Adverse Selection and the Role of Monetary Policy (preliminary)

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July 2007

Abstract

1. INTRODUCTION

We consider an environment where money is necessary to support exchange, and where an economic agents may be imperfectly informed about the future exchange opportunities of their trading partners. This leads to an inefficiency, relative to full information, and monetary policy can potentially alter this inefficiency. The role of monetary policy in this context depends critically on the ability of the central bank to intervene differentially across markets.

The basic structure of the model builds on Lagos and Wright (2005). Here, there is segmentation in centralized markets, and the price of goods in terms of money will in general differ across these markets. In the decentralized market, there is random bilateral matching and monetary exchange, and agents who meet will be privately informed concerning their centralized market location in the next period. Thus, there is asymmetric information concerning how trading partners value money. There are elements of the bargaining problem in the decentralized market that conform to the features of standard adverse selection environments, such as Maskin and Riley (1984). However, a key element of the problem is that cash constraints alter the outcomes, and in this way our analysis shares something with the work of Ennis (2007).

The model is also related to Lucas (1972). In Lucas's competitive environment, producers can be fooled by the central bank into producing more or less than what is optimal, as these producers have imperfect information about relative prices. In our model, buyers of goods are imperfectly informed concerning how sellers value the money offered in exchange for goods. This implies that contracts are distorted in order to induce self-selection, and these distortions will vary with monetary intervention by the central bank.

In general, prices will differ in equilibrium across the segmented centralized markets, and this creates a private-information inefficiency in decentralized trade. If the central bank can intervene in all centralized markets, then a Friedman rule equalizes prices across centralized markets and corrects the standard intertemporal monetary distortion, even if the central bank is constrained to making the same lump-sum money transfer to all agents. If there is financial trading (essentially a federal funds market) across centralized markets, then prices are equalized across markets and a Friedman rule is optimal, no matter who is on the receiving end of the central bank's lump-sum transfers.

The interesting case is the one where the central bank can intervene in only one centralized market. The central bank would like to intervene in such a way as to alter the distribution of money balances across centralized markets in each period, and to correct the intertemporal monetary distortion. However, the central bank cannot correct both the private information friction and the intertemporal monetary distortion simultaneously, and a Friedman rule is in general suboptimal.

2. THE MODEL

The basic structure of the model is derived from Lagos and Wright (2005), and we add some locational and informational frictions. Time is discrete and there is a continuum of agents with unit mass. Each agent is infinite-lived and maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - l_t],$$

where $\beta \in (0,1)$, c_t is consumption of the unique perishable consumption good, and l_t is labor supply. Assume that $u(\cdot)$ is twice continuously differentiable, strictly increasing, and strictly concave, with $u(0) = 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. Let q^* denote the solution to $u'(q^*) = 1$. Each agent possesses a technology which permits the production of one unit of the consumption good for each unit of labor supplied, and no agent can consume his or her own output.

In periods $t = 0, 2, 4, ...,$ agents are randomly allocated between two locations indexed by $i = 1, 2$. Let ρ denote the probability that an agent goes to location 1, and $1 - \rho$ the probability of going to location 2, where $0 < \rho < 1$. Goods and agents cannot be moved between the two locations. Exchange occurs competitively in even periods in each location. At the beginning of periods $t = 1, 3, 5, ...$, an agent learns whether he or she will be a buyer or a seller during the current period. For an agent who is in location i during period t, for t even, the probability of being a buyer in period $t + 1$ is α_i, and the probability of being a seller is $1 - \alpha_i$, where $0 < \alpha_i < 1$ for $i = 1, 2$. Assume that $\alpha_1 > \frac{1}{2}$, and that

$$\alpha_2 = \frac{1 - 2\alpha_1\rho}{2(1 - \rho)},$$

which guarantees that half the population consists of buyers (and the other half consists of sellers) during an odd period. We need to assume that

$$\alpha_1\rho < \frac{1}{2},$$

which assures that $\alpha_2 > 0$. Thus, agents in location 1 during an even period have a higher probability of being buyers during the next odd period than is the case for agents in location 2.

At the beginning of an odd period, each agent first learns whether he or she is a buyer or seller during the current period, and then each buyer is randomly matched with a seller. Trade is anonymous in these pairwise matches, so if exchange is to take place the seller must be willing to accept money for the consumption goods that he or she can produce. When an agent learns if he or she is a buyer or seller at the beginning of an odd period, he or she also learns what his or her location will be in the next even period, and this is assumed to be private information.

The setup of the model is illustrated in Figure 1.

CENTRALIZED EXCHANGE

Let $W_t^i(m)$ be the value function of an agent with m units of money at location i, for $t = 0, 2, 4, ...$, and let $V_t^i(m)$ be the value function of an agent with m units of money in decentralized market who resided in location i in period $t-1$ (before learning period t buyer/seller status and period $t+1$ location), for $t = 1, 3, 5, ...$ We then have

$$W_t^i(m) = \max_{\left(c_t, l_t^i, m_{t+1}^i\right) \in \mathbb{R}_+^3} \left[u(c_t) - l_t^i + \beta V_{t+1}^i\left(m_{t+1}^i\right) \right]$$

subject to

$$c_t + \phi_t^i m_{t+1}^i = l_t^i + \phi_t^i m + \phi_t^i \tau_t^i. \tag{1}$$

Here, ϕ_t^i is the value of money in units of consumption goods in location $i = 1, 2$, and τ_t^i is a lump-sum money transfer from the central bank which we allow at this stage to depend on the agent's location. Suppose there is an interior solution for c_t and l_t^i in every even period. Then, for each $i = 1, 2$, we have

$$W_t^i(m) = \phi_t^i m + W_t^i(0) \tag{2}$$

where

$$W_t^i(0) = u(q^*) - q^* + \phi_t^i \tau_t^i + \max_{m_{t+1}^i \in \mathbb{R}_+} \left[-\phi_t^i m_{t+1}^i + \beta V_{t+1}^i\left(m_{t+1}^i\right) \right] \tag{3}$$

Note from (2) that, as in Lagos and Wright (2005), the value function $W_t^i(m)$ is linear in m. Further, since agents are randomly allocated to locations 1 and 2 in even periods, the per capita stock of money must always be the same in each location in even periods. Ultimately we will show that, as in Lagos and Wright (2005), all agents (regardless of location) choose to hold the same quantity of nominal money balances at the end of any even period.

