}]$ with positive probability. B must bid $b^* \in (b-\epsilon, b]$ by the previous argument. If B instead bids $b^* + \epsilon$, his probability of winning increases discontinuously, while his expected payment when he wins goes up by an arbitrarily small amount. This is a profitable deviation for B, who will therefore not bid $b^* \in (b-\epsilon, b]$, a contradiction. Thus, there are no mass points in $(b-\epsilon, b]$.

Suppose both A and B bid $\underline{b}$ with positive probability. If B deviates to bidding $\underline{b} + \epsilon$, his probability of winning increases discontinuously, while his expected payment when he wins goes up by an arbitrarily small amount. This is a profitable deviation for B, who will therefore not bid $\underline{b}$. Thus, at most one player bids $\underline{b}$ with positive probability

Suppose $\underline{b} > 0$ and that A bids according to $F_A(b)$ .with $F_A(\underline{b}) > 0$. The payoff to B from bidding $\underline{b} + \epsilon$ is

$$\frac{1}{2}[1 + F_A(\underline{b}+\epsilon)](v_H - \underline{b} - \epsilon).$$

The payoff from bidding ϵ is

$$\frac{1}{2}(v_H - \epsilon).$$

Taking ϵ arbitrarily small ensures that bidding ϵ yields a higher expected payoff than bidding $\underline{b} + \epsilon$. Thus, it must be that $\underline{b} = 0$.

Using B's indifference relation, (2),

$$v_H - \bar{b} = (1-\alpha)v_H.$$

This implies $\bar{b} = \alpha v_H$.

2,3) Let $F_A(b)$ and $F_B(b)$ be the equilibrium bid distributions used by type v_H players A and B, respectively. To determine $F_A(b)$ and $F_B(b)$ I employ the typical mixed strategy approach: Use an indifference condition on player i to solve for player j's distribution. A type v_H player A must be indifferent over all bids in $[0, \bar{b}]$, so

$$\underbrace{(1-\beta)(v_H - b)}_{\text{Against } v_L} + \underbrace{\beta F_B(b)(v_H - b)}_{\text{Against } v_H} = (1-\beta)v_H + \beta F_B(0)v_H. \tag{1}$$

That is, A's expected payoff from bidding b must be identical to the expected payoff from bidding zero. Similarly, for a type v_H player B

$$\underbrace{(1-\alpha)(v_H - b)}_{\text{Against } v_L} + \underbrace{\alpha F_A(b)(v_H - b)}_{\text{Against } v_H} = (1-\alpha)v_H + \alpha F_A(0)v_H. \tag{2}$$

Also note that this indifference relation must hold at $\bar{b}$, so for A

$$v_H - \bar{b} = [(1-\beta) + \beta F_B(0)]v_H \tag{3}$$

while for B

$$v_H - \bar{b} = [(1-\alpha) + \alpha F_A(0)]v_H. \tag{4}$$

Equating the two right hand sides of (1) and (2) gives

$$1 - \beta + \beta F_B(0) = 1 - \alpha + \alpha F_A(0),$$

or

$$\beta F_B(0) - \alpha F_A(0) = \beta - \alpha. \tag{5}$$

If $\alpha \leq \beta$, then $F_A(0) = 0$ and $F_B(0) = 1 - \frac{\alpha}{\beta}$. The information about $F_A(0)$ and $F_B(0)$ can be used in conjunction with the indifference relations to solve for the equilibrium bid distributions.

4) Use the indifference relations (1) and (2) and the definition of $\bar{b}$.

5,6) The expected payment of a type v_H player A, when beliefs that A and B are type v_H are α and β, respectively, is

$$P^L(v_H \mid \alpha,\beta) = \int_0^{\bar{b}} b[(1-\beta)+\beta F_B(b)]F_A'(b)db.$$

Using the indifference relation (1), the integrand can be rewritten to yield

$$\begin{aligned} P^L(v_H \mid \alpha,\beta) &= \int_0^{\bar{b}} [(\alpha-\beta)v_H + \beta F_B(b)v_H]F_A'(b)db \\ &= (\alpha-\beta)v_H + \beta v_H \int_0^{\bar{b}} F_B(b)F_A'(b)db. \end{aligned}$$

Some straightforward integration shows

$$\int_0^{\bar{b}} F_B(b)F_A'(b)db = 1 - \frac{\alpha}{2\beta},$$

so

$$P^L(v_H \mid \alpha,\beta) = \frac{\alpha v_H}{2}.$$

A similar argument establishes $P^H(v_H \mid \alpha,\beta) = \frac{\alpha^2 v_H}{2\beta}$.

