Public Information and Monetary Policy

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PRELIMINARY AND INCOMPLETE

Abstract

We study monetary policy in a model where uncertainty can lead to a discrepancy between economic agents' beliefs and true fundamentals. The key feature in our model is that decisions are subject to a "beauty contest". Monetary policy transmits information about fundamentals. The public nature of this information can help agents to coordinate. This comes at a cost, however, since monetary policy may lead the private sector to coordinate on the wrong fundamentals. In addition, a shift in monetary policy may result in inflation. We discuss conditions under which monetary policy will be unambiguously welfare-improving. We offer an information-based (as opposed to the standard liquidity-based) argument for why higher nominal rate hikes occur less frequently that lower ones.

1 Introduction

We study monetary policy in a model where uncertainty can lead to a discrepancy between economic agents' beliefs and true fundamentals. Following the work of Morris and Shin (MS), the key feature in our model is that investment decisions are subject to a "beauty contest:" Additional consumption is more beneficial if more agents consume the good, and less beneficial otherwise.

Our model has three novel features relative to MS. First, we imbed their static structure in an infinite horizon model. Second, our model is subject to frictions that make money essential. Thus, it is meaningful to study questions related to monetary policy. Third, our environment involves an explicit role for prices, since trade takes place in Walrasian markets. Spatial separation prevents prices from being fully revealing which allows us to study the interplay between monetary and informational frictions.

A central assumption in our paper is that monetary policy is the only way for the central bank to credibly transmit its information about fundamentals. This information is not necessarily more precise relative to that of the private sector, but it can nevertheless induce a higher degree of coordination. This, however, comes at a cost, since monetary policy may actually lead the private sector to coordinate on the wrong fundamentals. In addition, our analysis discusses an additional cost from revealing information by the central bank, since monetary policy in that case creates a distortion and may result in higher inflation in the economy. Thus, monetary policy should be applied only if the central bank is fairly certain that the private sector's beliefs are away from true fundamentals.

Our model offers an information-based (as opposed to the standard liquidity-based) argument for why higher nominal rate hikes occur less frequently than lower ones. Extensions of the basic model can make the "beauty contest" feature arise endogenously through the introduction of a financial sector.

The paper proceeds as follows. The next two sections describe the environment and deal with the full information benchmark. Section 4 studies the private information case. Section 5 discusses welfare as a function of the quality of information in the economy. Section 6 studies the general case in which signal precision itself is a random variable.

2 The Environment

Our basic environment combines the monetary model of Lagos-Wright (2005), who in turn build on the model of Kiyotaki-Wright (1989), with the information-based model of Morris-Shin (2002). Other choices of a monetary model are also possible. Our choice was for the most tractable model in which money is essential.

Agents, Preferences, and technology. Time $t = 0, 1, ...,$ is infinite and discrete. The economy consists of a $[0, 1]$ continuum of infinite-lived agents. Agents are uncertain about their type. There are two possible types of agents, denoted by n and e, and each agent becomes a type n in each period with probability $\frac{1}{2}$. Agents discount the future at rate β. There is a benevolent central bank (CB) which has the ability to print money.

Trading stages. Agents populate a large number of locations or "islands." There is a continuum of agents in each island. Agents produce and consume two non-storable goods, q and x, that are traded sequentially in two stages within each period. There is no discounting between stages. The nature of trading is somewhat different in the two stages. In the first, agents are divided across islands, with each island containing an equal measure of buyers and sellers. In this stage, good q is traded within each island in isolation. Agents are anonymous so they trade via the use of money. In the second stage, good x is traded in a general market that involves all islands together. The second stage is modelled as a frictionless Walrasian market.

Starting with the second stage, utility (disutility) of consumption (production) of the x good is assumed to be linear and is denoted by $(-)x$. Given the linearity assumption, agents will use the second stage to even their money holdings. This dramatically improves tractability. Preferences and technology in the first stage are slightly more involved. Let $r \in (0, 1)$ be a constant. Let q denote the amount of good q consumed (always by an e agent) and g denote the amount of the good produced (always by an n agent) in a given island during the first stage. We use $\mathbf{q}$ to denote the profile of such consumptions across all islands. Finally, define $L_i = \int_0^1 (q_j - q_i)^2 dj$, and $\overline{L} = \int_0^1 L_j dj$.