DECENTRALIZED EXCHANGE

There will be four kinds of meetings that can occur between buyers and sellers during an odd period t, which we will index by (i, j), where $i, j = 1, 2$, with i denoting the type of the buyer, and j the type of the seller. Here, "type" refers to the period $t+1$ location of the agent. Let q_t^{ij} denote the quantity of goods provided by a type j seller to a type i buyer, in exchange for d_t^{ij} units of money. In a meeting between a buyer and a seller, let the buyer have m units of money, and assume that he or she makes a take-it-or-leave-it offer to the seller. Recall that type is private information, so the buyer does not know the location that the seller will take up next period. This information is critical, as the agent's location in the next centralized market will tell the buyer how the seller values the money that the buyer offers in exchange for goods.

The problem that the buyer faces when meeting a seller is much like the problem of a monopolist selling goods to heterogeneous buyers whose types are private information, as captured for example in the adverse selection model of Maskin and Riley (1984). A key difference in this problem, however, is that the money balances held by the buyer potentially constrain the array of contracts that can be offered to the seller.

Now, consider the problem faced by a buyer of type i. In general, this buyer will offer a choice of two contracts to the seller, (q_t^{i1}, d_t^{i1}) and (q_t^{i2}, d_t^{i2}), intended respectively for buyers of types 1 and 2. The utility received by the buyer from an accepted contract is $u(q_t^{ij}) - \beta\phi_{t+1}^i d_t^{ij}$, given (2). Buyer i then chooses the two contracts to maximize

$$\rho[u(q_t^{i1}) - \beta\phi_{t+1}^i d_t^{i1}] + (1-\rho)[u(q_t^{i2}) - \beta\phi_{t+1}^i d_t^{i2}]. \tag{4}$$

Each contract must be *individually rational* for each type of seller, or

$$-q_t^{ij} + \beta\phi_{t+1}^j d_t^{ij} \geq 0, \text{ for } j = 1, 2, \tag{5}$$

and each contract must be *incentive compatible* for each type of seller, or

$$-q_t^{ij} + \beta\phi_{t+1}^j d_t^{ij} \geq -q_t^{ik} + \beta\phi_{t+1}^j d_t^{ik}, \text{ for } j = 1, 2 \text{ and } k \neq j. \tag{6}$$

Further, the quantities of money that can be offered in exchange to each type of seller cannot exceed m, that is the *cash constraints*

$$d_t^{ij} \leq m, \text{ for } j = 1, 2, \tag{7}$$

must hold.

Now, conjecture that

$$\phi_{t+1}^1 > \phi_{t+1}^2, \tag{8}$$

which we will later show holds in equilibrium. We can then characterize the optimal contracts offered by a type i buyer with the following lemmas.

Lemma 1 *The optimal contract offered by a type i buyer to a type 2 seller yields zero surplus to the seller. That is, the individual rationality constraint holds with equality for the type 2 seller, or*

$$-q_t^{i2} + \beta\phi_{t+1}^2 d_t^{i2} = 0 \tag{9}$$

Proof. Suppose $-q_t^{i2} + \beta\phi_{t+1}^2 d_t^{i2} > 0$ at the optimum. Then, from (6) and (8), we have

$$\beta\phi_{t+1}^1 d_t^{i1} - q_t^{i1} \geq \beta\phi_{t+1}^1 d_t^{i2} - q_t^{i2} > \beta\phi_{t+1}^2 d_t^{i2} - q_t^{i2} > 0$$

so that the optimal contracts offered by the buyer to each seller give both sellers strictly positive surplus. This implies that both d_t^{i1} and d_t^{i2} can be reduced, holding constant q_t^{ij}, $j = 1, 2$, in such a way that constraints (5)-(7) continue to hold, while increasing the value of the objective function in (4). Thus the contracts are not optimal, a contradiction. ■

Lemma 2 *The incentive constraint for the type 1 seller binds at the optimum. That is,*

$$-q_t^{i1} + \beta\phi_{t+1}^1 d_t^{i1} = -q_t^{i2} + \beta\phi_{t+1}^1 d_t^{i2} \tag{10}$$

Proof. Suppose $-q_t^{i1} + \beta\phi_{t+1}^1 d_t^{i1} > -q_t^{i2} + \beta\phi_{t+1}^1 d_t^{i2}$ at the optimum. Then, given (8), we have

$$\beta\phi_{t+1}^1 d_t^{i1} - q_t^{i1} > 0,$$

which implies that d_t^{i1} can be reduced in such a way that the constraints (5)-(7) continue to hold, while increasing the value of the objective function in (4). Thus the contracts are not optimal, a contradiction. ■

Lemma 3 *The optimal contract offered to the type* 1 *seller gives the seller strictly positive surplus. That is, the individual rationality constraint for the type* 1 *seller holds as a strict inequality, or*

$$-q_t^{i1} + \beta\phi_{t+1}^1 d_t^{i1} > 0. \tag{11}$$

Proof. From (10), (8), and (9) we get

$$-q_t^{i1} + \beta\phi_{t+1}^1 d_t^{i1} = -q_t^{i2} + \beta\phi_{t+1}^1 d_t^{i2} > -q_t^{i2} + \beta\phi_{t+1}^2 d_t^{i2} = 0$$

■

Lemma 4 *At the optimum, the type* 1 *seller supplies more goods and receives more money in exchange than does the type* 2 *seller. That is, $q_t^{i1} \geq q_t^{i2}$ and $d_t^{i1} \geq d_t^{i2}$ at the optimum, and $q_t^{i1} > q_t^{i2}$ if and only if $d_t^{i1} > d_t^{i2}$.*

Proof. Adding the two incentive constraints, i.e. constraint (6) for $(j,k) = (1,2)$, $(2,1)$, we obtain

$$\beta(\phi_{t+1}^1 - \phi_{t+1}^2)(d_t^{i1} - d_t^{i2}) \geq \beta(\phi_{t+1}^1 - \phi_{t+1}^2)(d_t^{i2} - d_t^{i1}),$$

which, given (8) implies $d_t^{i1} \geq d_t^{i2}$. Then, it is immediate from equation (10) that $q_t^{i1} \geq q_t^{i2}$, and that $q_t^{i1} > q_t^{i2}$ if and only if $d_t^{i1} > d_t^{i2}$. ■