7) Expected revenues for the seller are

$$\begin{aligned} R(\alpha,\beta) &= (1-\alpha)(1-\beta)[0] + (1-\alpha)\beta[P^H(v_H \mid \alpha,\beta)] + \\ &\quad \alpha(1-\beta)[P^L(v_H \mid \alpha,\beta)] + \alpha\beta[P^H(v_H \mid \alpha,\beta) + P^L(v_H \mid \alpha,\beta)] \\ &= \alpha^2 v_H. \end{aligned}$$

Each term in the expression for $R(\alpha,\beta)$ is the probability of a certain state of the world (combination of player types) times the expected payment of players in that state of the world.

Proof of Proposition 4: Suppose player A of type v_H submits a zero bid with probability one. The payoff to a type v_H player B from bidding zero is

$$\pi^B(0 \mid v_H) = \underbrace{\frac{1}{2}\left[v_H + \pi^H\left(v_H \mid \frac{1}{3},1\right)\right]}_{\text{Against } v_L} + \frac{1}{2}\left[\underbrace{\frac{1}{2}\left\{v_H + \pi^H\left(v_H \mid \frac{1}{3},1\right)\right\}}_{\text{Beating } v_H} + \underbrace{\frac{1}{2}\left\{0 + \pi^L\left(v_H \mid \frac{1}{3},1\right)\right\}}_{\text{Losing to } v_H}\right].$$

The beliefs in the continuation game are computed as follows. If B wins with a zero bid, this is either because he beat a type v_L player who submitted no bid, or because he won the tie break against a type v_H player A bidding zero. The probability that A is type v_H given that B beat A with a zero bid is $\frac{1}{3}$. Similarly, if B lost to A, then B knows A will use the same thought process to determine new beliefs that B is type v_H. Simplification of $\pi^B(\cdot)$ by using part 4 of Proposition 1 yields

$$\pi^B(0 \mid v_H) = \frac{17 v_H}{12}.$$

If B deviates and bids $\epsilon > 0$, then he will win for sure while revealing himself to be type v_H. However, he would also reveal himself to be type v_H if he won auction one with a bid of zero. More importantly, B learns nothing about A's type. B's payoff from deviating is

$$\pi^B(\epsilon \mid v_H) = \underbrace{\frac{1}{2}\left[v_H - \epsilon + \pi^H\left(v_H \mid \frac{1}{2}, 1\right)\right]}_{\text{Against } v_L} + \underbrace{\frac{1}{2}\left[v_H - \epsilon + \pi^H\left(v_H \mid \frac{1}{2}, 1\right)\right]}_{\text{Against } v_H},$$

which simplifies to

$$\pi^B(\epsilon \mid v_H) = \frac{3v_H}{2} - \epsilon.$$

This is a profitable deviation for B, for suitable ϵ. Thus, there cannot be a pooling equilibrium.

Proof of Proposition 5: Suppose there is a pure separating equilibrium with first period bid distribution $F_1(b)$. By definition, $F_1(0) = 0$. If A follows $F_1(b)$, then B's expected payoff from bidding $b > 0$ when his type is v_H is

$$\begin{aligned} \pi^B(b \mid v_H) \quad = \quad & \underbrace{\frac{1}{2}\left[(v_H - b) + \pi^H(v_H \mid 0, 1)\right]}_{\text{Against } v_L} + \\ & \underbrace{\frac{1}{2}F_1(b)\left[(v_H - b) + \pi^H(v_H \mid 1, 1)\right]}_{\text{Beating } v_H} + \underbrace{\frac{1}{2}[1 - F_1(b)]\left[0 + \pi^L(v_H \mid 1, 1)\right]}_{\text{Losing to } v_H}. \end{aligned}$$