The payoffs of the two types of agents in each island are then defined as follows

$$U^n(g^n) = \begin{cases} -g^n, & g^n \geq 0 \\ g^n, & g^n \leq 0 \end{cases}, \tag{1}$$

$$U^e(\mathbf{q}) = \begin{cases} -(1-r)(q^e-\theta)^2 - r(L_i - \overline{L}) + q^e, & q^e \geq 0 \\ -(1-r)(q^e-\theta)^2 - r(L_i - \overline{L}) - q^e, & q^e \leq 0 \end{cases}, \tag{2}$$

where $\theta \in R$ denotes the underlying state. We assume that θ is *iid* across time. The first term in this preference structure captures the need to know the true state θ. The second and third terms capture complementarities in the consumption decisions. These are a reduced way of capturing the financial sector involvement in the economy.[1,2]

3 True Value of θ is Common Knowledge

As a benchmark, we first consider the case where there is no private information problem. In this case, the first-best involves all agents coordinating to produce and consume the same level of output. This can be decentralized as an equilibrium out-

[1] In particular, the above preference structure can be viewed as a reduced-form representation of the behavior of fund managers as described by Rajan (2005). In that case, as managers' performance is evaluated vis-a-vis their peers, they would tend to engage in correlated investments. This way, they would never under-perform their peers. In other words, their investment decisions are not necessarily based on fundamentals but rather on what other investors in the economy do. Rajan points out that particularly in the environment of ample liquidity (low interest rate levels), these agency problems can lead to excessive risk-taking on the part of fund managers (search for yield) and suggests that there is a new, "behavioral", channel of monetary policy transmission.

[2] Note that with this utility/cost structure, welfare is maximized when the economy shuts down, which we assume not to be feasible. An easy way to deal with this issue is to assume that $U(\mathbf{q}) = AI_{\{q>0 \text{ or } q<0\}} + U^e(\mathbf{q})$, where $U^e(\mathbf{q})$ is defined as above, and $A > g(0)$. This modification does not change any of the calculations that follow. In addition, it ensures that utility from consuming amount q is always higher than the cost of producing this amount.

come only if the CB follows monetary policy that is consistent with the Friedman rule.

3.1 The Planner's Problem

The planner maximizes the normalized welfare function

$$W(\mathbf{q},\theta) = \frac{1}{1-r}[\int_0^1 [U^n(g^n) + U^e(\mathbf{q})]di$$

$$= \frac{1}{1-r}[\int_0^1 -|g_i|)di - (1-r)\int_0^1 (q_i-\theta)^2 di - r\int_0^1 (L_i - \overline{L})di + \int_0^1 |q_i|)di$$

$$= -\int_0^1 (q^i - \theta)^2 di. \tag{3}$$

Claim 1 *The efficient allocation has agents in each island producing(consuming) $q = g = \theta$ in the first stage of each period. The amount of the general good, x, produced in the second stage is indeterminate.*

3.2 The Equilibrium Problem

As agents are not readily identifiable during the first stage, they need some type of record-keeping in order to transact. In this section we decentralize the above allocation using cash. Cash is provided exclusively by the CB. Let M be the per capita supply of cash. A transfer of cash, T, takes place at the *end* of the second stage of each period. The net stock of money grows as $M_{+1} = \gamma M$. We will concentrate our analysis on stationary monetary equilibria where $\phi M = \phi_{-1}M_{-1}$, where ϕ is the real price of money in terms of the numeraire good (x). Hence, γ also equals ϕ_{-1}/ϕ. We use $W(m)$ denote the discounted lifetime utility of an agent when he enters the second stage holding m units of cash, and $V(\widetilde{m})$ to denote the expected discounted lifetime utility from entering the first stage with money holdings $\widetilde{m}$. The function

$W(m)$ is then defined as

$$W(m) \;=\; \max_{x,m_{+1}} \{-x + \beta E V(\widetilde{m}_{+1} + T, \theta)\}$$
$$s.t. \quad \phi \widetilde{m}_{+1} + x = \phi m, \tag{4}$$

where x is net production of the general good. Given θ, the discounted lifetime utility of agents when they enter the first stage with $\widetilde{m}$ units of cash is

$$V(\widetilde{m}) = \frac{1}{2}\left[U^n(g^n) + W(m^n)\right] + \frac{1}{2}\left[U^e(\mathbf{q}) + W(m^e)\right]. \tag{5}$$