Thus, in spite of the cash constraints (7) that make this problem different from standard adverse selection problems in the literature, from lemmas 1-4 the solution will have some standard properties. The type 2 seller, who has a low value of money in the following period, receives zero surplus from the contract offered by the buyer, while the type 1 seller, who has a high value of money, receives strictly positive surplus. The incentive constraint binds for the type 1 seller, and larger quantities are exchanged between the buyer and a type 1 seller than between the buyer and a type 2 seller. These features allow us to solve the optimal contracting problem (4) subject to (5)-(7) in a more straightforward way. In particular, substitute in the objective function in (4) and in the cash constraints (7) for d_t^{i1} and d_t^{i2} using (9) and (10), and then solve the problem as

$$\max_{q_t^{i1},q_t^{i2}} \rho \left[u(q_t^{i1}) - \frac{\phi_{t+1}^i q_t^{i1}}{\phi_{t+1}^1} - \left(\frac{\phi_{t+1}^i}{\phi_{t+1}^2} - \frac{\phi_{t+1}^i}{\phi_{t+1}^1} \right) q_t^{i2} \right] + (1-\rho) \left[u(q_t^{i2}) - \frac{\phi_{t+1}^i q_t^{i2}}{\phi_{t+1}^2} \right] \tag{12}$$

subject to the cash constraints

$$q_t^{i1} + q_t^{i2} \left(\frac{\phi_{t+1}^1}{\phi_{t+1}^2} - 1 \right) \leq \beta \phi_{t+1}^1 m \tag{13}$$

$$q_t^{i2} \leq \beta \phi_{t+1}^2 m \tag{14}$$

From the proof of Lemma 3, since we have imposed (9) and (10), therefore both individual rationality constraints hold, and we need only check that the second incentive constraint, (6) for $(j,k) = (2,1)$, holds. In turn, from the proof of Lemma 4, we then only need to check that the solution has the property $q_t^{i1} \geq q_t^{i2}$.

Case 1: Cash Constraints Bind for Both Contracts

In this case the two contracts that the buyer offers the seller are both constraint by the quantity of money m that the buyer possesses. That is, (13) and (14) both hold with equality. Solving for q_t^{i1} and q_t^{i2} from (13) and (14) we obtain

$$q_t^{i1} = q_t^{i2} = \beta \phi_{t+1}^2 m, \tag{15}$$

and so, since the buyer gives up all his or her money balances irrespective of the seller's type, the payoff to the buyer as a function of m is

$$\psi_t^{i1}(m) = u(\beta\phi_{t+1}^2 m) \tag{16}$$

Thus, in this case the buyer is constrained to offering the same contract to each type of seller, and the type 1 seller who values money highly extracts some surplus from the buyer.

In Figure 2, we show the equilibrium contract in Case 1. Note that both equilibrium contracts involve a distortion from full-information quantities. In this case, the buyer has sufficiently low money balances that it is inefficient for him or her to induce the seller to reveal his or her type.

Case 2: Cash Constraint Binds Only for the Type 1 Seller

Recall from Lemma 4 that $d_t^{i1} \geq d_t^{i2}$ at the optimum, so if one cash constraint binds, it must be the one for the type 1 seller. Thus, substituting for q_t^{i1} in the (12) using (13) with equality, in case 2 we can write the buyer's optimization problem as

$$\max_{q_t^{i2}} \rho \left\{ u\left[-q_t^{i2}\left(\frac{\phi_{t+1}^1}{\phi_{t+1}^2} - 1\right) + \beta\phi_{t+1}^1 m\right] - \beta\phi_{t+1}^i m\right\} + (1-\rho)\left[u(q_t^{i2}) - \frac{\phi_{t+1}^i q_t^{i2}}{\phi_{t+1}^2}\right] \tag{17}$$

subject to (14). The first-order condition for an optimum is then

$$-\rho\left(\frac{\phi_{t+1}^1}{\phi_{t+1}^2} - 1\right) u'\left[-q_t^{i2}\left(\frac{\phi_{t+1}^1}{\phi_{t+1}^2} - 1\right) + \beta\phi_{t+1}^1 m\right] + (1-\rho)\left[u'(q_t^{i2}) - \frac{\phi_{t+1}^i}{\phi_{t+1}^2}\right] = 0. \tag{18}$$

Now, let $\varphi(q_t^{i2}, m)$ denote the function on the left-hand side of (18).

Proposition 5 *There is a unique $q_t^*(m)$ that solves $\varphi(q_t^*(m), m) = 0$, with $0 < q_t^*(m) < (\beta\phi_{t+1}^1 m)/\left(\frac{\phi_{t+1}^1}{\phi_{t+1}^2} - 1\right)$.*

Proof. Nonnegativity of consumption for the buyer implies that

$$0 \leq q_t^{i2} \leq (\beta\phi_{t+1}^1 m)/\left(\frac{\phi_{t+1}^1}{\phi_{t+1}^2} - 1\right).$$

Given (8), and the strict concavity of $u(\cdot)$, $\varphi(q_t^{i2}, m)$ is strictly decreasing in q_t^{i2} on $\left(0, (\beta\phi_{t+1}^1 m)/\left[\frac{\phi_{t+1}^1}{\phi_{t+1}^2} - 1\right]\right)$ for fixed $m > 0$. Further, $\lim_{q\to 0}\varphi(q, m) = \infty$, and $\lim_{q\to(\beta\phi_{t+1}^1 m)/\left(\frac{\phi_{t+1}^1}{\phi_{t+1}^2}-1\right)} \varphi(q, m) = -\infty$. ■

Proposition 6 *The solution $q_t^*(m)$ satisfies the cash constraint (14) if and only if $\varphi(\beta\phi_{t+1}^2 m, m) \leq 0$.*

Proof. Since $\varphi(q_t^{i2}, m)$ is strictly decreasing in q_t^{i2} and $\varphi(q^*(m), m) = 0$, therefore $q^*(m) \leq \beta\phi_{t+1}^2 m$ iff $\varphi(\beta\phi_{t+1}^2 m, m) \leq 0$. ■

Further, since at the case 2 optimum the quantity of money exchanged with the type 2 seller cannot exceed the quantity exchanged with the type 1 seller, from (10) we must have $q_t^{i1} \geq q_t^{i2}$, and so the incentive constraint for the type 2 seller is satisfied.

