Engines of Liberation: The Impact of Technological Progress in an Imperfect Competition Setting

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Abstract

We present some evidence from the U.S. Census about the market concentration in the home appliances sector (e.g., four-firm concentration ratio and Herfindahl-Hirshman index) which suggests that competition in this sector, rather than being perfect, is better described by an oligopoly structure. We develop a general equilibrium three-sector growth model (home, market, appliances) where the price of home appliances is endogenous and firms in the appliances sector interact strategically. We assess the qualitative importance of technological progress at home and in the market for the decline in the relative price of home appliances. Due to the presence imperfectly competitive markets, the price of home appliances declines relative to the market wage even when total factor productivity at home and in the market grow at the same rate. Finally, we calibrate our model to match key facts of the economy in 1900. We analyze the quantitative impact of changes in the relative price of home appliances on women's employment and the appliances adoption decisions under the following two (opposite) scenarios. First, technology at home and in the market grow at a common rate equal to the historical average value of total factor productivity. Second, technology at home grow at a faster rate. In the first case, our model captures slightly less than half of the decline in the appliance price and slightly more than half of the increase in employment rate of married women.

JEL code: O15, J22 - Key Words: Imperfect Competition, Technological Change, Home Appliances Sector

1 Introduction

According to Greenwood, Seshadri, and Yorukoglu (2005), Greenwood and Guner (2004), and Greenwood, Seshadri, and Vandenbroucke (2005), technological advances in the household sector played a major role in launching and sustaining the "household revolution" that took place in the U.S. and other developed countries during the 20th century. When markets are perfectly competitive and factors of production are free to move across sectors, the fraction of households that buys labor-saving home appliances (refrigerator, washer and dryer, vacuum cleaner, etc) increases when the price of these appliances declines relative to the household median income. In turn, since it takes less time to produce one unit of the home good, women have more time available to allocate to other activities, including leisure and work. The proposed mechanism is

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embedded in dynamic general equilibrium models of home production and is shown to explain a significant fraction of the increase in labor force participation of married women, changes in the marriage and divorce rates, and the baby-boom/burst episode.[1]

When markets are perfectly competitive and factors of production are free to move across sectors, however, the decline in the price of home appliances is identified by the differential between the rate of technological progress in the home appliances sector and the remaining sectors of the economy. The hypothesis that the growth rate of total factor productivity was higher in the appliances sector in the twentieth century is a valid one but requires either some theoretical justification, perhaps along the lines proposed by Acemoglu (1998, 2007), or some empirical verification. To this date, both of them remain to be done.

In this paper, we pursue another approach. We first present some evidence from the U.S. Census about the market concentration in the home appliances sector (e.g., four-firm concentration ratio and Herfindahl-Hirshman index) which suggests that competition in this sector, rather than being perfect, is better described by an oligopoly structure. We then develop a general equilibrium three-sector growth model (home, market, appliances), where the price of home appliances is endogenous and firms in the appliances sector interact strategically. We assess the qualitative importance of technological progress at home and in the market for the decline in the relative price of home appliances. Due to the presence imperfectly competitive markets, we find that the price of home appliances declines relative to the market wage even when total factor productivity at home and in the market grow at the same rate. Finally, we calibrate our model to match key facts of the economy in 1900. We analyze the quantitative impact of changes in the relative price of home appliances on women's employment and the appliances adoption decisions under the following two (opposite) scenarios. First, technology at home and in the market grow at a common rate equal to the historical average value of total factor productivity. Second, technology at home grow at a faster rate. In the first case, our model captures slightly less than half of the decline in the appliance price and slightly more than half of the increase in employment rate of married women.

The household decision problem in our model is fairly similar to the one in Greenwood, Seshadri, and Yorokoglu (2005) and Buttet (2007). Households take the market wage and the appliances price as given and must make two decisions: whether women work and whether to adopt labor-saving appliances. We extend their framework, however, by explicitly modeling firm's output and pricing decisions in the appliances sector. To create an environment with imperfect competition, we assume that the production function exhibits increasing returns to scale which creates natural entry barriers and ensures that only a limited number of firms enter the market in equilibrium.

Our model has interesting implications for the price of home appliances and women's employment decisions. First, we show that, when the technology at home is below a threshold level, the unit-cost of production of home appliances is very high and households prefer to operate the labor-intensive technology. As technological progress at home unfolds, however, the appliances industry becomes more competitive and the price of home appliances declines relative to the market wage. Second, as in the standard home production model of Benhabib, Rogerson, and Wright (1991), changes in the market real wage due to technological progress in the market sector have no impact on either women's employment decisions or the fraction of households that buys home appliances.

[1] Alternative explanations for the rise in employment of married women include from changes in the nature of jobs from "brawn to brain" (Galor and Weil 1996), a narrowing of the gender wage gap (Jones, McGrattan, and Manuelli 2003), the introduction of the pill (Goldin and Katz 2002), changes in social norms (Fernandes, Fogli, and Olivetti 2004), or increases in returns to experience favoring women over men (Olivetti 2006) to name only a few. Alternative explanations for the decrease in the marriage rate and the delay at age at first marriage include the introduction of the pill (Goldin and Katz 2002) or changes in the perceived value of marriage (Caucutt, Guner, and Knowles 2002).

This is because the price of home appliances and the market wage increase at the same rate and thus substitution and income effect cancel out. Third, as the price of home appliances declines relative to the market wage following technological progress in the appliances sector, the fraction of households that buys appliances increases and more women work. The latter result is new and interesting on its own because it implies that the price of home appliances can decline and women's employment rates can increase even when technology at home and in the market grow at the same rate.

To conduct our quantitative exercise, we calibrate our model borrowing parameters from Greenwood, Seshadri, and Yorokoglu (2005) for the home appliances production function, from Knowles (1999) for the standard deviation of male's income distribution, and from Jones (2002) for the technology growth rate in the market. Moreover, we choose the increasing returns to scale parameter of the production function in the appliances sector to match the employment of married women in 1900.

Since, to the best of my knowledge, there is no available measure of total factor productivity for the appliances sector that covers the entire 20th century, we consider two extreme cases. First, we assume that technology at home and in the market grow at a common rate equal to the historical value of total factor productivity. In this case, we find that our model captures slightly less than half of the decline in the appliance price and slightly more than half of the increase in employment rate of married women. Second, we fix the technology growth rate at home to match the negative growth rate of the ratio between the home appliances price and the market wage between 1900 to 2000. In that case, technology at home grows twice as fast as technology in the market place and the model captures all of the rise in employment rate of married women.

Comparing the results of the two experiments is instructive and suggests two avenues for future research. First, if one accepts the hypothesis that technology grew at the same rate at home and in the market (perhaps because the coming of electricity was a true aggregate shock that affected all sectors of the economy in a similar way), our model suggests that increases in technology alone can only account for half of the increase in the rise of employment of married women which leaves room for other explanations, including the decrease in the gender wage gap or changes in social norms. On the other hand, we find that our model can account for all of the increase in the rise of women's employment if one accepts the hypothesis that technology grew at a faster rate in the home appliances sector compared to the market. In this case, however, one is left with the arduous task of explaining the difference in technology growth rates across sectors.

We believe our paper makes an important contribution to the macroeconomic literature that studies the increase in employment of married women for the following reasons. First, we present a new mechanism where women's employment decisions and the price of home appliances are simultaneously determined by the rate of technological progress in the market and the appliances sector. We find that in the presence of imperfect competition in the appliances sector, the price of home appliances decreases even when technology grows at the same rate across sectors. In turn, the decline in the price of home appliances triggers a rise in women's employment rate and the diffusion of labor-saving appliances. Second, we show that our mechanism is quantitatively important. Assuming that technology in the market grow at a rate equal to the historical value of total factor productivity, we find that our model captures a significant fraction of the increase in women's employment and the decline in appliances price.

The remainder of the paper is organized as follows. In Section 2, we outline our model and present some supporting evidence for the presence of imperfect competition in the home appliances sector. In Section 3, we define a balanced growth path equilibrium for our economy and study the qualitative impact of technological progress at home and in the market on the price of home appliances and women's employment decisions. In Section 4, we propose a calibrated example. Finally, in Section 5, we offer some concluding remarks.

2 The Model

2.1 Household Decisions Problem

Time is discrete and infinite, $t = 1, 2, ...$ There is a continuum of households of mass one. Households are made up of a man and a woman, who derive utility from consuming market goods, c_{mt}, home goods, c_{ht}, and leisure, l_t. Households preferences are given by the discounted sum of utility flow:

$$U = \sum_{t=1}^{t=\infty} \delta^{t-1} U(c_{mt}, c_{ht}, l_t) \tag{1}$$

where $\delta \in (0, 1)$ is the discount rate and the period-t utility is of the Cobb-Douglas form:

$$U(c_{mt}, c_{ht}, l_t) = \alpha \ln(c_{mt}) + \beta \ln(c_{ht}) + (1 - \alpha - \beta) \ln(l_t) \tag{2}$$

with $(\alpha, \beta) \in [0, 1] \times [0, 1]$ and $\alpha + \beta \leq 1$.