Claim 2 *When the true value of θ is publicly observable, the Friedman rule decentralizes the efficient allocation of the previous section.*

4 Private and Public Signals about θ

Following MS, we now assume that the realization of θ is not observable. Instead, when the true state is θ, the CB receives a signal $y = \theta + \eta$, while agents in each island receive an island-specific signal $x_i = \theta + \varepsilon_i$. The noise terms η and ε_i are normally distributed with zero mean and variances σ_η^2 and σ_ε^2, respectively. Moreover, $E(\varepsilon_i \varepsilon_j) = 0$ holds for $i \neq j$.

First, consider the case where the central bank can credibly communicate its signal by making a public announcement. In this case, the equilibrium exhibits classical dichotomy and the optimal monetary policy is to follow the Friedman rule in each period. The following is central in what follows.

Assumption 1 *The CB can only credibly communicate its signal through monetary policy.*[3]

[3] Even if a public announcement by the CB is possible, it is plausible to argue that the economy is subject to "higher order" uncertainty. This is because agents might not be entirely sure about whether every other agent has heard the announcement, or if every other agent is sure that every other agent is sure... and so on. Our analysis remains valid in the presence of such higher order uncertainty. Signals through monetary policy can establish common knowledge as they result in different prices.

We assume that both the private signals and the signal through monetary policy arrive at the *end* of each second stage. Next, we contrast the welfare between the following two special cases. In the first case, the CB does not communicate its signal. Instead, it follows the Friedman rule in each period. In this case, the buyers and sellers in each island trade based on their island-specific signal only. Second, we consider the other extreme case whereby the CB communicates its signal (violating the Friedman rule in at least some periods), in which case agents make decisions based on both, their island-specific signals as well as the public signal contained in monetary policy.

Thus, the CB, which we model as a benevolent planner, faces a trade-off. Communicating its information *might* lead to better coordination of actions by the private sector. However, this *always* comes at the cost of a distortion associated with inflation. We assume that monetary policy takes the following form

$$M_{+1} = f(M, y). \tag{6}$$

A special case, which we study for simplicity, is the case where

$$M_{+1} = (\beta + 1_y)M, \tag{7}$$

where $1_y = 0$, if the CB follows the Friedman rule, while $1_y = H(y)$, if the CB reveals its signal. We assume that $H : R \to R_+$ is a homeomorphism[4] so that, having observed the money stock, private agents can infer the bank's signal, y. Let $\mathcal{I}^i$ denote the information available to each agent in a generic island, i, (note that there is no asymmetric information *within* an island). Hence, $\mathcal{I}^i = (y, x^i)$. Thus, the agents' choices of q^b, q^s, m^b, m^s in each island i must be measurable with respect to $\mathcal{I}^i$. Under this restriction, $W(m)$ and $V(m)$ are defined as before. Let $T(y)$ denote the monetary transfer prescribed by H. We then have that

$$\begin{aligned} W(m) \quad &= \quad \max_{x,m_{+1}} \{-x + \beta EV(\widetilde{m}_{+1} + T(y), \theta)\} \\ & \qquad s.t. \quad \phi\widetilde{m}_{+1} + x = \phi m. \end{aligned} \tag{8}$$

[4] A function H is a homeomorphism if it has the following properties: 1) H is a bijection; 2) H is continuous; 3) the inverse function H^{-1} is continuous.

As before, the discounted lifetime utility of agents when they enter the first stage with $\widetilde{m}$ units of cash is

$$V(\widetilde{m}) = \frac{1}{2}\left[U^n(g^n) + W(m^n)\right] + \frac{1}{2}\left[U^e(\mathbf{q}) + W(m^e)\right]. \tag{9}$$

The problem of a type n agent is

$$\max_{g^n}[U^n(g^n) + EW(\widetilde{m} + p|g^n|)]. \tag{10}$$

The FOC for the producer of type n with respect to g^n (when his constraint is not-binding) gives

$$1 - pE\left[\phi_{+1}\right] = 0. \tag{11}$$

Thus, the price in island i in this case is given by

$$p_i = \frac{1}{E_i\left[\phi_{+1}\right]}. \tag{12}$$

The type e consumer solves

$$\begin{aligned} &\max_{q^e}[U^e(\mathbf{q}) + E_iW(\widetilde{m} - p|q^e|)] \\ &s.t.\ \widetilde{m} - pq^e \geq 0. \end{aligned} \tag{13}$$