This last proposition gives us a necessary restriction on m for the optimum to have case 2 characteristics. That is, from (18), $\varphi(\beta\phi_{t+1}^2 m, m) \leq 0$ gives

$$\left(1 - \rho\frac{\phi_{t+1}^1}{\phi_{t+1}^2}\right) u'(\beta\phi_{t+1}^2 m) - (1-\rho)\frac{\phi_{t+1}^i}{\phi_{t+1}^2} \leq 0 \tag{19}$$

Now, assume for now (we will later establish conditions which guarantee that this holds) that

$$1 - \rho\frac{\phi_{t+1}^1}{\phi_{t+1}^2} > 0, \tag{20}$$

and let $\omega(m)$ denote the function on the left-hand side of inequality (19). Note that $\omega(m)$ is strictly decreasing and continuous in m with $\omega(0) = \infty$ and $\omega(m) < 0$ for m sufficiently large. Therefore, there is some $m_1 > 0$ such that $\omega(m_1) = 0$, $\omega(m) < 0$ for $m > m_1$ and $\omega(m) > 0$ for $m < m_1$. Therefore, if the optimum is case 2, then it is necessary that $m \geq m_t^{i1}$, where m_t^{i1} is the solution to

$$\left(1 - \rho\frac{\phi_{t+1}^1}{\phi_{t+1}^2}\right) u'(\beta\phi_{t+1}^2 m_t^{i1}) - (1-\rho)\frac{\phi_{t+1}^i}{\phi_{t+1}^2} = 0. \tag{21}$$

Finally, since when we have a case 2 optimum, the buyer gives up all of his or her cash balances to a type 1 seller and only some of his or her cash balances to a type 2 seller, the expected payoff to the buyer as a function of m is

$$\begin{aligned}\psi_t^{i2}(m) &= \rho u\left[-q_t^*(m)\left(\frac{\phi_{t+1}^1}{\phi_{t+1}^2}-1\right)+\beta\phi_{t+1}^1 m\right] \\ &\quad +(1-\rho)\left[u\left[q_t^*(m)\right]+\beta\phi_{t+1}^i\left(m-\frac{q_t^*(m)}{\beta\phi_{t+1}^2}\right)\right]\end{aligned} \tag{22}$$

We illustrate the equilibrium contracts in Figure 3. Here, note that the binding cash constraint implies that the contracts for both types are distorted from what would be achieved with full information. Relative to Case 1, the buyer has enough cash that he or she optimizes by inducing self-selection by the seller, but has insufficient cash to offer a non-distorted contract to the type 1 seller.

Case 3: Neither Cash Constraint Binds

In this case q_t^{i1} and q_t^{i2} are chosen by the buyer to solve (12) ignoring the cash constraints. The first-order conditions characterizing an optimum are

$$u'(q_t^{i1})-\frac{\phi_{t+1}^i}{\phi_{t+1}^1}=0, \tag{23}$$

and

$$(1-\rho)u'(q_t^{i2})-\left(\frac{\phi_{t+1}^i}{\phi_{t+1}^2}-\rho\frac{\phi_{t+1}^i}{\phi_{t+1}^1}\right)=0. \tag{24}$$

Now, let $\bar{q}_t^{i1}$ and $\bar{q}_t^{i2}$ denote the solutions to equations (23) and (24), respectively. First, note that (8) implies that $\bar{q}_t^{i1} > \bar{q}_t^{i2}$, which implies that the incentive compatibility constraint for the type 2 seller is satisfied. Further, note that $\bar{q}_t^{i1}$ would be the quantity traded in a full information contract between the buyer and a type 1 seller, unconstrained by the buyer's cash holdings. As well, given (8) $\bar{q}_t^{i2}$ is smaller than the quantity traded with a full information contract between the buyer and a type 2 seller, again unconstrained by the buyer's cash holdings. This is a standard feature of adverse selection models with two types, whereby the type 2 contract is distorted from what it would be with full information, so as to induce the type 1 seller to self-select.

The next step is to establish conditions on m that guarantee that there is a case 3 optimum. That is, we want m to be sufficiently large that neither cash constraint binds. Since $\bar{q}_t^{i1} > \bar{q}_t^{i2}$, a larger quantity of cash is traded in the type 1 contract, so if the cash constraint does not bind for the type 1 contract it will not bind for the other contract. Therefore, neither cash constraint binds if and only if, from (13),

$$m \geq \frac{\bar{q}_t^{i1}}{\beta\phi_{t+1}^1} + \bar{q}_t^{i2}\left(\frac{1}{\beta\phi_{t+1}^2} - \frac{1}{\beta\phi_{t+1}^1}\right), \tag{25}$$

and we let m_t^{i2} denote the quantity on the right-hand side of (25).

The payoff to the buyer if there is a case 3 optimum is

$$\begin{aligned} \psi_t^{i3}(m) \quad = \quad & \rho\left\{u\left(\bar{q}_t^{i1}\right) + \beta\phi_{t+1}^i\left[m - \frac{\bar{q}_t^{i1}}{\beta\phi_{t+1}^1} - \bar{q}_t^{i2}\left(\frac{1}{\beta\phi_{t+1}^2} - \frac{1}{\beta\phi_{t+1}^1}\right)\right]\right\} \\ & +(1-\rho)\left[u(\bar{q}_t^{i2}) + \beta\phi_{t+1}^i\left(m - \frac{\bar{q}_t^{i2}}{\beta\phi_{t+1}^2}\right)\right] \end{aligned} \tag{26}$$

In Figure 4, we show the equilibrium contracts in Case 3. Here, as cash constraints do not bind, the type 1 seller receives a contract that is not distorted, but the type 2 contract is distorted to induce self-selection, just as in Maskin and Riley (1984). In Figure 5, we show how contracts differ across the three cases. Note that, as the money held by the seller declines, the surplus received by the type 1 seller falls, and the distortion in each contract rises.

Odd-Period Value Functions

Now that we know the payoffs to the buyer as a function of the buyer's cash balances m, and the constraints on m that are necessary to obtain the cases 1-3 above, we can proceed to construct the value functions $V_t^i(m)$, for $i = 1, 2$. Recall that $V_t^i(m)$ gives the value of money at the beginning of period t (before learning buyer/seller status) of money balances m to an agent who resided in location i in period $t-1$, where t is an odd period.

