# Inflation, Money Demand and Credit* 

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#### Abstract

Past empirical studies find that the widerspread use of credit substitutes money as a means of payment. Since inflation is a tax on money, one would expect that people switch from money to credit more often as the inflation rate increases, which however is not supported by empirical evidence. This paper builds a model where money and credit coexist as means of payment. The model predicts that credit substitutes money at low rates of inflation, but complements money at high rates of inflation. Moreover, the introduction of credit improves the society's welfare only when inflation exceeds a certain level.


Key Words: Money, Credit, Money Demand, Inflation
JEL Classification Number: E41, E50

## 1 Introduction

Both money and credit are widely used in transactions. From BIS's report, the percentages of total number of transactions using cards with a credit function in 2005 are $23.4 \%$ in the U.S., $13 \%$ in UK and $24.8 \%$ in Canada. From Figure 1, one can see that the consumer revolving credit to GDP ratio has been increasing in recent decades in the U.S. and the money to credit ratio has been decreasing. It appears obvious that credit becomes more and more important as a means of payment.

[^0]

Figure 1: Credit to GDP Ratio and Credit to Money Ratio

As money is also an important means of payment, one may wonder how the introduction of credit affects money. In particular, how does money and credit interact? Does credit decrease money demand? How does credit affect the transmission of monetary policy? What is the effect of monetary policy on credit? These questions are interesting and important to the conduction of monetary policy in an economy where credit is a popular means of payment.

Previous empirical work has revealed that credit is likely to reduce money demand. For example, Duca and Whitesell (1995) estimate that for every $10 \%$ increase in the probability of owning a credit card, checking balances are reduced by $9 \%$ using U.S. household-level data. ${ }^{1}$ Credit seems to serve as a substitute for money. Following this line, one may expect to observe that people switch from money to credit more as the inflation rate becomes higher. Indeed, this type of effect is predicted by many past theoretical models. (See Aiyagari et al. (1998) or English (1999) for examples.) However, empirical observations do not seem to agree with these theoretical predictions. Reducing the inflation rate is perceived to be beneficial for the use of credit. Credit cards gained widespread popularity in Brazil following the success of reducing inflation to sustainable levels. The number of cards in force has grown by $88 \%$ between 2000 and 2004. In Colombia, as a result of lower inflation

[^1]and lending rates, the proportion of household with formal access to credit is expected to increase by $25 \%$ from 2004 to 2008. It is also documented that high inflation episodes delayed the adoption and widespread usage of credit cards in Turkey. Even in Australia, due to the lower inflation rates and thus lower cost of borrowing in 1990s, households' debts increased dramatically. ${ }^{2}$

More empirical evidence on inflation and credit is based on a broader measure of credit - the total private credit to GDP ratio. ${ }^{3}$ Using a sample of 97 countries, Boyd et al. (2001) conclude that inflation has a negative impact on credit. Later, Khan et al. (2006) also use a large cross-country sample, but find that there is a threshold effect of inflation on credit. Only when inflation exceeds this threshold, it has a negative impact on credit. I estimate the long run response of credit to GDP ratio to permanent shocks to inflation following Bullard and Keating (1995). The result is that a higher inflation rate increases the credit to GDP ratio for low inflation countries, but not for high inflation countries. ${ }^{4}$ Putting all the evidence together, credit tends to substitute money at low inflation rates, but not at high inflation rates.

To rationalize the above observations, one needs a model where money and credit coexist and interact with inflation. I propose a model that is able to replicate the evidence on inflation, money demand and credit. To allow credit to exist, I assume that competitive financial intermediaries can identify agents and has access to a record-keeping technology. There are two frictions associated with credit arrangements. First, arranging credit is costly. In a bilateral trade, if a buyer wants to use credit or not, he has to incur a fixed utility cost to make the seller and himself identified to the financial intermediary. When agents have heterogeneous preferences, the fixed cost of credit will endogenously determine whether an agent wants to use credit in a transaction. Second, the settlement of credit is available only at a particular time in each period, during which the financial intermediaries accept repayment of credit and settles debts. Due to the timing structure of the model, the settlement is "delayed" and money becomes the only means of repayment. Therefore, credit transactions are subject to inflation distortions.

These two features of the model allow some interesting interactions between money and credit.

[^2]Inflation tends to increase the fraction of agents using credit at low inflation rates, but decrease the fraction of agents using credit at high inflation rates. From a calibrated example, the introduction of credit lowers demand for money at low to moderate inflation rates. Having credit may or may not improve social welfare depending on the inflation rate. At low inflation rates, costly credit does not improve social welfare. When the inflation rate exceeds a certain threshold, costly credit becomes beneficial to the society.

The model is built on Lagos and Wright (2005). In monetary theory, frictions that render money essential make credit arrangements impossible. Several recent papers have attempted to allow the coexistence of money and credit. ${ }^{5}$ In general, some imperfections associated with credit should be incorporated to sustain the essentiality of money and permit the existence of credit. Sanches and Williamson (2008) adopt the notion of limited participation in the sense that only an exogenous subset of agents can use credit. The banks in Berentsen et al. (2006) can record financial history, but not goods transaction history so that credit takes the form of bank loans. However, bank loans in their paper can only be taken in the form of fiat money. Telyukova and Wright (2008) build a model where agents can use money and credit to explain the credit card debt puzzle. Their market structure determines that agents cannot use money and credit simultaneously. A more related paper is Chiu and Meh (2008). They study how banks as in Berentsen et al. (2006) affect the allocations and welfare in an economy where ideas (or projects) are traded among investors and heterogeneous entrepreneurs. The role of banking is similar to credit in this paper, but money is the only means of payment in their paper.

In terms of the model's prediction on inflation and credit, the paper's result is similar to Azariadis and Smith (1996). The key friction for their results is asymmetric information with using credit. This paper instead considers different frictions associated with credit. Several papers have used the notion of costly credit in the Cash-in-Advance model or the OLG model. With the fixed cost, it is not surprising that inflation always decreases money demand and increases credit demand. I label the effect of inflation on credit through the fixed cost channel as the fixed cost effect. The "delayed" settlement has been used in Ferraris (2006), where money and credit are complements. The general idea can be traced back to Stockman (1981), where he shows that inflation reduces

[^3]the capital stock if money and capital are complements. The delayed settlement effect of inflation on credit is that inflation should reduce credit. As credit is subject to both frictions in this paper, it turns out that the fixed cost effect dominates at low inflation rates and the delayed settlement effect dominates at high inflation rates. This prediction is consistent with the empirical evidence cited earlier.

The rest of the paper is organized as follows. Section 2 lays out the physical environment. Section 3 solves for the equilibrium and analyzes the equilibrium when the repayment of credit can be enforced. I calibrate the model in section 4 . Section 5 concludes.