The FOC for the consumer of type e with respect to q^e gives

$$U^{e\prime}(\mathbf{q}) - E_i[\phi_{+1}]p - \lambda^e p = 0. \tag{14}$$

Market clearing conditions in the first stage require

$$q^e = g^n. \tag{15}$$

Combining the FOC for q^e and g^n and using the equilibrium conditions, the equilibrium consumption is then given by:

$$q_i = (1-r)\,E_i(\theta) + rE_i(\overline{q}) + p\lambda_i^e. \tag{16}$$

Under the Friedman rule, this becomes

$$q_i = (1-r)\,E_i(\theta) + rE_i(\overline{q}). \tag{17}$$

One question is under what conditions the CB will find it optimal to reveal information about the state θ. As mentioned before, such revelation is costly, since a distortion is introduced in any period where the Friedman rule is violated. Thus, the CB must optimally choose whether the benefits from the additional information exceed the costs. The answer to this question depends in an interesting way on the relative informativeness of the public versus the private signals. Next, assume that

$$M_{+1} = (\beta + H(y))M. \tag{18}$$

The value function at the beginning of the first stage can be written as

$$V(\widetilde{m}) = \frac{1}{2}[U^n(g^n) + EW(\widetilde{m} + p|g^n|)] + \frac{1}{2}\left[U^e(\mathbf{q}) + E_i W(\widetilde{m} - p|q^e|)\right]. \tag{19}$$

The first order conditions with respect to $\widetilde{m}$ give

$$\begin{aligned} V'(\widetilde{m}) &= \frac{1}{2}\left[\frac{\partial g^n}{\partial \tilde{m}} + EW'(\widetilde{m} + pg^n)\left(1 + p\frac{\partial g^n}{\partial \tilde{m}}\right)\right] \\ &\quad + \frac{1}{2}\left[U^{e\prime}(\mathbf{q})\frac{\partial q^e}{\partial \tilde{m}} + EW'(\widetilde{m} - pq^e)\left(1 - p\frac{\partial q^e}{\partial \tilde{m}}\right)\right]. \end{aligned} \tag{20}$$

Since the budget constraint of each buyer is binding, we have $\frac{\partial q^e}{\partial \tilde{m}} = \frac{1}{p}$, while $\frac{\partial g^n}{\partial \tilde{m}} = 0$. Hence, the FOC simplifies to

$$V'(\widetilde{m}) = \frac{1}{2}\left[EW'(\widetilde{m} + pg^n)\right] + \frac{1}{2}\left[U^{e\prime}(\mathbf{q})\frac{1}{p}\right]. \tag{21}$$

Using the envelope condition, $W_m(m) = \phi$, we have that $EW'(\widetilde{m} - pq^n) = E\left[\phi_{+1}\right]$. The first order condition gives $\beta EV_m(\widetilde{m}_{+1}, \theta) = \phi$. Thus, we have

$$\frac{\phi}{\beta} = \frac{1}{2}E\left[\phi_{+1}\right] \; + \frac{1}{2}\left[U^{e\prime}(\mathbf{q})\frac{1}{p}\right]. \tag{22}$$

Substituting the expression for the prices in the first stage market in island i, we obtain

$$p = \frac{1}{E_i[\phi_{+1}]}. \tag{23}$$

Thus, the previous expression becomes

$$
\begin{aligned}
\frac{\phi}{\beta} &= \frac{1}{2}E\left[\phi_{+1}\right] \; + \frac{1}{2}E_i[\phi_{+1}]U^{e\prime}(\mathbf{q}), \text{ or,} && (24)\\
\frac{2\phi}{\beta E\left[\phi_{+1}\right]} &= 1 + U^{e\prime}(\mathbf{q}), \text{ or,} && (25)\\
\frac{2\phi}{\beta E\left[\phi_{+1}\right]} &= 1 + q_i - (1-r)\,E_i\left(\theta\right) - rE_i\left(\overline{q}\right) + 1, \text{ or,} && (26)\\
\frac{2\phi}{\beta E\left[\phi_{+1}\right]} - 2 &= q_i - (1-r)\,E_i\left(\theta\right) - rE_i\left(\overline{q}\right). && (27)
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
q_i &= (1-r)\,E_i\left(\theta\right) + rE_i\left(\overline{q}\right) + 2\left[\frac{\phi}{\beta E_i\left[\phi_{+1}\right]} - 1\right]\\
&= (1-r)\,E_i\left(\theta\right) + rE_i\left(\overline{q}\right) + 2\left[\frac{\gamma\left(y\right)}{\beta} - 1\right]. && (28)
\end{aligned}
$$