It is straightforward to show that, given (8), $m_t^{i1} < m_t^{i2}$ for $i = 1, 2$. Then, since a necessary condition for a case 2 optimum is that $m \geq m_t^{i1}$, and a necessary condition for a case 3 optimum is $m \geq m_t^{i2}$, we will have a case 1 optimum when $0 \leq m \leq m_t^{i1}$, a case 2 optimum when $m_t^{i1} \leq m \leq m_t^{i2}$, and a case 3 optimum when $m \geq m_t^{i2}$. Above, we calculated the payoffs to a buyer as a function of m in the three different cases. For a seller's payoff, note that seller does not give up any money balances no matter who he or she meets in the decentralized market, and the surplus received by the seller is independent of his or her money holdings. Therefore, we can write the odd-period value function as

$$V_t^i(m) = \alpha_i v_t(m) + (1-\alpha_i)\left\{\beta m[\rho\phi_{t+1}^1 + (1-\rho)\phi_{t+1}^2] + \sigma\right\}, \tag{27}$$

where

$$v_t(m) = \rho \sum_{k=1}^{3} I_t^{1k}(m)\psi_t^{1k}(m) + (1-\rho)\sum_{k=1}^{3} I_t^{2k}(m)\psi_t^{2k}(m) \tag{28}$$

In (27), σ is a constant, and in (28) the indicator functions $I_t^{ik}(m)$, for $k = 1, 2, 3$, are defined by

$$I_t^{i1}(m) = 1 \text{ if } 0 \leq m \leq m_t^{i1};\; I_t^{i1}(m) = 0 \text{ otherwise.}$$

$$I_t^{i2}(m) = 1 \text{ if } m_t^{i1} \leq m \leq m_t^{i2};\; I_t^{i2}(m) = 0 \text{ otherwise.}$$

$$I_t^{i3}(m) = 1 \text{ if } m \geq m_t^{i2};\; I_t^{i3}(m) = 0 \text{ otherwise.}$$

Proposition 7 *The function $v_t(m)$ is continuously differentiable for $m \geq 0$, concave for $m \geq 0$, and strictly concave for $0 \leq m < m_t^{22}$.*

Proof. Note that $v_t(\cdot)$ is clearly continuously differentiable at every point $m \geq 0$ except possibly at the critical points m_t^{i1}, m_t^{i2}, for $i = 1, 2$. It remains to show that $v_t(\cdot)$ is continuously differentiable at these points. Observe that

$$\frac{d\psi_t^{i1}}{dm} \to \beta\phi_{t+1}^2 u'\left(\beta\phi_{t+1}^2 m_t^{i1}\right)$$

as $m \to m_t^{i1}$ from below. On the other hand, using (18) and (21), we find that

$$\frac{d\psi_t^{i2}}{dm} \to \beta\phi_{t+1}^2 u'\left(\beta\phi_{t+1}^2 m_t^{i1}\right)$$

as $m \to m_t^{i1}$ from above. Therefore, we conclude that $v_t(\cdot)$ is continuously differentiable at m_t^{i1}. Consider now the critical point m_t^{i2}. As $m \to m_t^{i2}$ from below, we have

$$\frac{d\psi_t^{i2}}{dm} \to \beta\phi_{t+1}^i$$

where we have used (23). For any $m > m_t^{i2}$, it follows that

$$\frac{d\psi_t^{i3}(m)}{dm} = \beta\phi_{t+1}^i$$

so that we conclude that $v_t(\cdot)$ is continuously differentiable at m_t^{i2}.

To show that $v_t(\cdot)$ is concave, define $h_{i,t}(m) = \sum_{k=1}^{3} I_t^{ik}(m)\,\psi_t^{ik}(m)$, for $i = 1, 2$. We have the following: $h''_{i,t}(\cdot) < 0$ for any $m \in (0, m_t^{i1}) \cup (m_t^{i1}, m_t^{i2})$; $h''_{i,t}(\cdot) = 0$ for any $m > m_t^{i2}$; and $h'_{i,t}(\cdot)$ is continuous at m_t^{i1} and m_t^{i2}. This implies that $h_{i,t}(\cdot)$ is concave for $m \geq 0$ and strictly concave for $0 \leq m < m_t^{i2}$. ■

We illustrate the value function in Figure 6.

This proposition then implies that, from (3), and similarly to Lagos and Wright (2005), it is optimal for each agent in a given location in an even period to hold the same quantity of money at the end of the period. Since our assumptions guarantee that the quantity of money per capita is the same in each location in an even period, each agent in the economy holds the same quantity of money at the end of an even period.

CENTRAL BANK INTERVENTION IN BOTH CENTRALIZED MARKETS

Suppose that the central bank can make lump-sum transfers, but that these transfers are constrained to be the same in each location in a given period, as well as being identical across agents in a given location. This constraint could arise if, for example, the transfers are made electronically, an agent's location is private information, and the central bank has no memory of an agent's past transfers. Further, for simplicity assume that the money stock grows at a constant rate from one even period to the next. That is, let M_t denote the aggregate money stock during an even period t, where

$$M_{t+2} = \mu^2 M_t,$$

for $t = 0, 2, 4, ...$, with M_0 normalized to unity and $\mu > 0$. Note that there are no money transfers in odd periods while agents are engaged in decentralized exchange. The money transfer that each agent receives in an even period t is then

$$\tau_t^1 = \tau_t^2 = (\mu^2 - 1)M_{t-2}.$$

Now, confine attention to stationary equilibria having the property that $\phi_t^i = \frac{\phi^i}{\mu^t}$, for $i = 1, 2$, where ϕ^i is a constant for $i = 1, 2$. From (3) and (27), the following first-order conditions must be satisfied for each $t = 0, 2, 4, ...$,

$$\frac{\phi^i}{\mu^t} = \beta \left\{ \alpha_i v'_{t+1}(m_{t+1}^i) + \frac{(1-\alpha_i)\beta[\rho\phi^1 + (1-\rho)\phi^2]}{\mu^{t+2}} \right\}, \text{ for } i = 1, 2. \quad (29)$$

Then, imposing the equilibrium condition that $m_{t+1}^i = M_t = \mu^t$ for $i = 1, 2$, and rearranging, we get

$$1 = \frac{\alpha_i \beta \mu^t v'_{t+1}(\mu^t)}{\phi^i} + (1-\alpha_i)\frac{\beta^2}{\mu^2}\frac{[\rho\phi^1 + (1-\rho)\phi^2]}{\phi^i}, \text{ for } i = 1, 2. \quad (30)$$

Proposition 8 *If $\mu > \beta$, then $\phi^1 > \phi^2$ in a stationary equilibrium.*

Proof. If $\mu > \beta$, we must have $m_{t+1}^i < m_{t+1}^{22}$ for each $i = 1, 2$. To see this, note that, as $m \to m_{t+1}^{22}$ from below,

$$-\phi_t^i + \beta \frac{dV_{t+1}^i}{dm} \to \frac{1}{\mu^t}\left\{ -\phi^i + \frac{\beta^2}{\mu^2}\left[\rho\phi^1 + (1-\rho)\phi^2\right] \right\}$$

where we have used (23). When $\mu > \beta$, we have

$$\phi^i > \frac{\beta^2}{\mu^2}\left[\rho\phi^1 + (1-\rho)\phi^2\right]$$

for each $i = 1, 2$ in a stationary equilibrium. Therefore, the optimal choice of money balances in location i in an even period t is such that $m^i_{t+1} < m^{22}_{t+1}$ for each $i = 1, 2$. This means that at least one cash constraint must bind when $\mu > \beta$.