## 2 Environment

Time is discrete and runs forever. In each period, there are three subperiods. The first subperiod is characterized by decentralized trading and is labelled as decentralized market (hereafter DM). The second subperiod is characterized by centralized trading and is labelled as centralized market (hereafter CM). The third subperiod is called the overnight market. There is neither production nor consumption in the third subperiod. There are two permanent types of agents - buyers and sellers in the economy, each with measure 1. Buyers are those who want to consume in the DM and sellers are those who produce in the DM. For buyers, each of them receives a preference shock $\varepsilon$ at the beginning of each period, which determines the buyer's preference in the DM. The preference shock $\varepsilon$ is drawn from a c.d.f. $G(\varepsilon)$. I assume for simplicity that $\varepsilon \sim U(0,1]$. The preference shocks are iid across buyers and across time. The realization of the preference shocks is public information. There are two types of goods. Goods that are produced and consumed in the DM/CM are called DM/CM good. All goods are nonstorable.

What further distinguishes the DM and the CM is that trades in the DM are bilateral. Buyers and sellers are matched randomly according to a matching technology. All agents are anonymous and lack commitment. The probability for a buyer/seller to meet a seller/buyer is $\sigma$ with $0<\sigma \leq 1$. Given that a buyer and a seller meet, the terms of trade are determined by the buyer's take-it-or-leave-it offer. After exiting the DM, all agents enter into the CM. Buyers supply labor for production and consume the CM good. Sellers only consume the CM good. For simplicity, the production technology in the CM is assumed to be linear and 1 unit labor can be converted into 1
unit of the CM good.
The preference of a buyer with a preference shock $\varepsilon$ is

$$
\varepsilon u(q)+v(x)-h,
$$

where $\varepsilon u(q)$ is the buyer's utility from consuming $q$ units of the DM good. As usual, $u(0)=0$, $\lim _{q \rightarrow 0} u^{\prime}(q)=\infty$ and $u^{\prime \prime}(q)<0<u^{\prime}(q)$. In the CM, the buyer's utility from consuming $x$ units of the CM good is $v(x)$, where $\lim _{x \rightarrow 0} v^{\prime}(x)=\infty$ and $v^{\prime \prime}(x)<0<v^{\prime}(x)$. The buyer's labor supply is $h$. The preference of a seller is

$$
-c(q)+y,
$$

where $c(q)$ is the seller's disutility from producing $q$ units of the DM good with $c(0)=0, c^{\prime}(0)=0$, $c^{\prime}(q)>0$ and $c^{\prime \prime}(q) \geq 0$. The seller has a linear utility in the CM, where $y$ is the amount of consumption of the CM good. All agents discount between the overnight market and the next DM. The discount rate is $\beta$.

Now I consider a planner's problem as the benchmark allocation. Suppose that the planner weights all agents equally and is subject to the random matching technology. I restrict the attention to stationary allocations in what follows. In the DM of each period, given a buyer's preference shock $\varepsilon$ and the buyer meeting a seller, the planner instructs the seller to produce $q(\varepsilon)$ for the buyer. Those agents who do not find a match consume and produce nothing. In the CM of each period, the planner assigns the consumption of the CM good $x(\varepsilon), y$ and the labor supply $h(\varepsilon)$ subject to the resource constraint. Formally, the planner's problem is

$$
\begin{gather*}
\max _{q(\varepsilon), x(\varepsilon), y(\varepsilon), h(\varepsilon)} \sigma \int[\varepsilon u(q(\varepsilon))-c(q(\varepsilon))] d G(\varepsilon)+\int[v(x(\varepsilon))-h(\varepsilon)] d G(\varepsilon)+y  \tag{1}\\
\text { s.t. } \int x(\varepsilon) d G(\varepsilon)+y=\int h(\varepsilon) d G(\varepsilon) .
\end{gather*}
$$

The optimal allocation is characterized by:

$$
\begin{aligned}
v^{\prime}(x(\varepsilon)) & =1 \\
\varepsilon u^{\prime}(q(\varepsilon)) & =c^{\prime}(q(\varepsilon)) .
\end{aligned}
$$

Note that the buyer's optimal $x, x^{*}$ is given by $v^{\prime}\left(x^{*}\right)=1$ and is independent of $\varepsilon$. However, the optimal $q(\varepsilon)$ is increasing in $\varepsilon$. In fact, the optimal allocation features a slight indeterminacy. That is, given the quasi-linear preference structure, $h(\varepsilon)$ and $y$ are indeterminate as long as $\int h(\varepsilon) d G(\varepsilon)-$ $y=x^{*}$.

The planner's allocation cannot be implemented in the economy since agents are anonymous and lack commitment. As a result, money is essential. The aggregate money supply is controlled by the monetary authority. Let $M$ denote the aggregate money supply at any given date, the supply grows at a gross rate $\gamma>0 ; M_{+}=\gamma M$. Here the "plus" subscript denotes the "next period". I will consider $\gamma>\beta$ and $\gamma \rightarrow \beta$ from above. Money is injected (or withdrawn) via a lump-sum transfer (or tax) to each buyer at the beginning of the DM:

$$
\tau=(\gamma-1) M
$$

Besides money, there also exist competitive financial intermediaries. These financial intermediaries possess a record-keeping technology, which allow them to identify agents and keep track of goods market transaction histories. Clearly the availability of the record-keeping technology makes credit arrangements possible in this economy. To motivate the essentiality of money, there are two frictions associated with the record-keeping technology. The first friction is that the record-keeping technology or the financial intermediaries are not available in the CM. This restriction implies that agents may arrange credit transactions in the DM, but cannot settle their debts in the CM. As the financial intermediaries are available in the overnight market, buyers who have used credit in the DM repay their debts and sellers who have extended credit get repayment in the overnight market. In some sense, the settlement of debts is delayed. Without such a restriction, agents would want to settle their debts in the CM. Since goods are nonstorable, money becomes the only means of settlement. The second friction associated with the record-keeping technology is that it is costly. As all agents are anonymous, the buyer in a match in the DM can incur a fixed utility cost $k$ to make the pair identifiable to the financial intermediary so that the seller can extend credit to the buyer. ${ }^{6}$ Without incurring the fixed cost, the buyer and the seller remain anonymous and cannot make credit arrangements.

[^4]
## 3 Monetary Equilibrium with Enforcement

In this section, I assume that the financial intermediaries can enforce the repayment of credit and the monetary authority can impose lump-sum taxes.

### 3.1 Buyers

To facilitate the analysis, I start with buyers in the CM. Let $W^{b}(m, \ell)$ and $N^{b}(\hat{m}, \ell)$ be the value function for a buyer in the CM and the overnight market, respectively. The state variables for a buyer include the money holding $m$ and the amount of debt $\ell$ if the buyer used credit in the previous DM. The buyer chooses the labor supply, the consumption of the CM good and the money balance carried to the overnight market.

$$
\begin{gathered}
W^{b}(m, \ell)=\max _{x, h, \hat{m}}\left\{v(x)-h+N^{b}(\hat{m}, \ell)\right\} \\
\text { s.t. } x+\phi \hat{m}=\phi m+h .
\end{gathered}
$$

Here $\phi$ is the inverse of the price level (or the value of money). Substituting $h$ from the buyer's budget constraint, the unconstrained problem is

$$
W^{b}(m, \ell)=\phi m+\max _{x, \hat{m}}\left\{v(x)-x-\phi \hat{m}+N^{b}(\hat{m}, \ell)\right\} .
$$