At the Friedman rule, $1_y = 0$, so that $\phi_{+1}/\phi = 1/\beta$, and $\lambda_i^n = \lambda_i^e = 0$. Note that away from the Friedman rule, $\frac{\phi}{\beta E_i\left[\phi_{+1}\right]} > 1$, hence, $q \neq \theta$, even if the true value of θ is publicly revealed. In equilibrium, ϕ_{+1} can be inferred by every agent, so that $\frac{\phi}{E_i\left[\phi_{+1}\right]} = \frac{\phi}{\phi_{+1}} = \gamma$. In general, the expression for the equilibrium quantity in the first stage market in island i is given by

$$
q_i = (1-r)\,E_i\left(\theta|x_i, \gamma\left(y\right)\right) + rE_i\left(\overline{q}|x_i, \gamma\left(y\right)\right) + 2\left[\frac{\gamma\left(y\right)}{\beta} - 1\right]. \tag{29}
$$

4.1 Perfectly Informative CB signal ($\eta = 0$ and $y = \theta$)

Here we consider the benchmark case in which the CB receives a *perfectly informative* signal. In that case, the CB can communicate the exact value for θ to all agents through monetary policy by simply adopting the rule $\gamma\left(\theta\right) = \beta + H\left(\theta\right)$, where $H(\bullet) : \mathbb{R} \to [0, +\infty)$ is strictly monotone. Thus, $E\left(\theta|x_i, \gamma\left(\theta\right)\right) = \theta$, for all i. Then all agents produce the same q as given by

$$
\begin{aligned}
q &= (1-r)\,\theta + rE\left(\overline{q}|x_i, \gamma\left(\theta\right)\right) + 2\left[\frac{\gamma\left(\theta\right)}{\beta} - 1\right]\\
&= (1-r)\,\theta + rq + 2\left[\frac{\beta + H(\theta)}{\beta} - 1\right]. && (30)
\end{aligned}
$$

Assuming, for example, that $H(y = \theta) = h(\theta)\beta(1-r)/2$, the above gives

$$q = \theta + h(\theta). \tag{31}$$

At the other extreme, when the CB does not reveal its information, the equilibrium quantity in island i in the first stage market is given by

$$\begin{aligned} q_i &= (1-r) E(\theta | x_i) + r E(\bar{q} | x_i) \\ &= (1-r) E(\theta | x_i) + r q_i \\ &= E(\theta | x_i) \\ &= x_i. \end{aligned} \tag{32}$$

Thus, if the CB does not signal its information, period-t expected welfare in state θ is given by

$$\begin{aligned} E\left[W_t\left(q_t^i, \theta\right) \mid \theta\right] &= -\int_0^1 (q_t^i - \theta)^2 di \\ &= -\int_0^1 \varepsilon_i^2 di \\ &= -\sigma_\varepsilon^2. \end{aligned} \tag{33}$$

In contrast, when the CB signals through monetary policy (still assuming $\gamma(\theta) = \beta + H(\theta)$), period-$t$ expected welfare is given by

$$\begin{aligned} & -\int_0^1 h^2(\theta)\, di \\ =\ & -h^2(\theta). \end{aligned} \tag{34}$$

Hence, we have the following result.

Claim 3 *It is optimal for the central bank to deviate from the Friedman rule in period t if and only if $h^2(\theta) < \sigma_\varepsilon^2$.*

Note that the degree to which deviation from the Friedman rule distorts production away from the efficient output level depends on the fundamental θ. Since $h(\theta)$ is strictly increasing in θ in this example, the inequality above is more likely to hold the larger is the realization of θ and the noisier is the information held by the private sector. As the left hand side is always positive, if the precision of the private information is nearly perfect, it is never optimal for the CB to signal its information.