Notice that $v'_{t+1}(\cdot)$ is a decreasing function and that

$$v'_{t+1}(m) \geq \frac{\beta}{\mu^{t+2}}\left[\rho\phi^1 + (1-\rho)\phi^2\right]$$

for all $m \geq 0$. In fact, it holds with strict inequality when $m < m^{22}_{t+1}$. Therefore, whenever $\mu > \beta$, we have that

$$\beta\mu^t v'_{t+1}(m^i_{t+1}) > \frac{\beta^2}{\mu^2}\left[\rho\phi^1 + (1-\rho)\phi^2\right]$$

for each $i = 1, 2$. Since $\alpha_1 > \alpha_2$ and $m^i_{t+1} = \mu^t$ in equilibrium, it follows that

$$\begin{aligned}
& \alpha_1\beta\mu^t v'_{t+1}(\mu^t) + (1-\alpha_1)\frac{\beta^2}{\mu^2}\left[\rho\phi^1 + (1-\rho)\phi^2\right] \\
> \quad & \alpha_2\beta\mu^t v'_{t+1}(\mu^t) + (1-\alpha_2)\frac{\beta^2}{\mu^2}\left[\rho\phi^1 + (1-\rho)\phi^2\right]
\end{aligned}$$

so that $\phi^1 > \phi^2$ in a stationary equilibrium as claimed. ■

If $\mu > \beta$, this implies that some cash constraint must bind in equilibrium, and that a buyer faces a higher marginal payoff to holding money than does a seller in the decentralized market. Since an agent in location 1 in an even period has a higher probability of being a buyer in the next decentralized market, this agent then must have a higher expected marginal payoff to holding money in an even period. Since the quantities of money per capita are identical in the two locations in an even period, money must have a higher value in location 1 than in location 2 in equilibrium.

Thus, when the rate of money growth is larger than the discount rate, prices are different in the two locations, and we know that this induces a private information friction in monetary exchange in this model. That is, there is a friction here, in addition to what would occur with full information, due to the fact that a seller with a high value of money can extract some surplus from the buyer because the buyer needs to induce self-selection.

Proposition 9 *$\mu = \beta$ yields an optimal equilibrium allocation.*

Proof. As $\mu \to \beta$ from above, it follows that

$$\beta\mu^t v'_{t+1}(\mu^t) \to \rho\phi^1 + (1-\rho)\phi^2$$

From (30), it follows that

$$\phi^i = \rho\phi^1 + (1-\rho)\phi^2$$

for each $i = 1, 2$, which holds if and only if $\phi^1 = \phi^2 = \phi$. Then, as $\mu \to \beta$ from above, we have

$$\frac{\beta\mu^t v'_{t+1}(\mu^t)}{\phi} \to 1 = u'\left(q^*\right)$$

so that agents in both locations acquire enough money in order to get q^* in the next decentralized market in case they are buyers in a bilateral match. ■

Under a Friedman rule, all cash constraints are relaxed, and there is a stationary equilibrium where $\phi^1 = \phi^2$ so that prices are equalized in the two locations in even periods. The private information friction is eliminated and the economy collapses to essentially the same allocation studied by Lagos and Wright (2005), for the special case where buyers have all the bargaining power. The efficient quantity of output is produced and consumed in every bilateral match in the decentralized market.

FINANCIAL MARKET TRADE BETWEEN LOCATIONS

We have assumed that, in even periods, there is no trade between agents in location 1 and those in location 2. Here, we will continue to assume that neither goods or people can move across the two locations. However, we will permit a bond market in even periods where agents in the two locations can exchange outside money (say, in electronic form) for claims to money in the next even period. This of course requires that a bond issuer in period t can be found in period $t+2$ and that the financial claim can be enforced.

Assume a market in an even period t for two-period bonds, each of which sells for one unit of money and is a claim to R_{t+2} units of money in period $t+2$. We can then rewrite the budget constraint (1) of an agent in location i in an even period as

$$c_t + \phi_t^i m_{t+1}^i + \phi_t^i b_{t+2} = l_t^i + \phi_t^i m + \phi_t^i R_t b_t + \phi_t^i \tau_t^i, \tag{31}$$

where b_t denotes the quantity of bonds acquired by the agent that mature in period t. Given quasilinear utility, equilibrium requires that each agent in each location be indifferent about the bond holdings in any even period t, or

$$\phi_t^i = \beta^2 R_{t+2}\left[\rho\phi_{t+2}^1 + (1-\rho)\phi_{t+2}^2\right] \text{ for } i = 1, 2. \tag{32}$$

But these two conditions clearly imply that $\phi_t^1 = \phi_t^2$ in equilibrium, so that prices are equalized across the two locations. This economy then collapses to a basic Lagos-Wright structure with take-it-or-leave-it offers by buyers, and with no private information friction.

Now, if the aggregate money stock grows at a constant rate in a stationary equilibrium, as in the previous section, then (29) must hold, but now $\phi^1 = \phi^2 = \phi$ in equilibrium, and now the stocks of money in each location are endogenous. That is, in a stationary equilibrium, the quantity of money in location i is $M^i\mu^t$ in period t even, where from (30) and the equilibrium condition $\rho M^1 + (1-\rho)M^2 = 1$, we obtain

$$\alpha_1\beta\mu^t v_{t+1}'(\mu^t M^1) + (1-\alpha_1)\frac{\beta^2}{\mu^2}\phi = \alpha_2\beta\mu^t v_{t+1}'\left[\mu^t\frac{1-\rho M^1}{1-\rho}\right] + (1-\alpha_2)\frac{\beta^2}{\mu^2}\phi, \tag{33}$$

which solves for M^1, giving us the equilibrium distribution of money balances between locations 1 and 2.