The first order conditions are

$$
\begin{align*}
v^{\prime}(x) & =1  \tag{2}\\
\frac{\partial N^{b}(\hat{m}, \ell)}{\partial \hat{m}} & =\phi \tag{3}
\end{align*}
$$

Note that since the buyer cannot settle the debt in the CM, he carries his debt to the overnight market. The choice of $\hat{m}$ does not depend on $m$; however, it depends on $\ell$. The envelope conditions imply

$$
\begin{align*}
\frac{\partial W^{b}(m, \ell)}{\partial m} & =\phi  \tag{4}\\
\frac{\partial W^{b}(m, \ell)}{\partial \ell} & =\frac{\partial N^{b}(\hat{m}, \ell)}{\partial \ell} \tag{5}
\end{align*}
$$

Here $W^{b}(m, \ell)$ is linear in $m$.
For the buyer entering into the overnight market, the value function is

$$
N^{b}(\hat{m}, \ell)=\beta \int V_{+}^{b}\left(\hat{m}-\ell+\tau_{+}, 0 ; \varepsilon\right) d G(\varepsilon)
$$

Due to the quasilinear structure of the buyer's preference, the buyer should be indifferent between repaying the debt in the current overnight market or in any future overnight market. For simplicity, I assume that if the buyer has any debt, he repays in the current overnight market. The only activity for the buyer in the overnight market is to repay his debt. To simplify notations, let $z_{+}=\hat{m}-\ell+\tau_{+}$ be the money holding for a buyer at the beginning of the following period. Let $V_{+}^{b}\left(z_{+}, 0 ; \varepsilon\right)$ be the buyer's value function in the following DM, where $\varepsilon$ is the preference shock realized at the beginning of the following DM. Since $V_{+}^{b}\left(z_{+}, 0 ; \varepsilon\right)$ depends on $\varepsilon$, I take the expected value for the buyer in the overnight market and discount it by $\beta$. The envelope conditions give

$$
\begin{align*}
\frac{\partial N^{b}(\hat{m}, \ell)}{\partial \hat{m}} & =\beta \int \frac{\partial V_{+}^{b}\left(z_{+}, 0 ; \varepsilon\right)}{\partial z_{+}} d G(\varepsilon)  \tag{6}\\
\frac{\partial N^{b}(\hat{m}, \ell)}{\partial \ell} & =-\beta \int \frac{\partial V_{+}^{b}\left(z_{+}, 0 ; \varepsilon\right)}{\partial z_{+}} d G(\varepsilon) \tag{7}
\end{align*}
$$

Combining (3) and (5) with (6) and (7),

$$
\begin{equation*}
\frac{\partial N^{b}(\hat{m}, \ell)}{\partial \hat{m}}=\beta \int \frac{\partial V_{+}^{b}\left(z_{+}, 0 ; \varepsilon\right)}{\partial z_{+}} d G(\varepsilon)=-\frac{\partial N^{b}(\hat{m}, \ell)}{\partial \ell}=-\frac{\partial W^{b}(m, \ell)}{\partial \ell}=\phi \tag{8}
\end{equation*}
$$

Note that $W^{b}(m, \ell)$ is also linear in $\ell$.
After exiting the overnight market, each buyer realizes a preference shock $\varepsilon$. For a buyer with $\varepsilon$, if he only uses money in a trade, the value function in the DM is

$$
V_{+}^{b}\left(z_{+}, 0 ; \varepsilon\right)=\sigma\left[\varepsilon u\left(q_{+}\right)+W_{+}^{b}\left(z_{+}-d_{+}, 0\right)\right]+(1-\sigma) W_{+}^{b}\left(z_{+}, 0\right)
$$

where $\left(q_{+}, d_{+}\right)$are the terms of trade determined by the buyer's take-it-or-leave-it offer. With probability $\sigma$, the buyer meets a seller, spends $d_{+}$units of money and consumes $q_{+}$units of the DM good. With probability $1-\sigma$, the buyer is not matched and carries his money balance to the
following CM. If the buyer decides to use credit in a trade, his value function in the DM is

$$
V_{+}^{b}\left(z_{+}, 0 ; \varepsilon\right)=\sigma\left[\varepsilon u\left(q_{+}\right)-k+W_{+}^{b}\left(z_{+}-d_{+}, a_{+}\right)\right]+(1-\sigma) W_{+}^{b}\left(z_{+}, 0\right),
$$

where $\left(q_{+}, d_{+}, a_{+}\right)$are the terms of trade with $a_{+}$being the amount of debt the buyer incurs. Note that the buyer also incurs a utility cost $k$ by using credit.

### 3.2 Sellers

Let $W^{s}(m, \ell)$ be a seller's value function with money holding $m$ and debt $\ell$ in the CM. Since the seller is the creditor, $\ell$ should be either 0 or negative. The seller's value function is

$$
\begin{gathered}
W^{s}(m, \ell)=\max _{y, \hat{m}}\left\{y+N^{s}(\hat{m}, \ell)\right\} \\
\text { s.t. } y+\phi \hat{m}=\phi m,
\end{gathered}
$$

where $y$ is the seller's consumption in the CM and $N^{s}(\hat{m}, \ell)$ is the seller's value function in the overnight market. Substitute $y$ from the constraint to get the unconstrained problem,

$$
W^{s}(m, \ell)=\phi m+\max _{\hat{m}}\left\{-\phi \hat{m}+N^{s}(\hat{m}, \ell)\right\} .
$$

The first order condition with respect to $\hat{m}$ is

$$
\begin{equation*}
\frac{\partial N^{s}(\hat{m}, \ell)}{\partial \hat{m}} \leq \phi, \text { and } \hat{m}=0 \text { if } \frac{\partial N^{s}(\hat{m}, \ell)}{\partial \hat{m}}<\phi \tag{9}
\end{equation*}
$$

The envelope conditions yield

$$
\begin{align*}
\frac{\partial W^{s}(m, \ell)}{\partial m} & =\phi  \tag{10}\\
\frac{\partial W^{s}(m, \ell)}{\partial \ell} & =\frac{\partial N^{s}(\hat{m}, \ell)}{\partial \ell} . \tag{11}
\end{align*}
$$

Again, $W^{s}(m, \ell)$ is linear in $m$.