As in Greenwood, Seshadri, and Yorokoglu (2005), households must choose between two technologies to produce the home good. If the first (labor-intensive) technology is operated, the production of one unit of output requires the purchase of one unit of durable good (appliances) and a fraction $\rho\eta$ of household's total time endowment, with $0 < \rho\eta < 1$ and $\rho > 1$. In contrast, the second (labor-saving) technology only uses one unit of durable goods and a fraction $\frac{\eta}{\kappa}$ of households' time with $\kappa > 1$. More formally, the technology set for the production of the home good at time t is equal to:

$$\Omega_t = \{(-d_t, -n_t, c_{ht}) : c_{ht} \leq \min\{d_t, \zeta n_t\} \text{ and } (d_t, n_t) \geq 0\} \tag{3}$$

where d_t and n_t denote the stock of appliances and time inputs, respectively, and ζ is a fixed (labor productivity) parameter. Assuming that durable goods are lumpy, the labor-intensive technology is characterized by $(d_t, n_t, \zeta) = (1, \rho\eta, \frac{1}{\rho\eta})$, while a similar triplet for the labor-saving technology is given by $(d_t^{'}, n_t^{'}, \zeta') = (\kappa, \eta, \frac{\kappa}{\eta})$.

Whether households decide to operate the labor-saving technology depends of course on its price, p_{at}. We denote by $a_t \in \{0, 1\}$ the household adoption decision at time t and we let $a_t = 1$ when households operate the capital-intensive technology and $a_t = 0$ when households use the labor-intensive technology. Since the adoption decision only depends on the relative price between the two technologies, we assume that the labor-intensive technology can be operated at zero cost.

Labor supply decisions of men and women are made at the extensive margin, i.e. work in the market place is indivisible.[2] Households are endowed with two units of time, which they split up between market work, home work, and leisure. Men work with probability equal to one and we denote by $e_t \in \{0, 1\}$ women's employment decision. The length of the workweek is fixed and equal to t_w, which means that, conditional on working, individuals work a fraction t_w of their time. Finally, households labor is measured in efficiency units and we let λ_m and λ_f represent the permanent ability of men and women, respectively.

[2] Employment rates of married women increased over time rather than hours worked conditional on being employed (see Dora 2000). On the other hand, the employment rates of married men throughout the twentieth century stayed roughly constant. These two facts motivate our choice of modeling (i) labor supply at the extensive rather than the intensive margin and (ii) men working with probability equal to one.

We derive the budget and time constraints of the household under the following two assumptions. First, market and home goods are public goods within the household. Second, and without loss of generality, we use the price of the market good as the numeraire and impose $p_{mt} = 1$. Given the price of labor-saving appliances, p_{at}, and the market wage, w_t, households of type (λ_m, λ_f) choose a sequence of women's employment and the adoption decisions to maximize utility subject to the budget and time constraints:[3]

$$\begin{aligned} &c_{mt} + p_{at}a_t = w_t(\lambda_m + \lambda_f e_t) \\ &l_t + t_w(1 + e_t) + \eta(1 + (\rho - 1)(1 - a_t)) = 2 \\ &(e_t, a_t) \in \{0, 1\}^2 \end{aligned} \tag{4}$$

We denote by $s_t \in \{0, 1\}$ a variable that summarizes the household's adoption decisions up to time t. Households for which $s_t = 0$ never acquired any home appliances ($a_{t-i} = 0$ for all $i \geq 1$) while $s_t = 1$ if households bought durables at some point in the past. We consider the case where households can only buy and operate one unit of the home appliances and the stock of home appliances never depreciates which imply that $a_{t+i-1} = 0$ for all $i \geq 1$ if $s_t = 1$. Our assumptions are clearly restrictive as after all, home appliances have a finite life span and occasionally need to be replaced and households own a variety of home appliances.[4] Moreover, the quality and efficiency of home appliances has increased substantially during the twentieth century. Notwithstanding these criticisms, we carry on with our analysis keeping in mind that our model underestimates the demand for appliances. We let $\varphi = \frac{\lambda_f}{\lambda_m}$ denote the distance between the market ability of a wife to her husband.

Proposition 1 (Households' Optimal Decisions)**.** *There exists two real numbers $(\bar{\varphi}_1, \bar{\varphi}_2)$ such that, when $\bar{\varphi}_1 \leq \varphi \leq \bar{\varphi}_2$, the optimal employment and adoption decisions of households are given by:*

1. *when* $s_t = 1, \quad a(\varphi, \frac{p_{at}}{\lambda_m w_t}) = 0, \quad e(\varphi, \frac{p_{at}}{\lambda_m w_t}) = 1,$
2. *when* $s_t = 0, \quad e(\varphi, \frac{p_{at}}{\lambda_m w_t}) = a(\varphi, \frac{p_{at}}{\lambda_m w_t}) = 1 \Leftrightarrow \lambda_m \geq \frac{p_{at}}{w_t \phi(\varphi)}.$

The thresholds $(\bar{\varphi}_1, \bar{\varphi}_2)$ and the function ϕ are equal to:

$$\begin{aligned} \bar{\varphi}_1 &= (\frac{2 - \eta - t_w}{2 - \eta - 2t_w})^{\frac{(1-\alpha-\beta)}{\alpha}} - 1, \\ \bar{\varphi}_2 &= (\frac{2 - \rho\eta - t_w}{2 - \rho\eta - 2t_w})^{\frac{(1-\alpha-\beta)}{\alpha}} - 1 \\ \phi(\varphi) &= 1 - (\frac{2 - \rho\eta - t_w}{2 - \eta - 2t_w})^{\frac{(1-\alpha-\beta)}{\alpha}} (\frac{1}{\kappa})^{\frac{\beta}{\alpha}} + \varphi \end{aligned}$$

Proof. See the Appendix. □

According to the previous proposition, for given values of $\frac{p_{at}}{w_t}$ and φ, women work and household buy the labor-saving appliances when the male's market ability is greater than $\frac{p_{at}}{w_t \phi(\varphi)}$. When the

[3]Note that our model is not truly dynamic since households cannot accumulate either physical or human capital. We chose not to include capital in the model to be able to derive an analytical solution and because adding capital would not change our findings at least qualitatively. Quantitatively however, the presence of capital would affect our results through its impact on women's labor supply elasticity (e.g., see Attanasio, Low, and Sanchez-Marcos (2004)). We leave the important task of incorporating capital into the model for future research.

[4]Greenwood, Seshadri, and Yorokoglu (2005) consider a version of the model with divisible effort and durables. They find the qualitative and quantitative predictions of both models to be very similar.

male's market ability is less than the threshold $\frac{p_{at}}{w_t\phi(\varphi)}$, households cannot afford to buy the home appliances. As a result, women have to operate the labor-intensive technology to produce the home good and decide not to work because their marginal utility of leisure is greater than the wage offer they receive. The thresholds $(\bar{\varphi}_1, \bar{\varphi}_2)$ guarantee that the households' optimal decisions are not trivial and depend on the price of home appliances.

We derive the demand for home appliances and the aggregate labor supply of men and women assuming that matching of ability between men and women is perfectly assortative. This assumption implies that the ratio of women's to men's ability, φ, is constant across households. However, it does not preclude wage discrimination towards women as the ratio between women's and men's ability is not necessarily equal to one.

The demand for home appliances, $\mathcal{D}^a(\frac{p_{at}}{w_t})$, is equal to the measure of households which adopts the new technology:

$$\mathcal{D}^a(\frac{p_{at}}{w_t}) = \int_{\underline{\lambda}}^{+\infty} a(\frac{p_{at}}{\lambda_m w_t}) f(\lambda_m) d\lambda_m \tag{5}$$

where $\underline{\lambda} > 0$ and $f(\lambda_m)$ denote the lower bound and the probability density function, respectively, of men's market ability distribution. We assume that men's market ability follows a log-normal distribution truncated to the left with parameters μ and σ. Hence, the probability density function is equal to $f(\lambda_m) = \frac{1}{1-G(\underline{\lambda})} \frac{1}{\lambda_m \sqrt{2\pi}\sigma} e^{-\frac{(\ln(\lambda_m)-\mu)^2}{2\sigma^2}}$ with $G(\underline{\lambda}) = \int_0^{\underline{\lambda}} \frac{1}{\lambda_m \sqrt{2\pi}\sigma} e^{-\frac{(\ln(\lambda_m)-\mu)^2}{2\sigma^2}} d\lambda_m$. Finally, we let $F(x) = \int_{\underline{\lambda}}^x f(\lambda_m) d\lambda_m$ represent the cumulative distribution of men's ability.

Proposition 2.

$$\mathcal{D}^a(\frac{p_{at}}{w_t}) = F(\frac{p_{at-1}}{\phi(\varphi) w_{t-1}}) - F(\frac{p_{at}}{\phi(\varphi) w_t}) \tag{6}$$

Note that the demand for home appliances is non-increasing in the price of home appliances when the market wage is fixed. However, it shifts to the right when the market wage goes up. We let q_t denote the measure of households which operates home appliances at time t. Its law of motion is equal to:

$$q_t = q_{t-1} + \mathcal{D}^a(\frac{p_{at}}{w_t}) \tag{7}$$

Finally, we denote by $\mathcal{S}_f^w(\frac{p_{at}}{w_t})$ and $\mathcal{S}_m^w$, the aggregate supply of efficiency labor units from women and men, respectively. Since we assume that men work with probability one, $\mathcal{S}_m^w$ does not depend on either the market wage or the appliances price. It is equal to:

$$\mathcal{S}_m^w = \int_{\underline{\lambda}}^{+\infty} \lambda_m f(\lambda_m) d\lambda_m \tag{8}$$

On the other hand, the aggregate supply of efficiency labor units from women is equal to:

$$\mathcal{S}_f^w(\frac{p_{at}}{w_t}) = \varphi \int_{\underline{\lambda}}^{+\infty} \lambda_m e(\frac{p_{at}}{\lambda_m w_t}) f(\lambda_m) d\lambda_m \tag{9}$$

where the function $e(\frac{p_{at}}{\lambda_m w_t})$ denotes the optimal employment decision of married women and is characterized in Proposition 1.