Proposition 10 *If $\mu > \beta$, then $M^1 > M^2$ in equilibrium, and the equilibrium allocation is inefficient.*

Proof. If $\mu > \beta$, we have that $m^i_{t+1} < \mu^{t+2}q^* / (\beta\phi)$ and

$$\frac{\beta\mu^t v'_{t+1}(m^i_{t+1})}{\phi} > \frac{\beta^2}{\mu^2}$$

for each $i = 1, 2$. Since $\alpha_1 > \alpha_2$ and $m^i_{t+1} = M^i\mu^t$ for each $i = 1, 2$ in equilibrium, it follows from (33) that

$$v'_{t+1}(M^1\mu^t) < v'_{t+1}(M^2\mu^t)$$

Since $v'_{t+1}(\cdot)$ is strictly decreasing for $0 \leq m < \mu^{t+2}q^* / (\beta\phi)$, we have that $M^1 > M^2$. The fact that $\mu > \beta$ implies that it is not optimal for agents in each location to take enough money to the next decentralized market in order to get q^* in case they are buyers in a bilateral match. ■

Proposition 11 *If $\mu = \beta$ there is an optimal equilibrium allocation.*

Proof. When $\mu \to \beta$ from above, we have

$$\frac{\beta\mu^t v'_{t+1}(\mu^t M^i)}{\phi} \to 1 = u'(q^*)$$

for each $i = 1, 2$. This implies the efficient quantity is traded in each bilateral match in the decentralized market. ■

Just as in the previous section, a Friedman rule is optimal here, but trading in this cross-location bond market serves to equalize prices in the two locations by moving money balances to where they would otherwise have a higher value. Thus, there is no private information friction, even when money growth is higher than the Friedman rule rate. The bond market plays a role much like the federal funds market in the United States, except that in our model we have assumed that all economic agents have access to this market. Note that, given trading on the bond market, it is irrelevant what market the central bank intervenes in. Agents could receive money transfers from the central bank in location 1, location 2, or both locations, but the actions of the central bank can have no effect on the end-of-period distribution of money balances between locations 1 and 2 in an even period.

NO INTER-LOCATION TRADE, AND CENTRAL BANK INTERVENTION IN ONLY ONE LOCATION

In this section we will set up the environment so that the central bank will have more difficulty in dealing with the distortions arising from monetary exchange. In particular, assume that there is no trade between locations during an even period, and that the central bank can intervene at only one location, through lump-sum money transfers.

Central Bank Intervention Confined to Location 1

Here, let M_t^i denote the even-period t per-capita money stock at location i. Given that the central bank intervenes only at location 1, M_t^1 can be treated as exogenous, and we will have

$$M_{t+2}^2 = \rho M_t^1 + (1-\rho)M_t^2,\ t = 0, 2, 4, ... \tag{34}$$

Now, consider monetary policies such that $M_{t+2}^i = \mu^2 M_t^i$ for $t = 0, 2, 4, ...$, with $\frac{M_t^1}{M_t^2} = \delta$, where from (34), we have

$$\delta = \frac{\mu^2 - 1 + \rho}{\rho} \tag{35}$$

As should be clear, equation (35) reflects the fact that the central bank cannot independently determine the money growth rate and the distribution of money balances across the two locations.

Normalize M_0^1 to unity. Then, a stationary equilibrium is determined in a manner similar to the previous two sections, with ϕ^i for $i = 1, 2$ determined by

$$1 = \frac{\alpha_1 \beta \mu^t v'_{t+1}(\mu^t)}{\phi^1} + (1 - \alpha_1) \frac{\beta^2}{\mu^2} \frac{[\rho\phi^1 + (1-\rho)\phi^2]}{\phi^1}$$

and

$$1 = \frac{\alpha_2 \beta \mu^t v'_{t+1}\left(\mu^t \frac{\rho}{\mu^2 - 1 + \rho}\right)}{\phi^2} + (1 - \alpha_2) \frac{\beta^2}{\mu^2} \frac{[\rho\phi^1 + (1-\rho)\phi^2]}{\phi^2}$$

Here, prices can be equalized across locations in each period for some $\mu > 1$. This eliminates the private information friction, but there is a welfare loss from inflation. Note that a Friedman rule with equalized prices presumably does not work since this implies that agents have to indefinitely postpone taking leisure because someone is carrying unspent money balances forward in every period.

Central Bank Intervention Confined to Location 2

Here, we can just make some minor changes in the previous analysis. In this case, if we determine a similar stationary equilibrium to what we derived in the previous section, then the ϕ^i for $i = 1, 2$ are determined by

$$1 = \frac{\alpha_1 \beta \mu^t v'_{t+1}\left(\mu^t \frac{\rho}{\mu^2 - 1 + \rho}\right)}{\phi^1} + (1 - \alpha_1) \frac{\beta^2}{\mu^2} \frac{[\rho\phi^1 + (1-\rho)\phi^2]}{\phi^1}$$

and

$$1 = \frac{\alpha_2 \beta \mu^t v'_{t+1}(\mu^t)}{\phi^2} + (1 - \alpha_2) \frac{\beta^2}{\mu^2} \frac{[\rho\phi^1 + (1-\rho)\phi^2]}{\phi^2}$$

Here, we need $\mu < 1$ to get constant prices, and it is possible that constant prices are not feasible, as this would imply $\mu < \beta$. It seems clear that in the previous case the Friedman rule will never be optimal, but in this case it might.