For the seller in the overnight market, the value function is

$$
N^{s}(\hat{m}, \ell)=\beta \int V_{+}^{s}(\hat{m}-\ell, 0 ; \varepsilon) d G(\varepsilon) .
$$

If the seller extended any credit in the previous DM, the seller will receive the money back in the overnight market. I take the expected value function of the seller because the seller anticipates that a potential buyer he meets in the following DM may have a preference shock $\varepsilon$ drawn from $G(\varepsilon)$. The envelope conditions are

$$
\begin{align*}
& \frac{\partial N^{s}(\hat{m}, \ell)}{\partial \hat{m}}=\beta \int \frac{\partial V_{+}^{s}(\hat{m}-\ell, 0 ; \varepsilon)}{\partial(\hat{m}-\ell)} d G(\varepsilon)  \tag{12}\\
& \frac{\partial N^{s}(\hat{m}, \ell)}{\partial \ell}=-\beta \int \frac{\partial V_{+}^{s}(\hat{m}-\ell, 0 ; \varepsilon)}{\partial(\hat{m}-\ell)} d G(\varepsilon) \tag{13}
\end{align*}
$$

Now combining (11) with (12) and (13), I get

$$
\begin{equation*}
\frac{\partial N^{s}(\hat{m}, \ell)}{\partial \hat{m}}=\beta \int \frac{\partial V_{+}^{s}(\hat{m}-\ell, 0 ; \varepsilon)}{\partial(\hat{m}-\ell)} d G(\varepsilon)=-\frac{\partial N^{s}(\hat{m}, \ell)}{\partial \ell}=-\frac{\partial W^{s}(m, \ell)}{\partial \ell} . \tag{14}
\end{equation*}
$$

For the seller who may meet a buyer with $\varepsilon$, the value function in the following DM is

$$
\begin{equation*}
V_{+}^{s}(\hat{m}-\ell, 0 ; \varepsilon)=\sigma\left[-c\left(q_{+}\right)+W_{+}^{s}\left(\hat{m}-\ell+d_{+},-a_{+}\right)\right]+(1-\sigma) W_{+}^{s}(\hat{m}-\ell, 0) . \tag{15}
\end{equation*}
$$

If the seller meets a buyer, the seller sells $q_{+}$units of the DM good and receives $d_{+}$units of money and extends $a_{+}$units of money as credit if the buyer chooses to use credit.

### 3.3 Equilibrium

### 3.3.1 Take-it-or-Leave-it Offer

Before deriving the equilibrium conditions, I solve for the terms of trade in the DM. The terms of trade in a match are determined by the buyer's take-it-or-leave-it offer. ${ }^{7}$ There are two types of trades in the DM, depending on whether the buyer in a match wants to use credit or not.

[^5]Suppose that a buyer with $\varepsilon$ uses money only. Recall that $W_{+}^{b}$ and $W_{+}^{s}$ are linear in $m$. The buyer's problem is

$$
\begin{gathered}
\max _{q_{+}, d_{+}}\left[\varepsilon u\left(q_{+}\right)-\phi_{+} d_{+}\right] \\
\text {s.t. } c\left(q_{+}\right)=\phi_{+} d_{+} \\
d_{+} \leq z_{+}
\end{gathered}
$$

where $z_{+}$is the buyer's money holding. Let $\lambda_{1}$ and $\lambda_{2}$ be the Lagrangian multiplier associated with the two constraints.

$$
\mathcal{L}=\max _{q_{+}, d_{+}}\left[\varepsilon u\left(q_{+}\right)-\phi_{+} d_{+}\right]+\lambda_{1}\left[\phi_{+} d_{+}-c\left(q_{+}\right)\right]+\lambda_{2}\left(z_{+}-d_{+}\right) .
$$

It is straightforward that the solution is the following.

$$
\left\{\begin{array}{l}
\lambda_{2}=0:\left(q_{+}, d_{+}\right) \text {are given by } \varepsilon u^{\prime}\left(q_{+}\right)=c^{\prime}\left(q_{+}\right) \text {and } \phi_{+} d_{+}=c\left(q_{+}\right) \\
\lambda_{2}>0:\left(q_{+}, d_{+}\right) \text {are given by } d_{+}=z_{+} \text {and } c\left(q_{+}\right)=\phi_{+} d_{+}
\end{array}\right.
$$

Suppose that the buyer with $\varepsilon$ uses credit. From (8), $W^{b}$ is also linear in $\ell$. However, it is not clear that $W^{s}$ must be linear in $\ell$ at this stage. So I define the buyer's problem as

$$
\begin{gathered}
\max _{q_{+}, d_{+}, a_{+}}\left[\varepsilon u\left(q_{+}\right)-k-\phi_{+} d_{+}-\phi_{+} a_{+}\right] \\
\text {s.t. } c\left(q_{+}\right)=\phi_{+} d_{+}+W_{+}^{s}\left(0,-a_{+}\right)-W_{+}^{s}(0,0) \\
d_{+} \leq z_{+} .
\end{gathered}
$$

It is obvious that the seller's money holding does not appear in the above problem. The terms of trade with credit do not depend on the seller's money holding. Recall that the terms of trade with money only also do not depend on the seller's money holding. It follows from (15) that

$$
\begin{equation*}
\frac{\partial V_{+}^{s}(\hat{m}-\ell, 0 ; \varepsilon)}{\partial(\hat{m}-\ell)}=\sigma \phi_{+}+(1-\sigma) \phi_{+}=\phi_{+} . \tag{16}
\end{equation*}
$$

From (14) and (16),

$$
\begin{equation*}
\frac{\partial N^{s}(\hat{m}, \ell)}{\partial \hat{m}}=-\frac{\partial N^{s}(\hat{m}, \ell)}{\partial \ell}=-\frac{\partial W^{s}(m, \ell)}{\partial \ell}=\beta \phi_{+} . \tag{17}
\end{equation*}
$$

Two results follow from (17). First, $W^{s}$ is linear in $\ell$. Second, sellers choose $\hat{m}=0$. Since I focus on stationary equilibrium, one can show that the gross inflation rate $\frac{\phi}{\phi_{+}}=\gamma$. As I only consider $\gamma>\beta$ and $\gamma \rightarrow \beta$ from above, the second result is derived from (9) and (17).

Using (17), the Lagrangian is

$$
\mathcal{L}=\max _{q_{+}, d_{+}}\left[\varepsilon u\left(q_{+}\right)-k-\phi_{+} d_{+}-\phi_{+} a_{+}\right]+\lambda_{1}\left[\phi_{+} d_{+}+\beta \phi_{++} a_{+}-c\left(q_{+}\right)\right]+\lambda_{2}\left(z_{+}-d_{+}\right) .
$$

It turns out the inequality constraint is always binding. The solutions of ( $q_{+}, d_{+}, a_{+}$) are

$$
\begin{align*}
\frac{\varepsilon u^{\prime}\left(q_{+}\right)}{c^{\prime}\left(q_{+}\right)} & =\frac{\gamma}{\beta},  \tag{18}\\
d_{+} & =z_{+},  \tag{19}\\
\beta \phi_{++} a_{+} & =c\left(q_{+}\right)-\phi_{+} d_{+} . \tag{20}
\end{align*}
$$

It is interesting to note that $q_{+}$also depends on $\gamma$. In this economy, if a buyer chooses to use credit in the DM, he will accumulate money for debt repayment in the following CM. For the seller who extends the credit in the match, he won't be able to get paid in the following CM. The seller has to wait to get settled in the overnight market. After receiving the money in the overnight market, the seller carries the money to the next DM, but he cannot spend the money since he does not want to consume. So the seller actually spends the money one period after the buyer accumulates the money. There is an asymmetry between the time that the buyer accumulates the money for repayment and the time that the seller can actually use the money from repayment. When the rate of return of money is less than $\frac{1}{\beta}$, the buyer has to compensate the seller for the loss of value of money when using credit. From (20), the extra charge that the buyer pays is the nominal interest rate $i=\frac{\phi_{+}}{\beta \phi_{++}}-1=\frac{\gamma}{\beta}-1$. For any given $\varepsilon, q_{+}$is decreasing in $\gamma$. Credit transactions are subject to the inflation distortion.