2.2 Firm's Decision Problem

In this section, we describe firms' decision problem in the appliances and the market good sectors. First, we assume that the market good can be produced using a constant returns to scale technology that uses efficiency units of labor. The production function is linear and equal to:

$$f_m(l_{mt}) = A_{mt} l_{mt} \tag{10}$$

where A_{mt} and l_{mt} denote the technology level and labor input at time t, respectively. Taking the market wage as given, firms choose the output level, y_{mt}, and labor input to maximize profits:

$$\begin{aligned} \max_{(l_{mt},y_{mt})\in\Re^2_+} \quad & \Pi_{mt} = y_{mt} - w_t l_{mt} \\ \text{s.t.} \quad & y_{mt} \leq f_m(l_{mt}) \end{aligned} \tag{11}$$

The solution to the firm's problem is given by $w_t = A_{mt}$. Hence, the labor demand is perfectly elastic and firms' profits are equal to zero.

In Table 1, we present some data about the household appliances manufacturing sector following the North-American Industry Classification System (NACIS) of the Census Bureau. The household appliances manufacturing industry is divided into two main sub-sectors: small electrical appliances (e.g. fans and vacuum cleaners) and major appliances manufacturing (e.g., cooking appliances, refrigerator and home freezer, or laundry equipment) and the share of revenues of these two sub-sectors is equal to 21 and 79 percent, respectively.

The four-firm concentration ratios vary from 46.8 for small electrical appliances to 93.4 for laundry equipment and is equal to 62.2 for the entire industry. Similarly, the 50-firm Herfindahl-Hirshman index (HHI) ranges from 748.8 for small electrical appliances to 2096.3 for vacuum cleaners and HHI for the entire sector is equal to 1131.9.[5]

Based on the concentration ratios and HHI values, we model competition in the appliances industry as oligopolistic and assume that firms play a Cournot game with free-entry (see the previous footnote). We assume that the technology has increasing returns to scale which creates natural entry barriers in the industry. Hence, only a finite number of firms enter the industry in equilibrium and firms' production function is equal to:

$$f_a(l_{at}) = A_{at} l_{at}^{\theta_a} \tag{12}$$

where A_{at} and l_{at} denote the technology level and labor input at time t, respectively, and $\theta_a > 1$. Firm i's profit function is equal to:

$$\Pi_{a,i,t} = p_{at}(Y_{at}) y_{a,i,t} - w_t l_{a,i,t} \tag{13}$$

where Y_{at} denotes output in the appliances industry and $y_{a,i,t}$ represents output of firm i at time t. The appliances price can be found by inverting equation (6) and is equal to $p_{at}(Y_{at}) = w_t \phi(\varphi) F^{-1}(1-Y_{at})$, where F^{-1} denotes the inverse of the cumulative distribution function of men's ability.

Firms play a Cournot game. Let $Y_{a,-i,t}$ denote the output of all firms except for firm i, $Y_{a,-i,t} = Y_{at} - y_{a,i,t}$. Firm i chooses the output level and labor input to maximize profits taking as given $Y_{a,-i,t}$.

$$\begin{aligned} \max_{(l_{a,i,t},y_{a,i,t})\in\Re^2_+} \quad & \Pi_{a,i,t}(y_{a,i,t}, l_{a,i,t}, Y_{a,-i,t}) \\ \text{s.t.} \quad & y_{a,i,t} \leq f_a(l_{a,i,t}) \end{aligned} \tag{14}$$

[5]The N-firm concentration ratio is the percentage of market output generated by the N largest firms in the industry. Although there are no strict cutoffs or guidelines, an industry is considered perfectly competitive when the 4-firm concentration ratio is less than 15 percent; monopolistic competition when concentration is less than 40 percent; oligopolistic when concentration is greater than 40 percent; and monopolistic when the concentration ratio is around 100 percent. The 50-firm Herfindahl-Hirshman index (HHI) is a measure of the size of firms in relationship to the industry and is defined as the sum of the squares of the market shares of 50 largest firms. It is another indicator of the amount of competition. The industry is considered fairly competitive when HHI is less than 1,000; imperfectly competitive when HHI is less than 2000; and monopolistic when HHI is greater than 2000. Moreover, HHI are used by the AntiTrust Bureau to accept or reject merger proposals.

Tab. 1: The Household Appliances Manufacturing Indutry

SIC		Concentration Ratio[a] (4 largest firms)	Herfindahl-Hirshman [b] Index (50 largest firms)	Value of Shipments (in millions of dollars)
3352	Household Appliances Manufacturing	62.2	1131.9	22,269
33521	Small Electrical Appliances Manufacturing	46.8	748.8	4,623
335211	Electric Housewares and Household Fans	54.0	1024.5	2,641
335212	Household Vacuum Cleaner	77.9	2096.3	2,162
33522	Major Appliances Manufacturing	69.5	1427.3	17,645
335221	Cooking Appliances	48.3	856.5	4,327
335222	Refrigerator and Home Freezer	84.5	1988.5	5,491
335224	Laundry Equipment	93.4	-	4,404
335228	Other Major Appliances	52.9	1050.6	3,422

[a] The N-firm concentration ratio is the percentage of market output generated by the N largest firms in the industry. [b] The 50-firm Herfindahl-Hirshman index (HHI) is a measure of the size of firms in relationship to the industry and is defined as the sum of the squares of the market shares of the 50 largest firms - Source: Census Report on Manufacturing (2002)

Since $0 < Y_a < 1$, the first-order condition for firm i is equal to:

$$\frac{\partial \Pi_{a,i,t}}{\partial l_{a,i,t}} = 0 \Leftrightarrow p'_{at}(Y_{at}) f'_a(l_{a,i,t}) f_a(l_{a,i,t}) + p_{at}(Y_{at}) f'_a(l_{a,i,t}) = w_t \tag{15}$$

The number of firms operating in the market in the long-run, n_t, is determined by the free-entry condition: all firms in the market must make non-negative profits, while those outside the market must expect to make negative profits if they enter. The zero-profit condition is given by:

$$\Pi_{a,i,t} = p_{at}(Y_{at}) f_a(l_{a,i,t}) - w_t l_{a,i,t} = 0 \tag{16}$$

Since all n_t firms have the same first-order conditions, we restrict our analysis to symmetric equilibrium where $l_{a,i,t} = l_{a,t}$ for all i. From the inverse function theorem of calculus, the first-derivative of the appliances price is equal to $p'_{at}(Y_{at}) = -\frac{w_t \phi(\varphi)}{f(F^{-1}(1-Y_{at}))}$. Hence, we can rewrite the first-order condition and the free-entry condition in equations (15) and (16) as:

$$\theta_a A_{at} \phi(\varphi)(-\frac{A_{at} l_{at}^{\theta_a}}{f(x_t)} + x_t) = l_{at}^{1-\theta_a} \tag{17}$$

$$A_{at} \phi(\varphi) x_t = l_{at}^{1-\theta_a} \tag{18}$$

where $x_t = F^{-1}(1 - n_t A_{at} l_{at}^{\theta_a})$.

We denote by $(\hat{l}_{at}, \hat{n}_t)$ the labor demand and the number of entering firms that satisfies equations (17) and (18). Output in the appliances sector is equal to $\hat{Y}_{at} = \hat{n}_t A_{at} \hat{l}_{at}^{\theta_a}$. Note that equations (17) and (18) do not depend on either w_t or A_{mt}. Hence, changes in the market wage or productivity shocks in the market good sector have no impact on either output, firm's labor demand, or the number of entering firms.

In the Appendix, we show that equations (17) and (18) can be combined as:

$$A_{at} \phi(\varphi)^{\theta_a} x_t^{2\theta_a - 1} (\frac{\theta_a - 1}{\theta_a} f(x_t))^{\theta_a - 1} = 1 \tag{19}$$

Proposition 3. *Assuming that technology in the appliances sector grows at rate $z_a > 1$ ($A_{at+1} = z_a A_{at}$), equation (19) has:*

1. *no solution when $0 \leq A_{at} < \bar{A}_a$,*

2. *exactly one solution, $\hat{x}_0 = e^{(\mu + \frac{\sigma^2 \theta_a}{\theta_a - 1})}$, when $A_{at} = \bar{A}_a$,*

3. *two solutions, $\hat{x}_{1t}$ and $\hat{x}_{2t}$, when $A_{at} > \bar{A}_a$. The solutions are given by the following recursive formula:*

$$\begin{aligned} \ln(\hat{x}_{1t+1}) &= \ln(\hat{x}_0) - \sqrt{[\ln(\frac{\hat{x}_0}{x_{1t}})]^2 + \frac{2\sigma^2 \ln(z_a)}{\theta_a - 1}} \\ \ln(\hat{x}_{2t+1}) &= \ln(\hat{x}_0) + \sqrt{[\ln(\frac{\hat{x}_0}{x_{2t}})]^2 + \frac{2\sigma^2 \ln(z_a)}{\theta_a - 1}} \end{aligned} \tag{20}$$