After beating a type v_L player A, B knows it. Similarly, when both bid positive amounts each knows the other is type v_H. Now $\pi^H(v_H \mid 1, 1) = \pi^L(v_H \mid 1, 1) = 0$ and $\pi^H(v_H \mid 0, 1) = v_H$, so

$$\pi^B(b \mid v_H) = v_H - \frac{b}{2} + \frac{1}{2}F_1(b)(v_H - b).$$

Suppose B deviates by bidding zero. B will still beat a type v_L player A, and will fool a type v_H player A into thinking that B is type v_L. Thus, in auction two a type v_H player A will bid zero and lose to player B, who will bid $\epsilon > 0$. The expected payoff to B from this strategy is

$$\pi^B(0 \mid v_H) = \underbrace{\frac{1}{2}[v_H + \pi^H(v_H \mid 0, 1)]}_{\text{Against } v_L} + \underbrace{\frac{1}{2}[0 + (v_H - \epsilon)]}_{\text{Against } v_H} = \frac{3v_H}{2} - \frac{\epsilon}{2}.$$

If $\pi^B(b \mid v_H) = \pi^B(0 \mid v_H)$, then

$$v_H - \frac{b}{2} + \frac{1}{2}F_1(b)(v_H - b) = \frac{3v_H}{2} - \frac{\epsilon}{2}.$$

This inequality must hold for any b in the support of $F_1(b)$, $[0, \bar{b}]$, for some $\bar{b} > 0$. In fact, it must hold for

$b = \epsilon$. Substituting ϵ for b in the above inequality and rearranging implies

$$0 = \frac{v_H}{2} - \frac{1}{2}F_1(\epsilon)(v_H - \epsilon).$$

Taking ϵ arbitrarily small ensures that the right hand side of this inequality will be strictly positive, because $F_1(b)$ goes to zero as b goes to zero. This leads to the contradiction $0 > 0$, so it was incorrect to assume $\pi^B(b \mid v_H) = \pi^B(0 \mid v_H)$. Thus, B will deviate by bidding zero and there cannot exist a pure separating equilibrium.

Proof of Proposition 6: Suppose $F_1(b)$ is the first period bid distribution for the semi-pooling equilibrium. By definition, $F_1(0) > 0$. If A follows $F_1(b)$, then B's expected payoff from bidding $b > 0$ is

$$\begin{aligned}
\pi^B(b \mid v_H) \quad = \quad & \underbrace{\frac{1}{2}\left[(v_H - b) + \pi^H\left(v_H \mid \frac{F_1(0)}{1+F_1(0)}, 1\right)\right]}_{\text{Against } v_L} + \\
& \underbrace{\frac{1}{2}F_1(0)\left[(v_H - b) + \pi^H\left(v_H \mid \frac{F_1(0)}{1+F_1(0)}, 1\right)\right]}_{\text{Beating } v_H \text{ bidding zero}} + \\
& \underbrace{\frac{1}{2}[F_1(b) - F_1(0)][(v_H - b) + \pi^H(v_H \mid 1, 1)]}_{\text{Beating } v_H \text{ bidding positive}} + \\
& \underbrace{\frac{1}{2}[1 - F_1(b)][0 + \pi^L(v_H \mid 1, 1)]}_{\text{Losing to } v_H},
\end{aligned}$$

which when simplified yields

$$\begin{aligned}
\pi^B(b \mid v_H) \quad = \quad & \frac{1}{2}\left[(v_H - b) + \pi^H\left(v_H \mid \frac{F_1(0)}{1+F_1(0)}, 1\right)\right] + \\
& \frac{1}{2}F_1(0)\pi^H\left(v_H \mid \frac{F_1(0)}{1+F_1(0)}, 1\right) + \frac{1}{2}F_1(b)(v_H - b).
\end{aligned}$$