### 3.3.2 Money versus Credit

Having solved the terms of trade for money trades and credit trades, I proceed to find the condition that determines whether a buyer uses credit or not. First of all, notice that buyers carry the same amount of money in to the DM since money holdings are chosen before the realization of $\varepsilon \mathrm{s}$. That is, $z_{+}$degenerates for all buyers. For a buyer with a preference shock $\varepsilon$ in the DM, if he uses money only,

$$
V_{+}^{b}\left(z_{+}, 0 ; \varepsilon\right)=\sigma\left[\varepsilon u\left(q_{+}\right)-c\left(q_{+}\right)\right]+\phi_{+} z_{+}+W_{+}^{b}(0,0) .
$$

If the buyer uses credit,

$$
V_{+}^{b}\left(z_{+}, 0 ; \varepsilon\right)=\sigma\left[\varepsilon u\left(q_{+}\right)-\frac{\gamma}{\beta} c\left(q_{+}\right)+\left(\frac{\gamma}{\beta}-1\right) \phi_{+} z_{+}-k\right]+\phi_{+} z_{+}+W_{+}^{b}(0,0)
$$

Let $T(\varepsilon)$ be the net benefit of using credit for the buyer, where

$$
\begin{equation*}
T(\varepsilon)=\sigma\left[\varepsilon u\left(q_{+}^{c}\right)-\frac{\gamma}{\beta} c\left(q_{+}^{c}\right)+\left(\frac{\gamma}{\beta}-1\right) \phi_{+} z_{+}-k\right]-\sigma\left[\varepsilon u\left(q_{+}^{m}\right)-c\left(q_{+}^{m}\right)\right] . \tag{21}
\end{equation*}
$$

I use $q^{c}$ to denote the terms of trade with credit and $q^{m}$ to denote the terms of trade with only money, respectively.

Lemma 1 For a given $\gamma$, there exist two threshold values of $\varepsilon, \varepsilon_{0}$ and $\varepsilon_{1}$ such that

$$
\left\{\begin{array}{l}
0<\varepsilon<\varepsilon_{0}, \text { the buyer uses money and consumes } q^{*} \text { with } \varepsilon u^{\prime}\left(q^{*}\right)=c^{\prime}\left(q^{*}\right), \\
\varepsilon_{0}<\varepsilon<\varepsilon_{1}, \text { the buyer uses all the money, } \\
\varepsilon_{1}<\varepsilon<1 \text {, the buyer uses all the money and uses credit. }
\end{array}\right.
$$

Proof. Please see the Appendix.
Lemma 1 is very intuitive. If the buyer receives a very low $\varepsilon$, he has enough money at hand to afford $q^{*}$, which is the optimal consumption for him. Here $\varepsilon_{0}$ is the threshold that determines whether a buyer is liquidity constrained or not. For the buyer who receives an intermediate $\varepsilon$, the money may not be enough to afford $q^{*}$. The buyer is liquidity constrained. Using credit can relax the buyer's liquidity constraint, but it is costly. Therefore, buyers with intermediate $\varepsilon s$ find it optimal for them to not use credit because the benefit from using credit is not enough to cover
the fixed cost. For those buyers who have large $\varepsilon s$, paying the fixed cost to relax their liquidity constraint becomes optimal. The threshold $\varepsilon_{1}$ determines whether a buyer wants to use credit or not.

The decision of using credit is endogenous in this environment. Buyers use credit for large purchases. Empirically, the mean value of cash purchases is smaller than the mean value of credit purchases. In English (1999), the mean values of credit card purchases and cash purchases are $\$ 54$ and $\$ 11$, respectively. Klee (2008) documents that these mean values are $\$ 30.85$ and $\$ 14.2$.

### 3.3.3 Monetary Equilibrium

With different groups of buyers in terms of their choices in the DM, I can now characterize the equilibrium. To simplify notations, I define $\left(q_{0}, q_{1}\right)$ such that

$$
\frac{\varepsilon_{0} u^{\prime}\left(q_{0}\right)}{c^{\prime}\left(q_{0}\right)}=1 \text { and } \frac{\varepsilon_{1} u^{\prime}\left(q_{1}\right)}{c^{\prime}\left(q_{1}\right)}=\frac{\gamma}{\beta} .
$$

Notice that $c\left(q_{0}\right)=\phi_{+} z_{+}$is the transaction demand for money. In the overnight market, the expected marginal value of money is
$\beta \int \frac{\partial V_{+}^{b}\left(z_{+}, 0 ; \varepsilon\right)}{\partial z_{+}} d G(\varepsilon)=\beta \phi_{+}\left\{\int_{0}^{\varepsilon_{0}} d G(\varepsilon)+\int_{\varepsilon_{0}}^{\varepsilon_{1}}\left[\sigma \varepsilon \frac{u^{\prime}\left(q_{0}\right)}{c^{\prime}\left(q_{0}\right)}+(1-\sigma)\right] d G(\varepsilon)+\int_{\varepsilon_{1}}^{1}\left[\sigma \frac{\gamma}{\beta}+(1-\sigma)\right] d G(\varepsilon)\right\}$.
Using (8) and $\frac{d z_{+}}{d \dot{m}}=1$, the optimal $q_{0}$ is determined by

$$
\begin{equation*}
\varepsilon_{0}+\frac{1}{2} \frac{u^{\prime}\left(q_{0}\right)}{c^{\prime}\left(q_{0}\right)}\left(\varepsilon_{1}^{2}-\varepsilon_{0}^{2}\right)+\frac{\gamma}{\beta}\left(1-\varepsilon_{1}\right)=1+\frac{\gamma-\beta}{\beta \sigma} . \tag{22}
\end{equation*}
$$

In (22), the marginal benefit of 1 more unit of money equals to its marginal cost.

Lemma 2 When $\gamma$ is close to $\beta$ or big enough, $\varepsilon_{1}=1$.

Assumption 1: $k<u\left(q^{*}\right)-\frac{\gamma}{\beta} c\left(q^{*}\right)$ where $q^{*}$ is given by $u^{\prime}\left(q^{*}\right)=c^{\prime}\left(q^{*}\right)$.