5 Welfare and Signal Precision

We begin by deriving an expression for the production of q as a function of the respective precisions of the private signals and of monetary policy. As in Morris-Shin, it is useful to define the precision of the two types of information by

$$\alpha = \frac{1}{\sigma_\eta^2}, \delta = \frac{1}{\sigma_\varepsilon^2}. \tag{35}$$

Based on both signals, the expected value of θ by the private agents in island i if $1_y = y$ is given by

$$E_i(\theta) = \frac{\alpha y + \delta x_i}{\alpha + \delta}. \tag{36}$$

Following the same method as in Morris-Shin, in order to solve for the quantity produced in each island, we first conjecture that

$$q_i(\mathcal{I}_i) = \kappa x_i + (1-\kappa)y + (A + B\gamma(y)), \tag{37}$$

where $\gamma(y) = \beta + H(y)$.Then, the above two expressions imply

$$\begin{aligned} E_i(\overline{q}) &= \kappa\left(\frac{\alpha y + \delta x_i}{\alpha+\delta}\right) + (1-\kappa)y + (A + B\gamma(y)) \\ &= \left(\frac{\kappa\delta}{\alpha+\delta}\right)x_i + \left(1 - \frac{\kappa\delta}{\alpha+\delta}\right)y + (A + B\gamma(y)). \end{aligned} \tag{38}$$

Recall that, as a function of the information available in island i, the quantity produced is given by

$$q_i = (1-r)\,E_i(\theta|x_i, \gamma(y)) + rE_i(\overline{q}|x_i, \gamma(y)) + 2\left[\frac{\gamma(y)}{\beta} - 1\right]. \tag{39}$$

Combining the above two, we obtain

$$\begin{aligned} q_i &= (1-r)\left(\frac{\alpha y + \delta x_i}{\alpha+\delta}\right) + r\left[\frac{\kappa\delta}{\alpha+\delta}x_i + (1 - \frac{\kappa\delta}{\alpha+\delta})y + A + B\gamma(y)\right] + 2\left[\frac{\gamma(y)}{\beta} - 1\right] \\ &= \frac{\delta(r\kappa + 1 - r)}{\alpha+\delta}x_i + \left(1 - \frac{\delta(r\kappa+1-r)}{\alpha+\delta}\right)y + r(A + B\gamma(y)) + 2\left[\frac{\gamma(y)}{\beta} - 1\right]. \end{aligned} \tag{40}$$

Equating coefficients we obtain

$$\kappa = \frac{\delta(1-r)}{\alpha + \delta(1-r)}, \;\; A = -\frac{2}{1-r}, \;\; B = \frac{2}{\beta(1-r)}. \tag{41}$$

Thus, the equilibrium action as a function of the signal precisions is given by

$$q_i(\mathcal{I}_i) = \frac{\delta(1-r)}{\alpha+\delta(1-r)}x_i + \left(1 - \frac{\delta(1-r)}{\alpha+\delta(1-r)}\right)y - \frac{2}{1-r} + \frac{2}{\beta(1-r)}\gamma(y). \quad (42)$$

When $\gamma(y) = \beta + H(y)$, with $H(y) = h(y)\beta(1-r)/2$, we have

$$q_i(\mathcal{I}_i) = \frac{\delta(1-r)}{\alpha+\delta(1-r)}x_i + \left(1 - \frac{\delta(1-r)}{\alpha+\delta(1-r)}\right)y + h(y). \quad (43)$$

Note that, for the case where $\alpha \to \infty$ (perfectly informative CB signal), we get

$$q_i = \theta + h(y). \quad (44)$$

Simplifying the expression for $q_i(\mathcal{I}_i)$, we obtain

$$q_i(\mathcal{I}_i) = \frac{\delta(1-r)x_i + \alpha y}{\alpha+\delta(1-r)} + h(y). \quad (45)$$

In terms of θ, ε, and η, the equilibrium quantity in island i can be written as

$$q_i = \theta + \frac{\delta(1-r)\varepsilon_i + \alpha E(\eta|x_i, y)}{\alpha+\delta(1-r)} + h(y). \quad (46)$$