The payoff to a type v_H player B from bidding zero is

$$\begin{aligned}
\pi^B(0 \mid v_H) \quad = \quad & \frac{1}{2}\left[v_H + \pi^H\left(v_H \mid \frac{F_1(0)}{1+F_1(0)}, 1\right)\right] + \\
& \frac{1}{2}F_1(0)\left[\frac{1}{2}\left\{v_H + \pi^H\left(v_H \mid \frac{F_1(0)}{1+F_1(0)}, 1\right)\right\} + \frac{1}{2}\left\{0 + \pi^L\left(v_H \mid \frac{F_1(0)}{1+F_1(0)}, 1\right)\right\}\right] + \\
& \frac{1}{2}[1 - F_1(0)]\left[0 + \pi^L\left(v_H \mid \frac{F_1(0)}{1+F_1(0)}, 1\right)\right].
\end{aligned}$$

In equilibrium, $\pi^B(b \mid v_H) = \pi^B(0 \mid v_H)$, which permits computation of $F_1(b)$. To conserve space, let

$$\Omega = \pi^L\left(v_H \mid \frac{F_1(0)}{1+F_1(0)}, 1\right) = \pi^H\left(v_H \mid \frac{F_1(0)}{1+F_1(0)}, 1\right).$$

Simplifying $\pi^B(0 \mid v_H)$ gives

$$\pi^B(0 \mid v_H) = \frac{v_H}{2} + \frac{F_1(0)v_H}{4} + \Omega.$$

Equating $\pi^B(0 \mid v_H)$ and $\pi^B(b \mid v_H)$ yields

$$F_1(b)(v_H - b) = \Omega + b + \frac{F_1(0)v_H}{2} - F_1(0)\Omega.$$

Substituting zero for b and simplifying gives

$$F_1(0) = \frac{\sqrt{17}-3}{2},$$

which can then be used to show

$$F_1^{AA}(b) = \left(\frac{b}{v_H - b}\right) + \left(\frac{v_H}{v_H - b}\right)\left(\frac{\sqrt{17}-3}{2}\right).$$

Proof of Proposition 7: The expected payment of a type v_H player, when beliefs that A and B are type v_H are $\frac{1}{2}$ and $\frac{1}{2}$, respectively, is

$$P\left(v_H \mid \frac{1}{2}, \frac{1}{2}\right) = \int_0^{\bar{b}} b\left[\frac{1}{2} + \frac{1}{2}F_1(b)\right] F_1'(b)db.$$

I solve this integral directly, not using the indifference relation on the integrand as in the proof of Proposition 1. The result of this tedious integration[24] is

$$P(v_H \mid \alpha, \beta) = \left(\frac{5-\sqrt{17}}{4}\right)^2 v_H.$$

Expected revenues for the seller in auction one are

$$\begin{aligned} R_{S1}\left(\frac{1}{2}, \frac{1}{2}\right) &= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)[0] + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left[P\left(v_H \mid \frac{1}{2}, \frac{1}{2}\right)\right] + \\ &\quad \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left[P\left(v_H \mid \frac{1}{2}, \frac{1}{2}\right)\right] + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left[P\left(v_H \mid \frac{1}{2}, \frac{1}{2}\right) + P\left(v_H \mid \frac{1}{2}, \frac{1}{2}\right)\right] \\ &= \left(\frac{5-\sqrt{17}}{4}\right)^2 v_H. \end{aligned}$$

Let

$$\Omega = \frac{F_1(0)}{1+F_1(0)} = \frac{\sqrt{17}-3}{\sqrt{17}-1},$$

the probability that a player is type v_H given he bid zero in auction one. Ex ante expected revenues for the

[24]Which is available upon request.

seller in auction two are

$$
\begin{aligned}
R_{S2}\left(\frac{1}{2},\frac{1}{2}\right) &= \left(\frac{1}{4}\right)[0] + \left(\frac{1}{4}\right)[R(\Omega,1)] + \\
&\left(\frac{1}{4}\right)[R(\Omega,1)] + \left(\frac{1}{4}\right)[1-F_1(0)]^2 v_H + \left(\frac{1}{4}\right)\left(1-[1-F_1(0)]^2\right)R(\Omega,1).
\end{aligned}
$$