Definition 1 Given that repayment of credit can be enforced, when $\gamma$ is close to $\beta$ or big enough,

$$
\begin{aligned}
\varepsilon_{0}+\frac{1}{2} \frac{u^{\prime}\left(q_{0}\right)}{c^{\prime}\left(q_{0}\right)}\left(1-\varepsilon_{0}^{2}\right) & =1+\frac{\gamma-\beta}{\beta \sigma}, \\
\frac{\varepsilon_{0} u^{\prime}\left(q_{0}\right)}{c^{\prime}\left(q_{0}\right)} & =1 .
\end{aligned}
$$

Following Lemma 2, no buyer would want to use credit for some $\gamma$ s. When $\gamma$ is close to $\beta$, the rate of return of money is high enough so that there is no need to use credit. As $\gamma$ is higher, the terms of trade from using credit become worse. When $\gamma$ is big enough, the gain from using credit won't be able to cover the fixed utility cost $k$. In (21), $T(\varepsilon)$ is always negative. Intuitively, in the presence of a high $\gamma$, using credit involves high repayment, which is too costly for buyers. This exactly describes the consumer credit market in Brazil in the late 80s. ${ }^{8}$

Definition 2 Given that repayment of credit can be enforced, a monetary equilibrium with credit is characterized by $\left(\varepsilon_{0}, \varepsilon_{1}, q_{0}, q_{1}\right)$ satisfying

$$
\begin{aligned}
\varepsilon_{0}+\frac{1}{2} \frac{u^{\prime}\left(q_{0}\right)}{c^{\prime}\left(q_{0}\right)}\left(\varepsilon_{1}^{2}-\varepsilon_{0}^{2}\right)+\frac{\gamma}{\beta}\left(1-\varepsilon_{1}\right) & =1+\frac{\gamma-\beta}{\beta \sigma}, \\
\frac{\varepsilon_{0} u^{\prime}\left(q_{0}\right)}{c^{\prime}\left(q_{0}\right)} & =1, \\
\frac{\varepsilon_{1} u^{\prime}\left(q_{1}\right)}{c^{\prime}\left(q_{1}\right)} & =\frac{\gamma}{\beta}, \\
\varepsilon_{1} u\left(q_{1}\right)-\frac{\gamma}{\beta} c\left(q_{1}\right)-k & =\varepsilon_{1} u\left(q_{0}\right)-\frac{\gamma}{\beta} c\left(q_{0}\right) .
\end{aligned}
$$

Proposition 1 For $\gamma>\beta$ and $k>0$, there exists a unique monetary equilibrium. Inflation reduces $q_{0}$ and $\varepsilon_{0}$, i.e., $\frac{d q_{0}}{d \gamma}<0$ and $\frac{d \varepsilon_{0}}{d \gamma}<0$. The optimal monetary policy is the Friedman rule $(\gamma \rightarrow \beta)$.

In Proposition 1, $\varepsilon_{0}$ is decreasing in $\gamma$, which implies that more buyers are liquidity constrained when $\gamma$ increases. Inflation is a tax on money. It is common that inflation makes money less attractive as a means of payment. Since credit is also available as a means of payment in this economy, how would $\varepsilon_{1}$ respond to an increase in $\gamma$ ? The two frictions associated with using credit generate two channels through which $\gamma$ affects $\varepsilon_{1}$. Higher $\gamma$ s hurt the rate of return of money,

[^6]so it is more likely that buyers find credit beneficial with high $\gamma \mathrm{s}$. Due to the fixed cost alone, buyers would want to switch from money to credit as $\gamma$ is higher. Through the fixed cost channel, $\gamma$ decreases $\varepsilon_{1}$. The other friction associated with credit is the delayed settlement. From (18), $\gamma$ affects the marginal benefit from using credit. Since the repayment of credit becomes more costly when $\gamma$ is higher, buyers have less incentives to use credit. Through the delayed settlement channel, $\gamma$ increases $\varepsilon_{1}$.

Having analyzed these two channels, one might be interested in knowing the effect from which channel dominates. From Lemma $2, \varepsilon_{1}$ hits the boundary 1 with either a very low $\gamma$ or a very high $\gamma$. It seems that the total effect of $\gamma$ on $\varepsilon_{1}$ should be non-monotonic. It is likely that the effect displays a U-shape. Since it is hard to verify this guess analytically, I will rely on numerical work in the next section to further analyze the implications from the model.

In the model, the fixed cost $k$ of using credit is important to determine whether a buyer uses credit or not. A lower $k$ can be viewed as a result of financial innovation, which is likely to promote the use of credit and contract the transaction demand for money. Proposition 2 establishes the related results.

Proposition 2 In a monetary equilibrium with credit, the thresholds are increasing in $k$, i.e., $\frac{d \varepsilon_{0}}{d k}>$ 0 and $\frac{d \varepsilon_{1}}{d k}>0$. Moreover, $\frac{d q_{0}}{d k}<0$ and $\frac{d q_{1}}{d k}<0$.

By assumption 1, no one would like to use credit if $k$ is too big. The economy would function as the one where money is the only means of payment. From proposition 2, the introduction of credit lowers $q_{0}$, which in turn lowers the transaction demand for money. As credit becomes more easily accessible, the transaction demand for money is lower. Note that it does not follow that the total real money demand must be lower as $k$ decreases. The total real money demand is

$$
\phi_{+} M_{+}=c\left(q_{0}\right)+\frac{\sigma}{\beta} \int_{\varepsilon_{1}}^{1}\left[c\left(q^{c}(\varepsilon)-c\left(q_{0}\right)\right] d G(\varepsilon),\right.
$$

where $c\left(q_{0}\right)$ reflects the transaction demand for money. Since money is the only means for repayment, the second term represents the repayment demand for money. It may increases as $\varepsilon_{1}$ decreases. Therefore, the overall effect of $k$ on the real money demand is ambiguous.

To discuss the effect of monetary policy on the aggregate welfare, I define the aggregate welfare in this economy as $\mathcal{W}$, where

$$
\begin{align*}
(1-\beta) \mathcal{W}= & \underbrace{\sigma\left\{\int_{0}^{\varepsilon_{0}}\left[\varepsilon u\left(q^{*}(\varepsilon)\right)-c\left(q^{*}(\varepsilon)\right)\right] d G(\varepsilon)+\int_{\varepsilon_{0}}^{\varepsilon_{1}}\left[\varepsilon u\left(q_{0}\right)-c\left(q_{0}\right)\right] d G(\varepsilon)+\int_{\varepsilon_{1}}^{1}\left[\varepsilon u\left(q^{c}(\varepsilon)\right)-c\left(q^{c}(\varepsilon)\right)\right] d G(\varepsilon)\right\}}_{1} \\
& +\underbrace{v\left(x^{*}\right)-x^{*}}_{2}-\underbrace{\sigma\left(\frac{1-\beta}{\beta}\right) \int_{\varepsilon_{1}}^{1}\left[c\left(q^{c}(\varepsilon)-c\left(q_{0}\right)+k\right] d G(\varepsilon)\right.}_{3} . \tag{23}
\end{align*}
$$

Note that the aggregate welfare is also buyers' aggregate welfare since sellers in this economy earn 0 surplus from trades and their aggregate welfare is 0 . The first and second terms in the aggregate welfare function are standard. What's new in (23) is the third term, which is the production distortion and the utility cost from using credit in the following sense. After a seller extends credit to a buyer at the beginning of a certain period, he receives the money as payment at the end of the period and has to wait to the next period to spend the money. To compensate the seller for the distortions from the inflation and the discount factor, the buyer has to pay the nominal interests. Since buyers receive monetary transfer from the monetary authority in each period, the actual extra payment the buyer has to accumulate by working is the real interest rate. This part is reflected in the third term, which can be viewed as the production distortion from using credit. When $\gamma$ increases, the first term decreases and the second term does not change. However, it is not clear that how $\gamma$ affects the third term. Analytically, I can show that $\frac{d \mathcal{W}}{d \gamma}<0$ when $\frac{d \varepsilon_{1}}{d \gamma}>0$. For a more general analysis, I leave it in the next section.