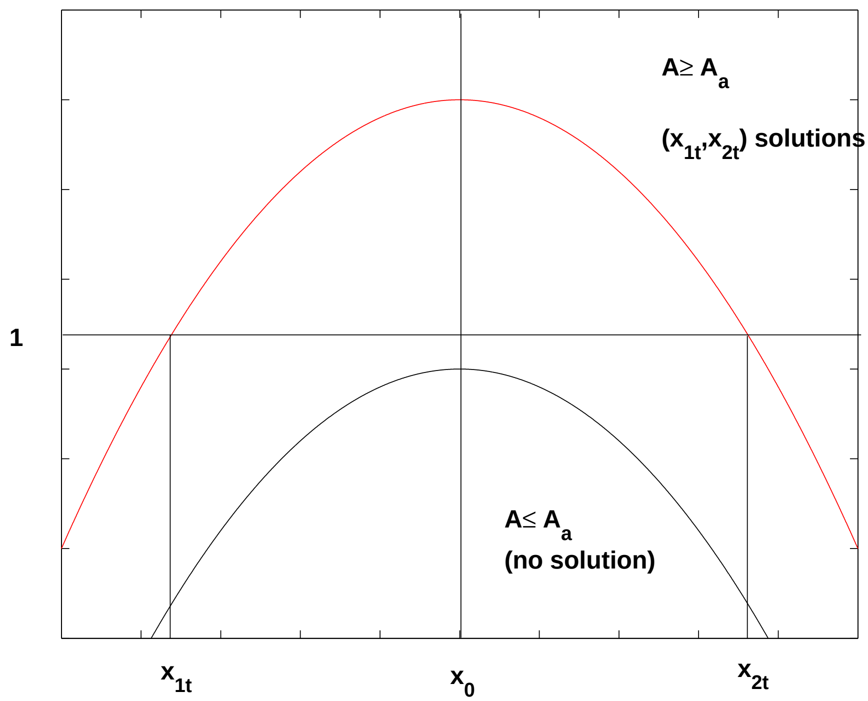
The threshold technology level is equal to:

$$\bar{A}_a = (\phi(\varphi) e^{(\frac{\sigma^2 \theta_a}{2(\theta_a - 1)} + \mu)})^{-\theta_a} (\frac{\theta_a - 1}{\theta_a} \frac{1}{1 - G(\underline{\lambda})} \frac{1}{\sqrt{2\pi}\sigma})^{1-\theta_a} \tag{21}$$

Proof. See the Appendix. □

The results the previous theorem are illustrated in Figure 1. When the technology level is below $\bar{A}_a$, the unit-cost of production of labor-saving appliances is very high and households prefer to operate the labor-intensive technology. However, as technological progress unfolds, the price of labor-saving appliances gradually declines and a positive fraction of the population, $\hat{x}_{it}$, adopts the new technology.[6] Moreover, notice that once the initial condition $\hat{x}_0$ is fixed, the entire sequence, $\{\hat{x}\}_{t=1}^{+\infty}$, is completely determined by the parameters $(\mu, \sigma, z_a, \theta_a)$.

Fig. 1: Market Ability of the Marginal Household - $\hat{x}_t$



To gain intuition about the multiplicity of solutions when $A_{at} > \bar{A}_a$, we write equation (19) as:

$$\hat{x}^{2\theta_a - 1}(f(\hat{x}))^{\theta_a} = (\frac{\theta_a}{\theta_a - 1})^{\theta_a - 1}\frac{1}{A_{at}\phi(\varphi)^{\theta_a}} \tag{22}$$

When f is the log-normal probability density function, the function in the left-hand side of equation (22) has a $\bigcap$-shape and reaches its maximum at $\hat{x}_0 = e^{(\mu + \frac{\sigma^2\theta_a}{\theta_a - 1})}$. Since $\bar{A}_a$ is defined as:

$$\hat{x}_0^{2\theta_a - 1}(f(\hat{x}_0))^{\theta_a} = (\frac{\theta_a}{\theta_a - 1})^{\theta_a - 1}\frac{1}{\bar{A}_a\phi(\varphi)^{\theta_a}} \tag{23}$$

and the right-hand side of equation (22) decreases with A_{at}, equation (22) has two solutions when $A_{at} > \bar{A}_a$. The solution $\hat{x}_{1t} < \hat{x}_0$ has the most economic appeal since it decreases with the market wage, which implies that a larger fraction of agents adopts the labor-saving technology as the market wage goes up. The solution $\hat{x}_{2t} > \hat{x}_0$, on the other hand, has counter-intuitive implications since it is increasing in the market wage. In the rest of the paper, we focus on the solution $\hat{x}_{1t}$ and we define output in the appliances sector, $\hat{Y}_{at} = 1 - F(\hat{x}_{1t})$.

[6] The model predicts that rich households adopt the labor-saving appliances first, which is line with the US data (see Day (1992), Table 8, p.319).

Corollary 1. *Output in the appliances sector, $\hat{Y}_{at}$, is equal to:*

1. *0 when $0 \leq A_{at} < \bar{A}_a$,*

2. *$1 - F(\hat{x}_{1t})$ when $A_{at} \geq \bar{A}_a$*

When $0 \leq A_{at} < \bar{A}_a$, the marginal benefit of producing the labor-saving appliances is always smaller than the marginal cost. Hence, output is equal to zero (i.e. no household buys new appliances) and the appliances price is greater than $w_t\phi(\varphi)\overline{\lambda}_m$. When $A_{at} \geq \bar{A}_a$, a positive measure of households, $1 - F(\hat{x}_{1t})$, buys the new appliances as marginal benefits increase with technological progress.

We derive the comparative statics of aggregate output and the appliances price with the appliances sector total factor productivity in the next proposition. We let $\hat{\gamma}_t = \frac{p_{at}(\hat{Y}_{at})}{w_t}$ denote the price markup of firms in the appliances sector.

Proposition 4. 1. $\frac{\partial \hat{Y}_{at}}{\partial A_{at}} \geq 0$ *and* $\frac{\partial \hat{\gamma}_t}{\partial A_{at}} \leq 0$.

2. $\frac{\partial \hat{Y}_{at}}{\partial A_{mt}} = 0$ *and* $\frac{\partial \hat{\gamma}_t}{\partial A_{mt}} \leq 0$.

Proof. See the Appendix. □

The intuition for the previous proposition is simple. First, as technological progress in the appliances sector unfolds, firms' unit-cost of production decreases. Hence, the market ability of the marginal household that adopts the new technology decreases over time, $\frac{\partial \hat{x}_{1t}}{\partial A_{at}} \leq 0$. Since $\hat{Y}_{at} = 1 - F(\hat{x}_{1t})$ and $\hat{\gamma}_t = \phi(\varphi)\hat{x}_{1t}$, output in the appliances sector increases with technological progress, while the firms' markup decreases because the appliances sector becomes more competitive. Second, the set of equations (17) and (18) that determines labor and output in the appliances sector does not depend on the technology level in the market sector. Hence, $\frac{\partial \hat{Y}_{at}}{\partial A_{mt}} = 0$. Finally, the firms' markup decreases with A_{mt} since the appliances price is unaffected but the market wage increases.

Notice that since $\hat{\gamma}_t = \phi(\varphi)F^{-1}(1 - \hat{Y}_{at})$, the markup depends on the shape of the inverse of men's ability cumulative distribution function. Moreover, the fact that the markup declines as technological progress unfolds in the appliances sector validates the prediction of our model along two dimensions. First, the appliances price to the market wage ratio did indeed decline at an average rate of 8.3 percent per year between 1900 and 1980 (see Greenwood, Seshadri, and Yorokoglu (2005) or Gordon (1990)). Second, and more importantly, the decline in $\hat{\gamma}_t$ is the single exogenous force driving the rise in women's employment and the diffusion of labor-saving appliances in Greenwood, Seshadri, and Yorokoglu (2005). In Section 4 of our paper, we use a calibrated version of our model to study the quantitative impact of technological improvements on the firms' markup and women's employment decisions.

3 Definition and Properties of Equilibrium

In this section, we define an equilibrium for our economy. We assess the general equilibrium effects of increases in productivity on women's aggregate employment as well as output and prices in the appliances sector.

Definition 1. *For all $t = 1, 2, ...$ and for exogenous sequences of A_{mt} and A_{at}, a general equilibrium for our economy is a list of prices (p^*_{at}, w^*_t) and allocations $(l^*_{mt}, l^*_{at}, n^*_t)$ such that:*

1. $w^*_t = A_{mt}$,

2. n_t^* *and* l_{at}^* *are determined by equations* (17) *and* (18)*,*

3. $p_{at}^* = w_t^* \phi(\varphi) F^{-1}(1 - n_t^* A_{at} {l_{at}^*}^{\theta_a})$*,*

4. Labor market clears:

$$l_{mt}^* + l_{at}^* = \mathcal{S}_m^w + \mathcal{S}_f^w(\frac{p_{at}^*}{w_t^*}) \tag{24}$$

where labor supply of men and women is determined by equations (8) *and* (9)*, respectively.*

We examine the impact of productivity shocks in the market good sector on allocations and prices assuming that A_{mt} grows at rate $z_m > 1$. First, the equilibrium market wage also grows at rate z_m since $w_t^* = A_{mt}$. Second, output in the appliances sector, Y_{at}^*, stays constant since changes in the market wage have no impact on either the number of entering firms or labor demand in the appliances sector (see Proposition 4). Third, the demand for appliances function shifts to the right. This implies that the appliances price grows at rate z_m since $p_{at}^* = A_{mt}\phi(\varphi)F^{-1}(1 - Y_{at}^*)$. Finally, productivity shocks in the market sector have no impact on women's labor supply (see Benhabib, Rogerson, and Wright (1991) and Greenwood, Seshadri, and Yorokoglu (2005)).