In order to investigate the effects of increased accuracy of the signals, recall that period-t welfare in this economy, (given a public signal y) is given by

$$\begin{aligned}
E\left[W_t(\mathbf{q}_t, \theta_t, y_t) \mid \theta_t\right] &= -\int_0^1 (q_t^i - \theta_t)^2 di \\
&= -\left[\int_0^1 h^2(y)\,di + \frac{\alpha+\delta(1-r)^2}{[\alpha+\delta(1-r)]^2} + \frac{\alpha\int_0^1 E(\eta|x_i, y)\,h(y)\,di}{\alpha+\delta(1-r)}\right] \\
&= -\left[\int_0^1 h^2(y)\,di + \frac{\alpha+\delta(1-r)^2}{[\alpha+\delta(1-r)]^2} + \frac{\alpha h(y)\int_0^1 E(\eta|x_i, y)\,di}{\alpha+\delta(1-r)}\right] \\
&= -h^2(y) - \frac{\alpha+\delta(1-r)^2}{[\alpha+\delta(1-r)]^2}. \quad (47)
\end{aligned}$$

In addition, recall from the previous section that, if the CB does not signal its information, period-t expected welfare in state θ is given by

$$E\left[W_t\left(q_t^i, \theta\right) \mid \theta\right] = -\sigma_\varepsilon^2. \quad (48)$$

Thus, it is optimal that the CB reveals its information if and only if

$$h^2(y) + \frac{\alpha+\delta(1-r)^2}{[\alpha+\delta(1-r)]^2} < \frac{1}{\delta}, \quad (49)$$

or after some manipulation if and only if

$$h^2(y) < \alpha \frac{\alpha + \delta(1-2r)}{\delta[\alpha + \delta(1-r)]^2} \equiv \kappa \tag{50}$$

Hence, (since $h(y) > 0$ for all y), we have the following.

Claim 4 *It is optimal for the central bank to reveal its information in period t if and only if $h(y) < \sqrt{\kappa}$.*

This finding suggests that it is possible for the central bank to design monetary policy so that the above equality holds for all y. In this case, it is always optimal for the CB to reveal information. Also, note that the higher the precision of the private information, the smaller the chance that the above inequality holds.

Next, suppose the CB finds it optimal to reveal its information. How does the period-t welfare change as the precision of the CB's information changes? Similarly, does smaller noise in the private signal necessarily lead to a higher period-t welfare?

Let us first consider the impact of changes in the precision of the private signal. We have

$$\begin{aligned} \frac{\partial E[W_t \mid \theta_t, y]}{\partial \delta} &= -\frac{(1-r)}{[\alpha + \delta(1-r)]^3}\left[(1-r)[\alpha + \delta(1-r)] - 2\left[\alpha + \delta(1-r)^2\right]\right] \\ &= -\frac{(1-r)}{[\alpha + \delta(1-r)]^3}\left[\alpha(1-r) + \delta(1-r)^2 - 2\alpha - 2\delta(1-r)^2\right] \\ &= -\frac{(1-r)}{[\alpha + \delta(1-r)]^3}\left[-\alpha(1+r) - \delta(1-r)^2\right] > 0 \end{aligned} \tag{51}$$

Thus, we have the following.

Claim 5 *Period-t welfare is always increasing in the precision of the private signal δ.*

Like in MS, an increase in the public signal's precision is beneficial only in some cases. More precisely, we have

$$\frac{\partial E[W_t \mid \theta_t, y]}{\partial \alpha} = -\frac{\alpha + \delta(1-r) - 2\left[\alpha + \delta(1-r)^2\right]}{[\alpha + \delta(1-r)]^3} = \frac{\alpha - \delta(2r-1)(1-r)}{[\alpha + \delta(1-r)]^3}. \tag{52}$$

Hence, an increase in the precision of the public signal is welfare improving if and only

$$\frac{\delta}{\alpha} \leq \frac{1}{(2r-1)(1-r)}. \tag{53}$$

Claim 6 *Period-t welfare is increasing in the precision of the public signal, α, if and only if* $\frac{\delta}{\alpha} \leq \frac{1}{(2r-1)(1-r)}$.

6 Signaling Precisions

Here we consider the case where the public signal precision itself is a random variable. In this case, the CB might want to use monetary policy in order to signal both the realized value of its signal and its confidence in the signal.