## 4 Quantitative Analysis

In this section, I rely on numerical analysis to further study the implications from the model. To calibrate the model, I consider some specific functional forms of $u(q), c(q)$ and $v(x)$ that are commonly used in the literature. For the utility functions in the DM and the CM , I let $u(q)=\frac{1}{\rho} q^{\rho}$ and $v(x)=B \log x$, where $0<\rho<1$. In terms of $c(q)$, I use a very simple cost function with $c(q)=q$. Since the DM is characterized by random matching, the matching technology that I specify is the urn-ball matching function, where $\sigma=1-e^{-1}$. There are four parameters $(\beta, B, \rho, k)$ to be calibrated.

For the time preference parameter $\beta$, the value is chosen to match the annual real interest rate 0.04 . For the rest three parameters, I consider three different calibration strategies. The first strategy is to simply fit the model's money demand to the U.S. money demand data by nonlinear least square. The data covers 1999 - 2000 annual nominal interest rate and the real demand for money (or the inverse of the velocity of money). ${ }^{9}$ The real money demand predicted by the model is

$$
L(i)=\frac{M}{P Y}=\frac{c\left(q_{0}\right)+\frac{\sigma}{\beta} \int_{\varepsilon_{1}}^{1}\left[c\left(q^{c}(\varepsilon)-c\left(q_{0}\right)\right] d G(\varepsilon)\right.}{Y_{c}+\sigma\left\{\int_{0}^{\varepsilon_{0}} c\left(q^{*}(\varepsilon)\right) d G(\varepsilon)+\int_{\varepsilon_{0}}^{\varepsilon_{1}} c\left(q_{0}\right) d G(\varepsilon)+\int_{\varepsilon_{1}}^{1} c\left(q^{c}(\varepsilon)\right) d G(\varepsilon)\right\}} .
$$

where

$$
\begin{aligned}
Y_{c}= & x+\sigma\left\{\int_{0}^{\varepsilon_{0}} c\left(q^{*}(\varepsilon)\right) d G(\varepsilon)+\int_{\varepsilon_{0}}^{\varepsilon_{1}} c\left(q_{0}\right) d G(\varepsilon)+\int_{\varepsilon_{1}}^{1} c\left(q^{c}(\varepsilon)\right) d G(\varepsilon)\right\} \\
& +\frac{\sigma(\beta-1)}{\beta} \int_{\varepsilon_{1}}^{1}\left[c\left(q^{c}(\varepsilon)-c\left(q_{0}\right)\right] d G(\varepsilon) .\right.
\end{aligned}
$$

The calibrated parameters from the best fit are in Table 1. However, this calibration strategy imposes the restriction that the fixed cost parameter is constant over the one hundred years, which may not be realistic. Considering that credit transactions rarely exist in the beginning of 1900s, it is hard to believe that the cost of using credit should be the same in 1900 as the cost today.

| Parameters | $\rho$ | $B$ | $k$ |
| :---: | :---: | :---: | :---: |
| Calibrated Values | 0.4732 | 1.4436 | 0.0739 |

Table 1: Calibration Results - Strategy 1

To accommodate the financial innovation associated with credit, I build in a time variant $k$ in the calibration. In particular, the cost of credit in year $t$ is modeled as $k /(t-1899)$. With this time variant cost, the cost is $k$ in year 1900, $k / 2$ in year 1901, etc. The calibration results are reported in Table 2.

| Parameters | $\rho$ | $B$ | $k$ |
| :---: | :---: | :---: | :---: |
| Calibrated Values | 0.4978 | 1.3390 | 3.9521 |

Table 2: Calibration Results - Strategy 2

[^7]The estimated cost of credit is 3.9521 in 1900. Together with the other calibrated parameter values, the model predicts that there is no credit transaction in 1900. In contrast, the estimated cost of credit is 0.0391 in 2001 and the implied credit to GDP ratio is $6.98 \% .^{10}$ It is interesting to note that allowing the time variant cost of credit makes the model fits the money demand data much better. The left panel in Figure 2 shows the actual money demand and the predicted money demand. The right panel provides the actual money demand and the predicted money demand from a model where there is no credit. It is clear that the lower cost of credit in recent years generates lower money demand, which improves the fit of the model. This calibration strategy may be better than the previous one to some extent, but it may not be ideal for the current model because the model makes predictions about the steady states whereas the time variant cost seems to capture the fluctuations.


Figure 2: Money Demand - Actual versus Fitted

To avoid the above problem, I use a third calibration strategy, where I calibrate $\rho$ and $B$ separately from $k$. The first step is to calibrate $\rho$ and $B$ using money demand data from 1900-1968. From Figure 1, the consumer revolving credit to GDP ratio is close to 0 around 1970. It is reasonable

[^8]to view the years before 1969 as when the credit is not available. Based on this assumption, I can calibrate ( $\rho, B$ ) first without worrying about $k$. Besides, calibrating $(\rho, B)$ together using money demand data has been widely used in the literature. The second step is to calibrate $k$. Since the model's prediction is about the steady states. I arbitrarily choose two time spans for two steady states. I use the money demand data from $1969-1972$ to calibrate one $k$ and from $1997-2000$ to calibrate the other $k .{ }^{11}$ As expected, estimated $k$ from $1969-1972$ is much higher than the $k$ from 1997 - 2000. I label the former $k$ as $k_{H}$ and the latter $k$ as $k_{L}$ in Table 3. By building in the interaction between money and credit, the money demand data show that credit is less costly in more recent years. For comparison purposes, I also calibrate an average $k, k_{\text {avg }}$ using the money demand data from $1969-2000$. As expected, $k_{\text {avg }}$ lies between $k_{H}$ and $k_{L}$.

| Parameters | $\rho$ | $B$ | $k_{H}$ | $k_{L}$ | $k_{\text {avg }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Calibrated Values | 0.3866 | 1.4882 | 0.0851 | 0.016 | 0.0634 |

Table 3: Calibration Results - Strategy 3

Based on the calibrated parameters, I study the effect of monetary policy. All 3 strategies produce similar qualitative predictions in terms of the effect of inflation. Here I present the results from the third strategy. Figure 3 and figure 4 show the effects of changing $\gamma$ with $k=k_{H}$ and $k=k_{L}$. The upper-left panel is the effect of inflation on the threshold $\varepsilon_{1}$. This is the primary interest of this paper. The lower-left panel is the effect of inflation on the credit to GDP ratio.

The upper-right panel and the lower-right panel are about comparisons between an economy with credit and an economy without credit. The model predicts that the aggregate welfare and the aggregate output are decreasing in the inflation rate. This is not so surprising although the model introduces a channel where inflation may potentially increases the output level at low rates through more buyers using credit. It seems that the effect from this channel is not strong enough. I choose to show the difference in welfare between an economy with credit and an economy without credit in the upper-right panel. In other words, the upper-right panel measures the improvement of welfare by having credit in the economy for various inflation rates. The lower-right panel plots the total demand for money in the credit economy and the no-credit economy.