We next study the effect of productivity shocks in the appliances sector assuming that A_{at} grows at rate $z_a > 1$. First, the market wage is unaffected since $w_t^* = A_{mt}$. Second, assuming that there is a positive measure of households that adopts the labor-saving appliances, output in the appliances sector goes up while labor demand eventually decreases (see Proposition 4). There is no simple relationship, however, that links output growth rate to z_a since $\frac{Y_{at+1}}{Y_{at}} = \frac{1-F(\hat{x}_{t+1})}{1-F(\hat{x}_t)}$. Third, the increase in output and the constancy of the market wage imply that the appliances price decreases since $p_{at}^* = w_t^*\phi(\varphi)F^{-1}(1-Y_{at}^*)$. Since $\frac{p_{at+1}^*}{p_{at}^*} = \frac{\hat{x}_{t+1}}{\hat{x}_t}$, the rate of decline for the appliances price is related to z_a through the recursive equation (20). Finally, since women's employment and the fraction of households that adopts labor-saving appliances are both equal to $1 - F(\frac{p_{at}^*}{w_t^*\phi(\varphi)})$, they increase at the same rate as output.

When the rate of technological progress is the same across sectors, the impact of increases in total factor productivity on the appliances price is ambiguous since the equilibrium market wage and output in the appliances sector both increase. In the Appendix, we show that the appliances price elasticity is positively related to the curvature of the demand function $c_d(\hat{x}_t)$:

$$\frac{A_t}{p_{at}^*}\frac{\partial p_{at}^*}{\partial A_t} = 1 - \frac{1}{2\theta_a - 1 - (\theta_a - 1)c_d(\hat{x}_t)} \tag{25}$$

where $c_d(x) = -\frac{x\mathcal{D}^{a''}(x)}{\mathcal{D}^{a'}(x)}$. When men's ability distribution is log-normally distributed, the curvature of men's ability distribution is low when the fraction of households that adopt technology is low (high $\hat{x}_t$). Hence, the appliances price initially declines following increases in A_t. As the measure of households that adopts labor-saving increases, the curvature of men's ability distribution increases, which implies that the appliances price goes up. We summarize this discussion in the following proposition.

Proposition 5. $\frac{\partial p_{at}^*}{\partial A_t} \leq 0$ *if and only if* $\ln(\hat{x}_t) \geq \mu + \sigma^2$*.*

Proof. See the Appendix. □

Finally, we look at the impact of technological progress on the output share of the appliances sector. In our economy, gross domestic product at time t is equal to:

$$Y_t^* = Y_{mt}^* + p_{at}^* Y_{at}^* \tag{26}$$

where $Y^*_{mt} = A_t l^*_{mt}$ and $Y^*_{at} = A_t n^*_t {l^*_{at}}^{\theta_a}$. The next proposition analyzes the impact of technological progress on the ratio between the stock value of home appliances and GDP.

Proposition 6. *The ratio* $\frac{p^*_{at} Y^*_{at}}{Y^*_{mt}}$ *is equal to:*

1. *0 when* $A_t \leq \bar{A}$,
2. $\frac{\phi(\varphi)\hat{x}_t(1-F(\hat{x}_t))}{\mathcal{S}^w_m + \mathcal{S}^w_f(\phi(\varphi)\hat{x}_t) - (A_t \phi(\varphi)\hat{x}_t)^{\frac{1}{1-\theta_a}}}$ *when* $\bar{A} < A_t \leq \bar{\bar{A}}$,
3. $\frac{\phi(\varphi)\underline{\lambda}_m}{\mathcal{S}^w_m(1+\varphi)}$ *when* $A_t > \bar{\bar{A}}$.

where the technology level threshold $\bar{\bar{A}}$ *is given by:*

$$\frac{1}{\bar{\bar{A}}} = (\phi(\varphi)\underline{\lambda}_m)^{\theta_a}(\frac{\theta_a - 1}{\theta_a} \frac{1}{\sqrt{2\pi}\sigma} \frac{1}{1 - G(\underline{\lambda}_m)} e^{-\frac{(\ln(\underline{\lambda}_m)-\mu)^2}{2\sigma^2}})^{\theta_a - 1} \tag{27}$$

Proof. See the Appendix. □

Since all sectors grow at the same rate on a balanced growth path, the ratio $\frac{p^*_{at} Y^*_{at}}{Y^*_{mt}}$ must be constant. The previous proposition implies that the economy is on a balanced growth path only when $A_t > \tilde{\tilde{A}}$. In this case, all women work and $Y^*_{at} = 1$ since all households adopt the labor-saving technology. Moreover, the ratio of the appliances price and the market wage is constant and us equal to $\frac{p^*_{at}}{w^*_t} = \phi(\varphi)\underline{\lambda}_m$. Gross domestic product, the market good and the appliance sectors all grow at rate z. Since the men's market ability distribution is log-normal truncated to the left at $\underline{\lambda}_m$, men's labor supply is equal to:

$$\mathcal{S}^w_m = \frac{1 - G(\underline{\lambda}_m e^{-\sigma^2})}{1 - G(\underline{\lambda}_m)} \times e^{(\frac{\sigma^2}{2} + \mu)} \tag{28}$$

where $G(x) = \int_0^x \frac{1}{\sigma\sqrt{2\pi}\lambda_m} e^{-\frac{(\ln(\lambda_m)-\mu)^2}{2\sigma^2}} d\lambda_m$ for any positive x. As a result, the long-run ratio between the stock value of home appliances and GDP is positive and equal to:

$$(\frac{p^*_a Y^*_a}{Y^*_m})^{ss} = \frac{\phi(\varphi)\underline{\lambda}_m(1 - G(\underline{\lambda}_m))}{(1+\varphi)(1 - G(\underline{\lambda}_m e^{-\sigma^2}))e^{(\frac{\sigma^2}{2} + \mu)}} \tag{29}$$

4 A Calibrated Example

In this section we evaluate the quantitative properties of our model. That is, we assess the impact of the growth rate of technological progress at home and in the market sector on the price of home appliances, and thus indirectly on women's employment rates and the diffusion of home appliances in the economy.

We divide the calibration into two distinct steps. First, we use oft-cited references to pin down the following parameters of our model: the income distribution $(\underline{\lambda}_m, \mu, \sigma)$, the home technology parameters, (η, ρ, κ), and the length of the workweek, t_w. In the second step, we choose the increasing returns to scale parameter θ to match the employment rate of married women in 1900. We then use our calibrated model as an economic laboratory and run two distinct experiments as follows. First, we assume that technology at home and in the market grows at a common rate equal to the historical average growth rate of total factor productivity in the twentieth century.

Second, we assume that technology at home grows faster. We now describe the first and second steps of the calibration and the two experiments in more detail.

First, we choose a baseline value of $\sigma = 0.9$ slightly higher than Knowles (1999)'s estimate of the standard deviation of men's income distribution. We set the lower bound for the distribution of men's market ability $\underline{\lambda}_m = 0.01$ and normalize the mean of men's ability distribution to be equal to one by choosing $\mu = -\frac{\sigma^2}{2}$.

Second, Jones (2001) estimates per capita growth rate of output in the US between 1960 to 1997 to be equal to 1.4 percent per year. Accordingly, we fix the growth rate of technology in the market $z_m = 1.014^5$.

Third, we borrow the home technology and preferences parameters from Greenwood, Seshadri, and Yorokoglu (2005). Adults are endowed with a total of 112 hours per week (excluding sleep) to be divided between work, house chores and leisure. In the model, the number of hours spent on the job t_w and on house chores η is fixed. In the data, the average length of the workweek is equal to 40 hours and the number of hours spent on house chores decreased from 58 in 1900 to 18 were in 1975 (see Greenwood, Seshadri, and Yorokoglu (2005)). Accordingly, we fix $t_w = \frac{40}{112} = 0.36$, $\eta = \frac{18}{112} = 0.16$, and $\rho = \frac{58}{18} = 3.22$. Moreover, they find that, after controlling for inflation, the per-capita stock of appliances was equal to \$66 in 1925 and increased to \$528 in 1980. As a result, we set $\kappa = \frac{\$528}{\$66} = 8$. We fix $\varphi = 0.6$ since Blau and Kahn (2000) find that the observed gap in earning between men and women was stable in the 20th-century around 60 percent. The preferences parameters are set to $\alpha = 0.33$ and $\beta = 0.2$ which implies that in 1900, the price of appliances is about 3.5 times the market wage.

Finally, we choose the returns to scale parameter θ_a to match the 1900 employment rate of married women which is equal to five percent. As a result, we set the initial condition for the sequence $x_0^* = e^{\frac{\sigma^2(\theta_a+1)}{2(\theta_a-1)}} = [F]^{-1}(0.95)$ which yields $\theta_a = 1.32$. Notice that the parameter θ_a is well identified by the initial level of employment of married women and in particular, is not a function of the technology growth rate either at home or in the market sector. Moreover, once the initial condition x_0^* is fixed, the entire sequence, $\{x_t^*\}_{t=1}^{+\infty}$, is completely determined by the parameters $(\mu, \sigma, z_a, \theta_a)$ (see equation (20)).