Straightforward calculation yields

$$
R_{S2}\left(\frac{1}{2},\frac{1}{2}\right) = \left(\frac{49-11\sqrt{17}}{4(\sqrt{17}-1)^2}\right) v_H.
$$

Proof of Proposition 8: Suppose a type v_H player A is following $F_1(b)$, the first period symmetric Nash equilibrium bid distribution. A type v_H player B's payoff from bidding $b > 0$ is

$$
\begin{aligned}
\pi^B(b \mid v_H) &= \underbrace{\frac{1}{2}\left[(v_H - b) + \pi^H\left(v_H \mid \frac{F_1(b)}{1+F_1(b)}, 1\right)\right]}_{\text{Against } v_L} + \\
&\underbrace{\frac{1}{2}F_1(b)\left[(v_H - b) + \pi^H\left(v_H \mid \frac{F_1(b)}{1+F_1(b)}, 1\right)\right]}_{\text{Beating } v_H} + \\
&\underbrace{\frac{1}{2}[1-F_1(b)]\left[\frac{\int_b^{\bar{b}} \pi^L\left(v_H \mid \frac{F_1(x)}{1+F_1(x)}, 1\right) F_1'(x)dx}{1-F_1(b)}\right]}_{\text{Losing to } v_H}.
\end{aligned}
$$

This may be simplified as

$$
\pi^B(b \mid v_H) = \frac{1}{2}[1+F_1(b)](v_H - b) + \frac{v_H}{2} + \frac{1}{2}\int_b^{\bar{b}} \frac{F_1'(x)v_H}{1+F_1(x)}dx.
$$

In equilibrium, $\pi^B(b \mid v_H)$ and $\pi^B(0 \mid v_H)$ must be equal. Equating the two implies

$$
\frac{1}{2}[1+F_1(b)](v_H - b) + \frac{v_H}{2} + \frac{1}{2}[\ln(2) - \ln(1+F_1(b))] = \frac{v_H}{2} + \frac{v_H}{2} + \frac{1}{2}\ln(2).
$$

Minor algebra yields

$$
F_1(b) = \frac{b}{v_H - b} + \left(\frac{b}{v_H - b}\right)\ln(1+F_1(b)),
$$

which is the desired result. □

Proof of Theorem 3: When all bids are announced, a bidder's expected payoff is the expected payoff from bidding zero in auction one. Straightforward calculation using the profit function shows that a type v_H bidder expects to earn $1.280776406 v_H$ when all bids are announced. A type v_H bidder's expected payoff when only the winner's bid is announced or when no bids are announced is $1.34657359v_H$. Therefore, the auctioneers do better when all bids are announced.

Proof of Proposition 9: Seller 1 earns $\frac{v_H}{2}$ half the time (when merged with a type v_H player) and zero half the time. So ex ante expected profits for seller 1 are $\frac{v_H}{4}$. Ex ante expected profits of seller 2 when there is no bid transmission between the upstream and downstream players are

$$\begin{aligned} R_{S2}(\frac{1}{2},\frac{1}{2}) &= \left(\frac{1}{4}\right)[0]+\left(\frac{1}{4}\right)[0]+\left(\frac{1}{4}\right)\left[P^H\left(\frac{1}{2},1\right)\right]+\left(\frac{1}{4}\right)\left[P^H\left(\frac{1}{2},1\right)+P^L\left(\frac{1}{2},1\right)\right] \\ &= \frac{v_H}{8}. \end{aligned}$$

Proof of Proposition 10: As in Proposition 9, seller 1 earns $\frac{v_H}{4}$. Ex ante expected profits of seller two when there is bid transmission between the upstream and downstream players are

$$\begin{aligned} R_{S2}\left(\frac{1}{2},\frac{1}{2}\right) &= \left(\frac{1}{4}\right)[0]+\left(\frac{1}{4}\right)[0]+\left(\frac{1}{4}\right)[0]+\left(\frac{1}{4}\right)[v_H] \\ &= \frac{v_H}{4}. \end{aligned}$$

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Figure 1