To study this issue, suppose that the precision of the public signal, α, can take on one of the two values: $\{\alpha_L, \alpha_H\}$, where $\alpha_L < \alpha_H$. The probability that $\alpha = \alpha_L$ is denoted by π. As before, the CB receives signal y about the value of θ. In addition, we assume that the CB knows the realization of α. The CB can choose to signal the value of y and α to the public through monetary policy. This can be accomplished via a rule that takes the following form:

$$\gamma(y,\alpha) = \beta + H(y,\alpha) = \beta + h(y,\alpha)\,\beta(1-r)/2, \tag{54}$$

with $H(y,\alpha) = 0$, if the CB does not reveal its information in state (y,α). The analysis from Section 5 continues to apply. More precisely,

$$q_i(\mathcal{I}_i) = \frac{\delta(1-r)x_i + \alpha y}{\delta(1-r)+\alpha} + h(y,\alpha). \tag{55}$$

Given $h(y,a)$, two cases are possible, as we showed in the previous section. First, it could be optimal not to reveal any information. This is true if the following inequality is satisfied for all y and both α_L and α_H:

$$\int_0^1 h^2(y,\alpha)\,di + \frac{\alpha + \delta(1-r)^2}{[\alpha+\delta(1-r)]^2} + \frac{\alpha}{\alpha+\delta(1-r)}\int_0^1 \eta h(y,\alpha)\,di \geq \frac{1}{\delta}. \tag{56}$$

Alternatively, it could be optimal to reveal the CB's information. This is true if

$$\int_0^1 h^2(y,\alpha)\,di + \frac{\alpha+\delta(1-r)^2}{[\alpha+\delta(1-r)]^2} + \frac{\alpha}{\alpha+\delta(1-r)}\int_0^1 \eta h(y,\alpha)\,di < \frac{1}{\delta} \tag{57}$$

holds for some y and α.

In what follows, we will only consider the case where the last inequality is satisfied for all pairs (y,α). Since the CB needs to signal both y and α to the public, it must be the case that $H(y,\alpha_L) \neq H(y',\alpha_H)$ for any y, y'. Since $H(y,\alpha)$ is strictly increasing given α, this inequality implies that $H(\infty,\alpha_j) < H(-\infty,\alpha_i)$ must hold for $i \neq j \in \{L,H\}$. One candidate policy for $i \neq j \in \{H,L\}$ is

$$h(y,\alpha) = \begin{cases} h(y) \text{ if } \alpha = \alpha_i \\ h(y) + \Delta \text{ if } \alpha = \alpha_j \end{cases}, \tag{58}$$

where $\Delta \geq h(\infty)$. For this policy, we have the following.

Proposition 7 *Suppose $h(y)$ is welfare maximizing when there is only one value for the precision of the public signal. Let*

$$h(y,\alpha) = \begin{cases} h(y) \text{ if } \alpha = \alpha_i \\ h(y) + \Delta \text{ if } \alpha = \alpha_j \end{cases}, \tag{59}$$

where $\Delta \geq h(\infty)$. Then, $h(y,\alpha)$ is also welfare maximizing.

Note that, whenever $\Delta > 0$, the monetary distortion will be larger if $\alpha = \alpha_j$ as compared to $\alpha = \alpha_i$, for the same realization of y. Therefore, the monetary policy rule that maximizes welfare has to take into account the relative frequency of the precision realizations. We state this in the next Proposition.

Proposition 8 *If $\pi > 1/2$, welfare is maximized if $h(y,\alpha_L) = h(y)$ and $h(y,\alpha_H) = h(y) + \Delta$. If $\pi \leq 1/2$, welfare is maximized if $h(y,\alpha_H) = h(y)$ and $h(y,\alpha_L) = h(y) + \Delta$.*

The last Proposition obtains an interesting interpretation in the context of monetary policy. Suppose that "most of the time" the precision of the CB's signal is low relative to that of the private sector; i.e., π is close to 1. Then, given signal y, a benevolent CB will find it optimal to choose $h(y, \alpha_L) = h(y)$. On the other hand, when the CB frequently receives a high quality signal (π is low), then it optimally adopts $h(y, \alpha_L) = h(y) + \Delta$. This interpretation implies that higher nominal rate hikes occur less frequently that lower ones, which is in line with actual observations.

7 Discussion

To be added.