Tab. 2: Baseline Parameters

$\underline{\lambda}_m$	σ	z_m	θ_a	t_w	η	ρ	κ	φ	α	β
0.01	0.9	1.014^5	1.32	0.36	0.16	3.22	8.0	0.60	0.33	0.2

Since, to the best of my knowledge, there is no available measure of total factor productivity for the appliances sector that covers the entire period of interest, we consider two extreme cases. First, we assume that technology at home and in the market grow at a common rate equal to the historical value of total factor productivity ($z_a = z_m$). Between 1900 to 2000, our model predicts that the home appliances price to the market wage ratio declined at an average 3.8 percent per year while employment rate of married women increased from 5 percent to 42 percent (see Figures 2 and 3). As a result, our model capture slightly less than half of the decline in the appliance price and slightly more than half of the increase in employment rate of married women (see Table 3).

Notice that the implied elasticity for women's employment relative to the appliance price is around unity which is higher compared to Greenwood, Seshadri, and Yorokuglu (2005) which comes from the following.[7] Remember from equation (9) that women's labor supply is equal to:

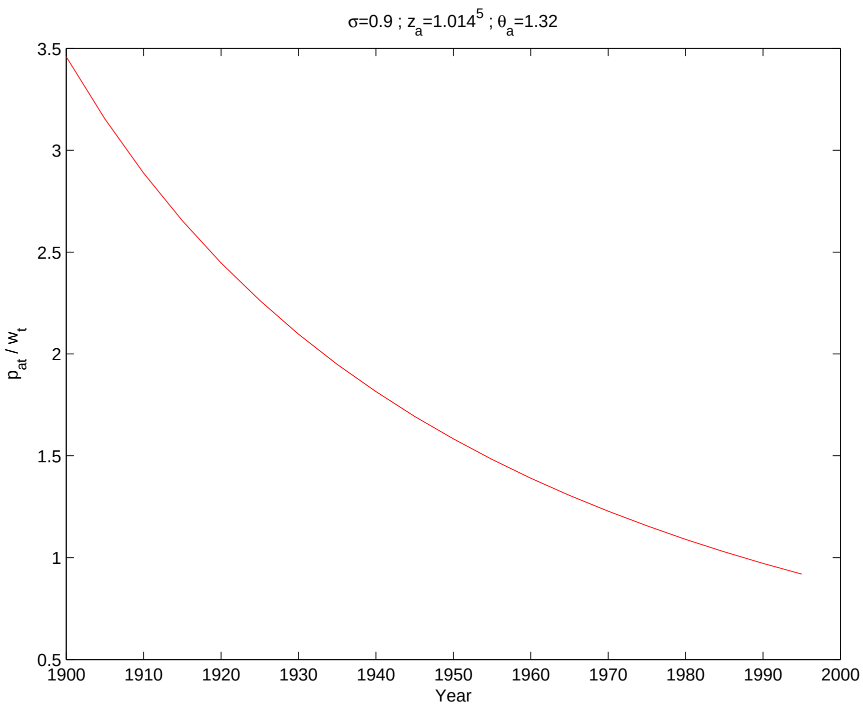
$$\mathcal{S}_f^w(\frac{p_{at}}{A_t}) = \varphi \int_{\frac{p_{at}}{A_t\phi(\varphi)}}^{\overline{\lambda}_m} \lambda_m f(\lambda_m) d\lambda_m \tag{30}$$

When the appliances price is endogenous, women's labor supply elasticity is equal to:

$$\frac{\partial \mathcal{S}_f^w}{\partial A_{at}} = \frac{\partial \mathcal{S}_f^w}{\partial A_{at}}|_{p_{at}} - \frac{\varphi}{\phi(\varphi)} f(\frac{p_{at}}{A_t\phi(\varphi)}) \frac{p_{at}}{A_t} \frac{\partial p_{at}}{\partial A_t} \tag{31}$$

where $\frac{\partial \mathcal{S}_f^w}{\partial A_{at}}|_{p_{at}}$ denotes women's labor supply elasticity when the appliances price is fixed. We know from Proposition 5 that the appliances price initially decreases as long as $\ln(\hat{x}_t) \geq \mu + \sigma^2$. Initially, when the measure of households that adopts the new technology is low, women's labor supply is very responsive to changes in productivity. As the fraction of households that adopts labor-saving appliances becomes larger than $1 - F(e^{\mu+\sigma^2})$, the appliances price increases, which implies that women's labor supply becomes less responsive to increases in productivity.

Fig. 2: Home Appliances Price Relative to the Market Wage - $z_a = z_m$



In the second experiment, we fix the technology growth rate at home to match the negative growth rate of the ratio between the home appliances price and the market wage between 1900 to 2000. We find that technology at home would have to grow twice as fast as technology in the market place ($z_a = 1.028^5$). Moreover, the model captures all of the rise in employment rate of married women (see Table 3 and Figure 4).

Comparing the results of the two experiments is instructive and suggests two avenues for future research. First, if one accepts the hypothesis that technology grew at the same rate at home and

[7] See also recent estimates of women's employment in Cavalcanti and Tavares (2006).

Fig. 3: Employment Rate of Married Women - $z_a = z_m$

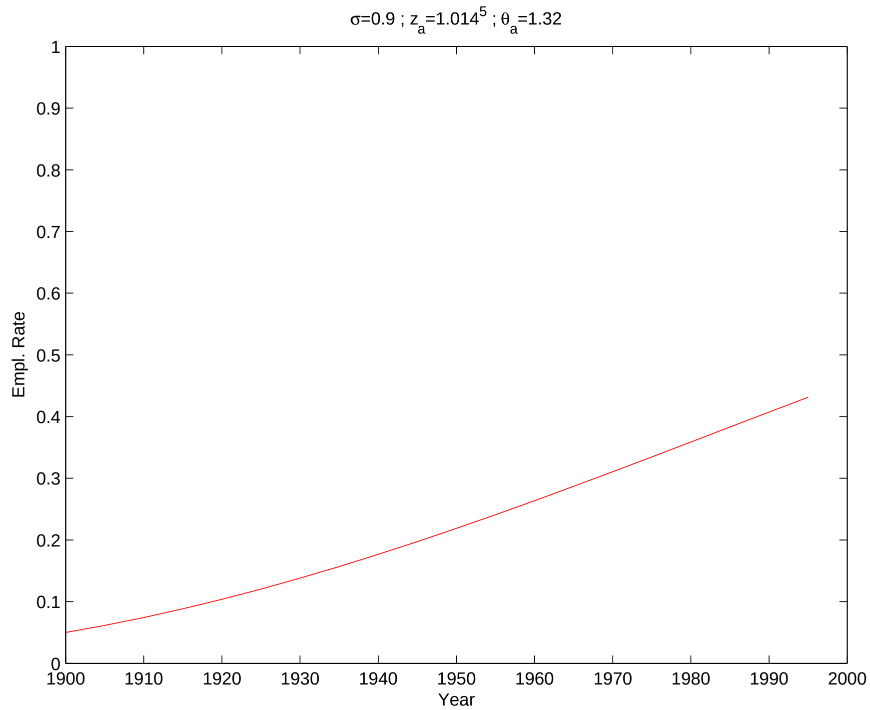
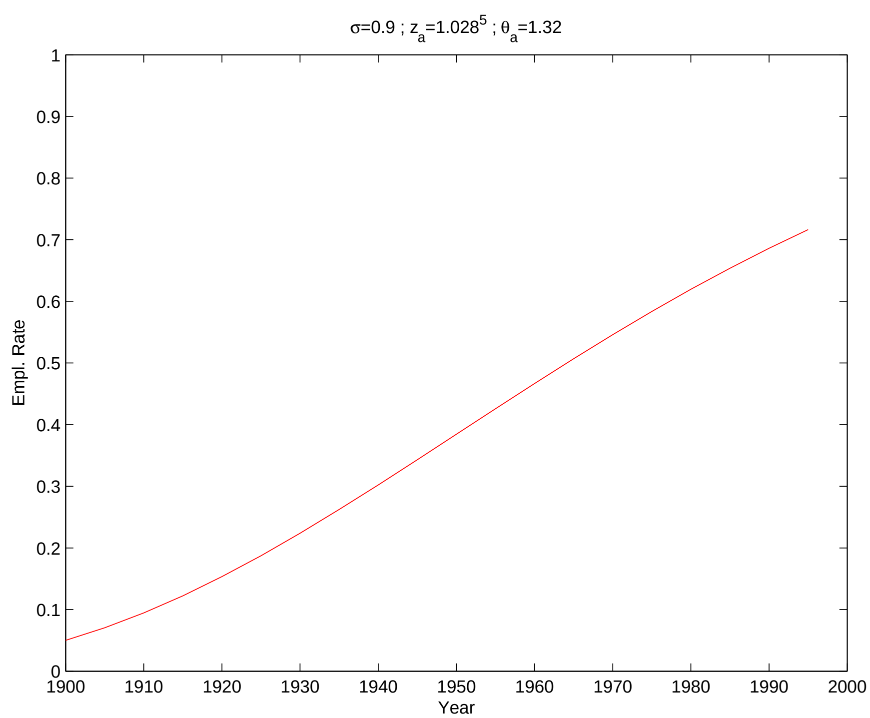


Fig. 4: Employment Rate of Married Women - $z_a > z_m$



in the market (perhaps because the coming of electricity is a true aggregate shock and affected on all sectors of the economy similarly), our model suggests that increases in technology alone can only account for fifty percent of the increase in the rise of employment of married women which leaves room for other explanation, for example the decrease in the gender wage gap or changes in social norms. On the other hand, we find that our model can account for all of the increase in the rise of women's employment if one accepts the hypothesis that technology grew at a faster rate in the home appliances sector compared to the market. In this case, however, one is left with the task of explaining why technology at home grew at a faster rate than technology in the market.