[^9]

Figure 3: the Effect of Inflation - High $k$


Figure 4: the Effect of Inflation - Low $k$

There are several interesting findings from Figure 3 and Figure 4. I first discuss the common findings from these two figures. It is clear that inflation induces more buyers to use credit at low inflation rates and less buyers to use credit at high inflation rates. Moreover, the credit to GDP ratio predicted by the model has an inverse U-shape against inflation. As discussed in the previous section, inflation has two effects on $\varepsilon_{1}$. The fixed cost effect implies that inflation makes
more buyers use credit. This is because high inflation makes more buyers liquidity constrained so that more buyers may find using credit beneficial enough to cover the fixed cost. The delayed settlement effect on the other hand lowers buyers' incentives to use credit because of the high cost of repayment. It turns out that the fixed cost effect dominates the delayed settlement effect at low inflation rates, but the delayed settlement effect dominates the fixed cost effect at high inflation rates.

Comparing to an economy without credit, credit lowers money demand at low to moderate inflation rates, but slight increases money demand at high inflation rates. One can show that the transaction demand in an economy with credit is always lower. As the repayment of credit also requires money, money demand from the repayment channel may increase as the inflation rate increases. It seems that credit substitutes money at low to moderate inflation rates and complements money at high inflation rates. The first half of the result can be supported by the empirical work on U.S. data since the inflation rates in the U.S. have been low to moderate in recent decades. See Duca and Whitesell (1995) for an example. The latter half of the result, however, has not been verified empirically.

Both Figure 3 and Figure 4 reveal that having credit does not always benefit the society in terms of the aggregate welfare. From (23), credit improves welfare by relaxing the liquidity constraint for some buyers, but hurts welfare because of the fixed cost and the production distortion. Besides these direct effects, credit affects welfare through the general equilibrium effect. As analyzed above, credit may lower demand for money and thus the value of money, which will generate a negative externality on agents who use money. On the other hand, credit may increase money demand and the value of money, which will generate a positive externality on agents who use money. The general equilibrium effect implies that credit may hurt welfare at low to moderate inflation rates, but improve welfare at high inflation rates. Similar results appear in Chiu and Meh (2008). From the numerical results, credit improves welfare when the inflation rate is not too low.

Comparing Figure 3 and Figure 4, it is clear that a lower cost of credit promotes the use of credit. Using the average inflation rate 6.335 from 1969 - 1972 and the average inflation rate 5.731 between 1997 - 2000, the predicted credit to GDP ratio are $0.22 \%$ and $10.22 \%$. From Figure 1, the consumer revolving credit to GDP ratio is less than $1 \%$ around 1970 and is about $6.5 \%$ in 2000 . The model seems to capture very well the trend in the increase use of credit although the predicted
ratio is a little higher than the actual ratio in 2000.
In terms of welfare, more costly credit makes credit less beneficial. One can see that the threshold for credit to be welfare improving is higher in Figure 3 than it is in Figure 4. It remains a little puzzling that credit does not always improve welfare as one may expect. A possible explanation is that the frictions associated with credit are too severe. In the real world, after sellers receive repayment, they could put the money into their saving accounts to avoid the inflation distortion if they cannot use the money right away. This type of argument can be built into the model by allowing a fraction of agents to settle in the CM and the rest to settle in the overnight market. While this is a nice extension, the current model still serves as a benchmark to analyze the effect of inflation on credit in a world where credit is not entirely free of the inflation distortion.

## 5 Conclusion

Both money and credit are widely used as means of payment. It is important to understand how credit affects money demand and hence the transmission of monetary policy. I construct a model where money and credit coexist as means of payment. There are two frictions associated with using credit - a fixed utility cost and the delayed settlement. In this environment, buyers' choices of means of payment are endogenous. Credit lowers money demand at low to moderate inflation, but slightly increases money demand at high inflation rates. Inflation increases the fraction of buyers using credit at low inflation rates, whereas lowers the fraction of buyers using credit at high inflation rates. The predicted effect of inflation on credit is broadly consistent the existing empirical evidence.

There are several extensions of the paper that worth pursuing. As mentioned in the previous section, it may be more realistic to assume that only a fraction of agents are subject to the delayed settlement associated with credit arrangements. With this modification, credit should be more beneficial in terms of improving social welfare. The pricing mechanism in the DM is the buyer's take-it-or-leave-it offer. It would be interesting to study if these results can be generalized to other pricing mechanisms, especially price posting, which seems to be more realistic. Lastly, there exist many studies on the long run effect of inflation on credit. Most of them use the total private credit to GDP ratio as the measure of credit. To test the implications from this paper, a better measure
of credit should be the consumer credit to GDP ratio. This empirical study is left for future work.

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## A Appendix

TO BE ADDED


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[^1]:    ${ }^{1}$ Money in this paper refers to a perfectly divisible and liquid asset that earns 0 interest rate. Since the interest rate on demand deposits is almost 0 in the U.S.,

[^2]:    ${ }^{2}$ The inflation rate was around $8 \%$ on average in Australia in the 1980 s and was reduced to around $3 \%$ in the 1990s.
    ${ }^{3}$ Total private credit might be too broad comparing to consumer credit. Given that [1] it is hard to get data on consumer credit for a large sample of countries and long enough time span; and [2] different measures of credit tend to highly comove, which can be seen from the U.S. data, it seems that it is useful and reasonable to review these evidences.
    ${ }^{4}$ Please see the Appendix for the detailed description of the estimation.

[^3]:    ${ }^{5}$ Many papers have attempted to rationalize the coexistence of inside and outside money. See Cavalcanti and Wallace (1998), Kocherlakota and Wallace (1998), Mills (2007), Sun (2007a, 2007b).

[^4]:    ${ }^{6}$ One may argue that sellers actually pay the cost of using credit in the real world. The model can be modified to have the seller pay the fixed cost in a match. All the main results go through.

[^5]:    ${ }^{7}$ It will be interesting to generalize the buyer's bargaining power from 1 to less than 1 . It will also be interesting to study other pricing mechanisms that have been used in the literature such as competitive pricing and price posting. In this paper, I only focus on the buyer's take-it-or-leave-it offer to get the main intuition from the model and leave those extensions for future work.

[^6]:    ${ }^{8}$ It is documented that due to the long time delay in credit card charges clearing through the banking system, sellers normally add on a 20 to 30 percent surcharge to the price of the purchased item. In this way, vendors can protect themselves from the depreciation of money during the time the vendors are waiting to be paid by the credit card companies.

[^7]:    ${ }^{9}$ The original data is from Craig and Rocheteau (2007).

[^8]:    ${ }^{10}$ The predicted credit to GDP ratio from the model is

    $$
    \frac{\sigma \int_{\varepsilon_{1}}^{1}\left[c\left(q^{c}(\varepsilon)-c\left(q_{0}\right)\right] d G(\varepsilon)\right.}{Y_{c}+\sigma\left\{\int_{0}^{\varepsilon_{0}} c\left(q^{*}(\varepsilon)\right) d G(\varepsilon)+\int_{\varepsilon_{0}}^{\varepsilon_{1}} c\left(q_{0}\right) d G(\varepsilon)+\int_{\varepsilon_{1}}^{1} c\left(q^{c}(\varepsilon)\right) d G(\varepsilon)\right\}}
    $$

[^9]:    ${ }^{11}$ I can use longer time spans to calibrate the steady state $k$. It turns out that the result that $k$ becomes lower in more recent time spans is very robust.