Tab. 3: Baseline Simulation Results

	1900 Data	Model	2000 Data	Model	1900-2000 Data	Model
$z_a = 1.014^5$						
Employment Rates	0.05	0.05	0.71	0.42	+0.66	+0.37
Annual Growth Rate $\frac{p_{at}}{w_t}$	-	-	-	-	-0.083	-0.038
$z_a = 1.028^5$						
Employment Rates	0.05	0.05	0.71	0.76	+0.66	+0.71
Annual Growth Rate $\frac{p_{at}}{w_t}$	-	-	-	-	-0.083	-0.083

5 Concluding Remarks

In this paper, we studied the impact of technological progress and market structure for the rise in women's employment and the diffusion of appliances in the US economy. We proposed a new mechanism where women's employment decisions feed back onto firm's decisions and the market price in the appliances sector. We find that in presence of imperfect competition, the home appliances price decreases even when the growth rate of technological progress is the same across sectors. We then showed that our mechanism is quantitatively important. Assuming that technology at home and in the market grow at a common rate equal to the historical value of total factor productivity, we found that our model captures about one-half of the increase in women's employment and the decline in appliances price.

There are several ways in which our analysis can be improved. The first, and perhaps the most urgent one, would be to extend our model to include either physical or human capital. Note that the essence of our results does not depend on the presence of capital, at least qualitatively. However, the presence of capital would alter our quantitative findings through its effect on women's labor supply elasticity.

A second and interesting avenue for further research would be to estimate total factor productivity in the appliances sector. This is likely to be a difficult task, however, due to the lack of industry level data on factors of production and factor prices which covers the entire twentieth century. Absent the possibility of directly estimating the production function, one is left with the arduous task of finding a clever instrument or a proxy to measure total factor productivity in the home appliances sector. We leave these two important tasks for future research.

6 Appendix: Proofs

6.1 Proposition 1

Proof. We conduct the proof in three steps. First, we evaluate the household's utility for different possible choices. Second, we use the indirect utility to derive two threshold functions conditional on the household's employment and adoption decisions. Finally, we use the conditional thresholds to derive households' optimal decision.

Given its employment and adoption decision, the utility of a household of type (λ_m, λ_f) is equal to:

$$\begin{aligned}
&\text{Do not work; do not adopt: } U^{NW,NA} = \alpha \ln(w_t\lambda_m) + (1-\alpha-\beta)\ln(2-t_w-\rho\eta) \\
&\text{Do not work; adopt: } U^{NW,A} = \alpha \ln(w_t\lambda_m - p_{at}) + \beta\ln(\kappa) + (1-\alpha-\beta)\ln(2-t_w-\eta) \\
&\text{Work; do not adopt: } U^{W,NA} = \alpha \ln(w_t(\lambda_m+\lambda_f)) + (1-\alpha-\beta)\ln(2(1-t_w)-\rho\eta) \\
&\text{Work; adopt: } U^{W,A} = \alpha \ln(w_t(\lambda_m+\lambda_f) - p_{at}) + \beta\ln(\kappa) + (1-\alpha-\beta)\ln(2(1-t_w)-\rho\eta)
\end{aligned} \tag{32}$$

Comparing the above choices pairwise, we can derive the following "conditional" thresholds. First, given the household's adoption decision, a_t, women work ($e_t = 1$) if and only if $\varphi \geq \phi^w(\frac{p_{at}}{\lambda_m w_t}; a_t)$ where:

$$\begin{aligned}
\phi^w(\frac{p_{at}}{\lambda_m w_t}; 0) &= (\frac{2-\rho\eta-t_w}{2-\rho\eta-2t_w})^{\frac{(1-\alpha-\beta)}{\alpha}} - 1 \quad (U^{NW,NA} = U^{W,NA}) \\
\phi^w(\frac{p_{at}}{\lambda_m w_t}; 1) &= (1 - \frac{p_{at}}{\lambda_m w_t})[(\frac{2-\eta-t_w}{2-\eta-2t_w})^{\frac{(1-\alpha-\beta)}{\alpha}} - 1] \quad (U^{NW,A} = U^{W,A})
\end{aligned} \tag{33}$$

Second, given women's employment decision, e_t, households adopt the technology ($a_t = 1$) if and only if $\chi_t \leq \phi^a(\varphi; e_t)$ where:

$$\begin{aligned}
\phi^a(\varphi; 0) &= 1 - (\frac{2-\rho\eta-t_w}{2-\eta-t_w})^{\frac{(1-\alpha-\beta)}{\alpha}}(\frac{1}{\kappa})^{\frac{\beta}{\alpha}} \quad (U^{NW,NA} = U^{NW,A}) \\
\phi^a(\varphi; 1) &= (1+\varphi)[1 - (\frac{2-\rho\eta-2t_w}{2-\eta-2t_w})^{\frac{(1-\alpha-\beta)}{\alpha}}(\frac{1}{\kappa})^{\frac{\beta}{\alpha}}] \quad (U^{W,A} = U^{W,NA})
\end{aligned} \tag{34}$$

Notice that $\phi^w(0;0)$ does not depend on the price of home appliances. As a result, it is always optimal for women to work whenever $\varphi > \phi^w(0;0)$. In order to guarantee that women's employment decision is not trivial and depends on the appliances price, we let $\bar{\varphi}_2 = (\frac{2-\rho\eta-t_w}{2-\rho\eta-2t_w})^{\frac{(1-\alpha-\beta)}{\alpha}} - 1$ and we assume that $\varphi < \bar{\varphi}_2$. Moreover, we let $\bar{\varphi}_1 = (\frac{2-\eta-t_w}{2-\eta-2t_w})^{\frac{(1-\alpha-\beta)}{\alpha}} - 1$.

When $\bar{\varphi}_1 < \varphi < \bar{\varphi}_2$, we can show that the following is true:

$$\begin{aligned}
\frac{p_{at}}{\lambda_m w_t} &< \phi^a(\varphi; 0) \Rightarrow a_t = e_t = 1 \\
\frac{p_{at}}{\lambda_m w_t} &> \phi^a(\phi^w(0;0); 1) \Rightarrow a_t = e_t = 0
\end{aligned} \tag{35}$$

When $\phi^a(\varphi; 0) \leq \frac{p_{at}}{\lambda_m w_t} \leq \phi^a(\phi^w(0;0); 1)$, we can use the conditional thresholds to rule out most of the choices. In the end, we must compare the alternatives (NW, NA) and (W, A). We have:

$$\phi^a(\varphi; 0) \leq \frac{p_{at}}{\lambda_m w_t} \leq \phi^a(\phi^w(0;0); 1) \Rightarrow \Big(a_t = e_t = 1 \Leftrightarrow \frac{p_{at}}{\lambda_m w_t} \leq \phi(\varphi)\Big) \tag{36}$$

where $\phi(\varphi) = 1 - (\frac{2-\rho\eta-t_w}{2-\eta-2t_w})^{\frac{(1-\alpha-\beta)}{\alpha}}(\frac{1}{\kappa})^{\frac{\beta}{\alpha}} + \varphi$.

Notice that $\phi(\phi^w(0;1)) > \phi^a(0;0)$ and $\phi(\phi^w(0;0)) = \phi^a(\phi^w(0;0);1)$. As a result, we have:

$$\forall(\lambda_m, p_{at}, w_t), \quad e(\frac{p_{at}}{\lambda_m w_t}) = a(\frac{p_{at}}{\lambda_m w_t}) = 1 \Leftrightarrow \lambda_m \geq \frac{p_{at}}{w_t \phi(\varphi)} \tag{37}$$

This completes the proof of the Proposition. □

6.2 Proposition 3

Proof. For all $(x,t) > 0$, let $g(x,t) = A_{at}\phi(\varphi)^{\theta_a} x^{2\theta_a - 1}(\frac{\theta_a - 1}{\theta_a} f(x))^{\theta_a - 1}$ where the function f denotes the probability density function of the log-normal distribution with parameter μ and σ. The function g is differentiable and its first-derivative satisfies:

$$\frac{x}{\theta_a - 1}\frac{1}{g(x)}\frac{\partial g(x,t)}{\partial x} = \frac{2\theta_a - 1}{\theta_a - 1} - c_d(x) \tag{38}$$

where $c_d(x) = -\frac{xf'(x)}{f(x)}$.

Since men's ability distribution is log-normally distributed with parameters μ and σ, the function $c_d(x)$ is equal to:

$$c_d(x) = 1 + \frac{\ln(x) - \mu}{\sigma^2} \tag{39}$$

Let $\hat{x}^0$ such that $c_d(\hat{x}^0) = \frac{2\theta_a - 1}{\theta_a - 1}$. Since the function $c_d(x)$ is increasing in x, $\frac{\partial g(x,t)}{\partial x} > 0$ for $x \in (0, \hat{x}^0)$ and $\frac{\partial g(x,t)}{\partial x} < 0$ for $x > \hat{x}^0$. Hence, for any t, the function g reaches its maximum at $\hat{x}^0 = e^{(\frac{\sigma^2\theta_a}{\theta_a - 1} + \mu)}$. Moreover, we have:

$$\begin{aligned} \ln(\hat{x}_0) &= \frac{\sigma^2\theta_a}{\theta_a - 1} + \mu \\ g(\hat{x}^0, t) &= A_{at}\phi(\varphi)^{\theta_a}(\frac{\theta_a - 1}{\theta_a}\frac{1}{\sqrt{2\pi}\sigma})^{\theta_a - 1} e^{\theta_a(\frac{\sigma^2\theta_a}{2(\theta_a - 1)} + \mu)} \end{aligned} \tag{40}$$

Let $\frac{1}{\bar{A}_a} = \frac{g(\hat{x}^0, t)}{A_{at}}$. When $0 \leq A_{at} < \bar{A}_a$, we have $g(\hat{x}^0, t) < 1$, which implies that $g(x,t) < 1$ for all x. Hence, equation (19) has no solution. When $A_{at} = \bar{A}_a$, equation (19) is equivalent to $x^{2\theta_a - 1} f(x)^{\theta_a - 1} = (\hat{x}^0)^{2\theta_a - 1} f(\hat{x}^0)^{\theta_a - 1}$. Hence, it has a unique solution $x = \hat{x}^0$. Finally, $g(\hat{x}^0, t) > 1$ when $A_{at} > \bar{A}_a$ since the function g shifts upward with A_{at}. Since for any t, $\lim_{x \to 0} g(x,t) = \lim_{x \to +\infty} g(x,t) = 0$, equation (19) has two solutions $\hat{x}_{1t}$ and $\hat{x}_{2t}$.

To complete the proof, we show that when $A_{at} > \bar{A}_a$, the solutions $(\hat{x}_{1t}, \hat{x}_{2t})$ are given by equation (20). Starting with equation (19), we have:

$$z x_{t+1}^{\theta_a} e^{-(\theta_a - 1)\frac{(\ln(x_{t+1}) - \mu)^2}{2\sigma^2}} = x_t^{\theta_a} e^{-(\theta_a - 1)\frac{(\ln(x_t) - \mu)^2}{2\sigma^2}} \tag{41}$$

where $z = \frac{A_{at+1}}{A_{at}}$.

Taking logs:

$$\ln(z) + \theta_a \ln(\frac{x_{t+1}}{x_t}) - \frac{\theta_a - 1}{2\sigma^2}\Big((\ln(x_{t+1}) - \mu)^2 - (\ln(x_t) - \mu)^2\Big) = 0 \tag{42}$$

Multiplying by $-\frac{2\sigma^2}{\theta_a - 1}$, we obtain a polynomial expression of second degree in $\ln(x_{t+1})$:

$$(\ln(x_{t+1}))^2 - 2\ln(\hat{x}_0)\ln(x_{t+1}) - \Big(\frac{2\sigma^2 \ln(z)}{\theta_a - 1} + (\ln(x_t))^2 - 2\ln(\hat{x}_0)\ln(x_t)\Big) = 0 \tag{43}$$

The reduced-form discriminant for this polynomial is equal to:

$$\Delta_t^2 = (\ln(\hat{x}_0))^2 + \frac{2\sigma^2 \ln(z)}{\theta_a - 1} + (\ln(x_t))^2 - 2\ln(\hat{x}_0)\ln(x_t) \tag{44}$$

Since $\Delta_t^2 = \frac{2\sigma^2 \ln(z)}{\theta_a - 1} + (\ln(\hat{x}_0) - \ln(x_t))^2 > 0$, the quadratic polynomial has two roots. Given x_t, the roots are given by the following recursive equation:

$$\begin{aligned} \ln(x_{1t+1}) &= \ln(\hat{x}_0) - \sqrt{\frac{2\sigma^2 \ln(z)}{\theta_a - 1} + (\ln(\frac{\hat{x}_0}{x_t}))^2} \\ \ln(x_{2t+1}) &= \ln(\hat{x}_0) + \sqrt{\frac{2\sigma^2 \ln(z)}{\theta_a - 1} + (\ln(\frac{\hat{x}_0}{x_t}))^2} \end{aligned} \tag{45}$$

□

6.3 Proposition 4

Proof. We first show that $\frac{\partial \hat{x}_{1t}}{\partial A_{at}} \leq 0$. When $A_t \geq \bar{A}_a$, output is equal to $Y_{at} = 1 - F(\hat{x}_{1t})$ where $\hat{x}_{1t}$ is a solution to equation (19). Taking logarithm and differentiating equation (19) with respect to A_{at}, we get:

$$\frac{1}{A_{at}} + \frac{2\theta_a - 1}{\hat{x}_t}\frac{\partial \hat{x}_t}{\partial A_{at}} + (\theta_a - 1)\frac{f^{'}(\hat{x}_t)}{f(\hat{x}_t)}\frac{\partial \hat{x}_t}{\partial A_{at}} = 0 \tag{46}$$

After rearranging, we have:

$$\frac{\partial \hat{x}_t}{\partial A_{at}}(\frac{2\theta_a - 1}{\theta_a - 1} - c_d(\hat{x}_t)) = -\frac{\hat{x}_t}{(\theta_a - 1)A_{at}} \tag{47}$$

where $c_d(x) = -\frac{xf'(x)}{f(x)} = 1 + \frac{\ln(x) - \mu}{\sigma^2}$.

The right-hand side of the previous expression is negative since $\theta_a > 1$. Since the function c_d is increasing in x, we have $c_d(\hat{x}_{1t}) < c_d(\hat{x}_0) = \frac{2\theta_a - 1}{\theta_a - 1}$ since $\hat{x}_{1t} < \hat{x}_0$. Hence, the expression inside brackets in the left-hand side of equation (47) is positive, which implies that $\frac{\partial \hat{x}_t}{\partial A_{at}} \leq 0$.

Since output in the appliances sector and the firms' markup are equal to $\hat{Y}_{at} = 1 - F(\hat{x}_{1t})$ and $\hat{\gamma}_t = \phi(\varphi)\hat{x}_{1t}$, respectively, it follows that $\frac{\partial \hat{Y}_{at}}{\partial A_{at}} \geq 0$ and $\frac{\partial \hat{\gamma}_t}{\partial A_{at}} \leq 0$. □

6.4 Proposition 5

Proof. Assuming that $\bar{A}_a \leq A_t$, the appliances price is equal to $p_{at} = A_t\phi(\varphi)\hat{x}_t$ where $\hat{x}_t$ is determined by the solution to equation (19). The first-derivative of the appliances price with respect to the technology level is equal to:

$$\frac{\partial p_{at}}{\partial A_t} = \frac{p_{at}}{A_t} + A_t\phi(\varphi)\frac{\partial \hat{x}_t}{\partial A_t} \tag{48}$$

We substitute $\frac{\partial \hat{x}_t}{\partial A_t}$ from equation (47) into the previous expression to get:

$$\frac{A_t}{p_{at}}\frac{\partial p_{at}}{\partial A_t} = 1 - \frac{1}{2\theta_a - 1 - (\theta_a - 1)c_d(\hat{x}_t)} \tag{49}$$

The appliances price elasticity is negative when $c_d(\hat{x}_t) \geq 2$. Since $c_d(x) = 1 + \frac{\ln(x) - \mu}{\sigma^2}$, it is easy to check that $\frac{\partial p_{at}}{\partial A_t} \leq 0$ if and only if $\hat{x}_t \geq e^{(\mu + \sigma^2)}$. □

6.5 Proposition 6

Proof. Since firms earn zero profits, the value of output in the market good sector at time t is equal to $Y_{mt} = w_t^* l_{mt}^* = w_t^*(\mathcal{S}_m^w + \mathcal{S}_f^w(\phi(\varphi)\hat{x}_t) - l_{at}^*)$. From the free entry condition in the appliances sector in equation (18), the labor demand in the appliances sector is given by $l_{at}^* = (A_t\phi(\varphi)\hat{x}_t)^{\frac{1}{1-\theta_a}}$. Since $w_t^* = A_t$, the output value in the market good sector is equal to:

$$Y_{mt} = A_t(\mathcal{S}_m^w + \mathcal{S}_f^w(\phi(\varphi)\hat{x}_t) - (A_t\phi(\varphi)\hat{x}_t)^{\frac{1}{1-\theta_a}}) \tag{50}$$

The appliances price at time t is equal to $p_{at}^* = A_t\phi(\varphi)\hat{x}_t$. We know from Corollary 1 that $Y_{at} = 0$ when $A_t \leq \bar{A}$; $Y_{at} = 1 - F(\hat{x}_t)$ when $\bar{A} < A_t \leq \bar{\bar{A}}$; and $Y_{at} = 1$, when $A_t > \bar{\bar{A}}$. Since $\lim_{t\to\infty} \hat{x}_t = \underline{\lambda}_m$ and $\theta_a > 1$, we have $\lim_{t\to\infty}(A_t\phi(\varphi)\hat{x}_t)^{\frac{1}{1-\theta_a}} = 0$. Hence, the ratio $\frac{p_{at}Y_{at}}{Y_{mt}}$ is equal to:

1. 0 when $A_t \leq \bar{A}$,
2. $\frac{\phi(\varphi)\hat{x}_t(1-F(\hat{x}_t))}{\mathcal{S}_m^w + \mathcal{S}_f^w(\phi(\varphi)\hat{x}_t) - (A_t\phi(\varphi)\hat{x}_t)^{\frac{1}{1-\theta_a}}}$ when $\bar{A} < A_t \leq \bar{\bar{A}}$,
3. $\frac{\phi(\varphi)\underline{\lambda}_m}{\mathcal{S}_m^w(1+\varphi)}$ when $A_t > \bar{\bar{A}}$.

□

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