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Figure 1: Timing

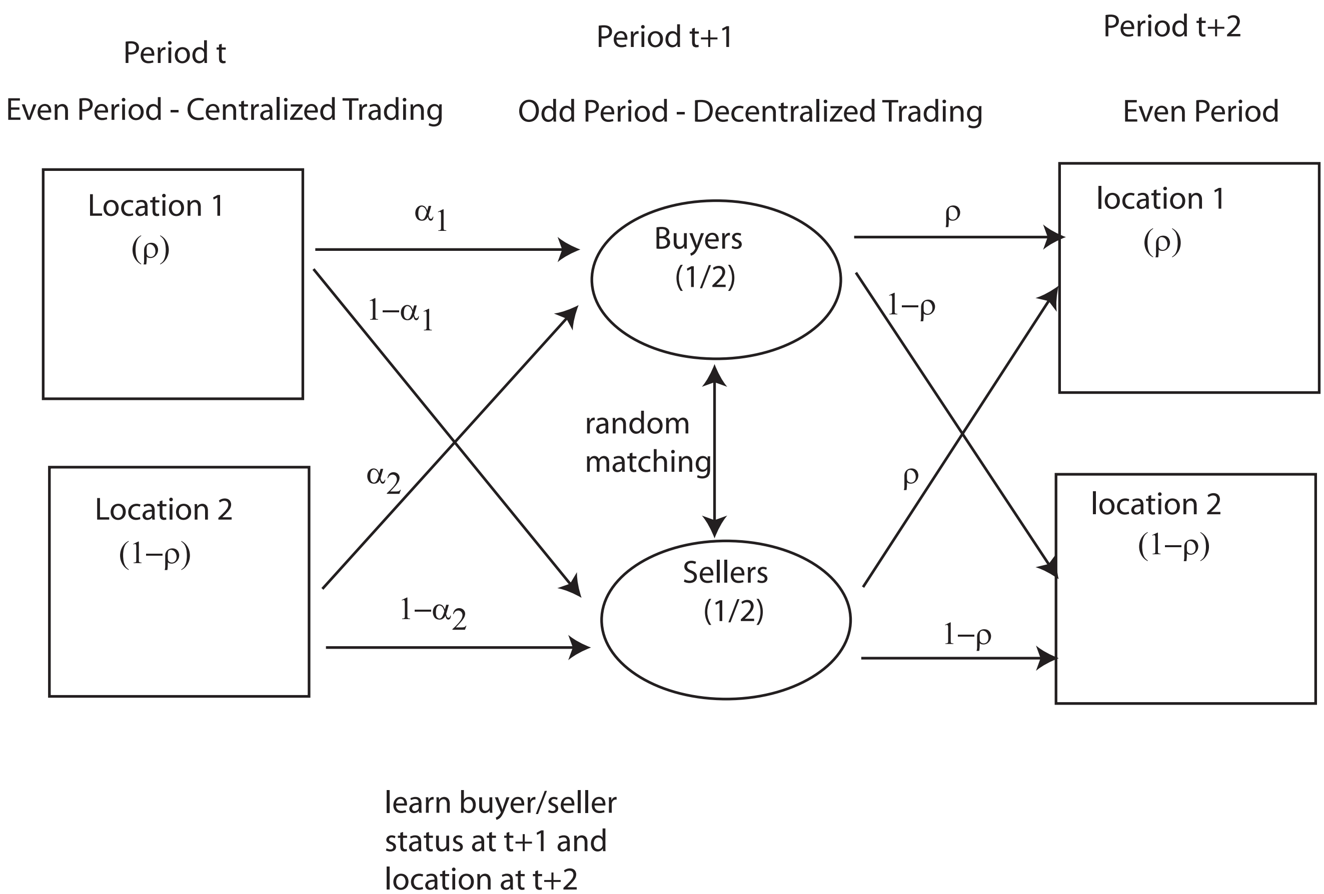

Figure 2: Case 1

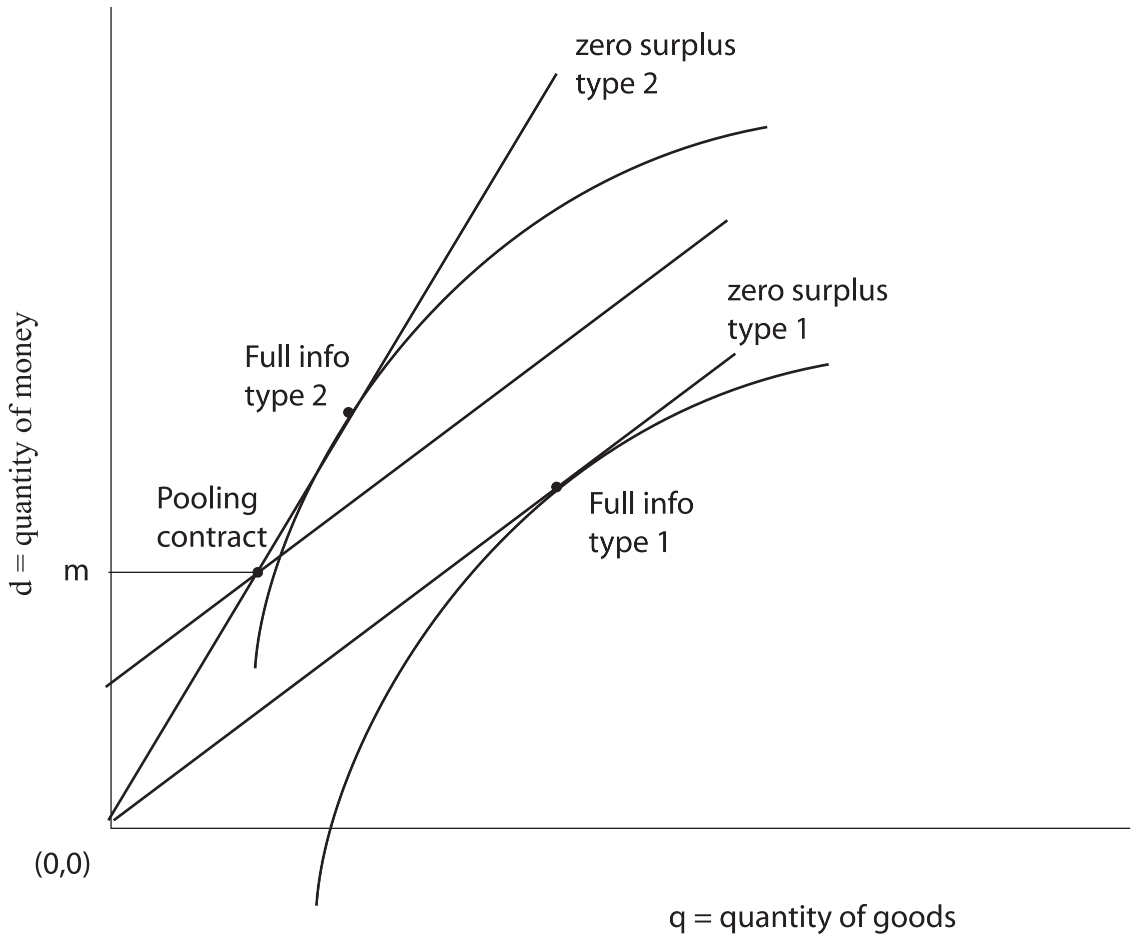

Figure 3: Case 2

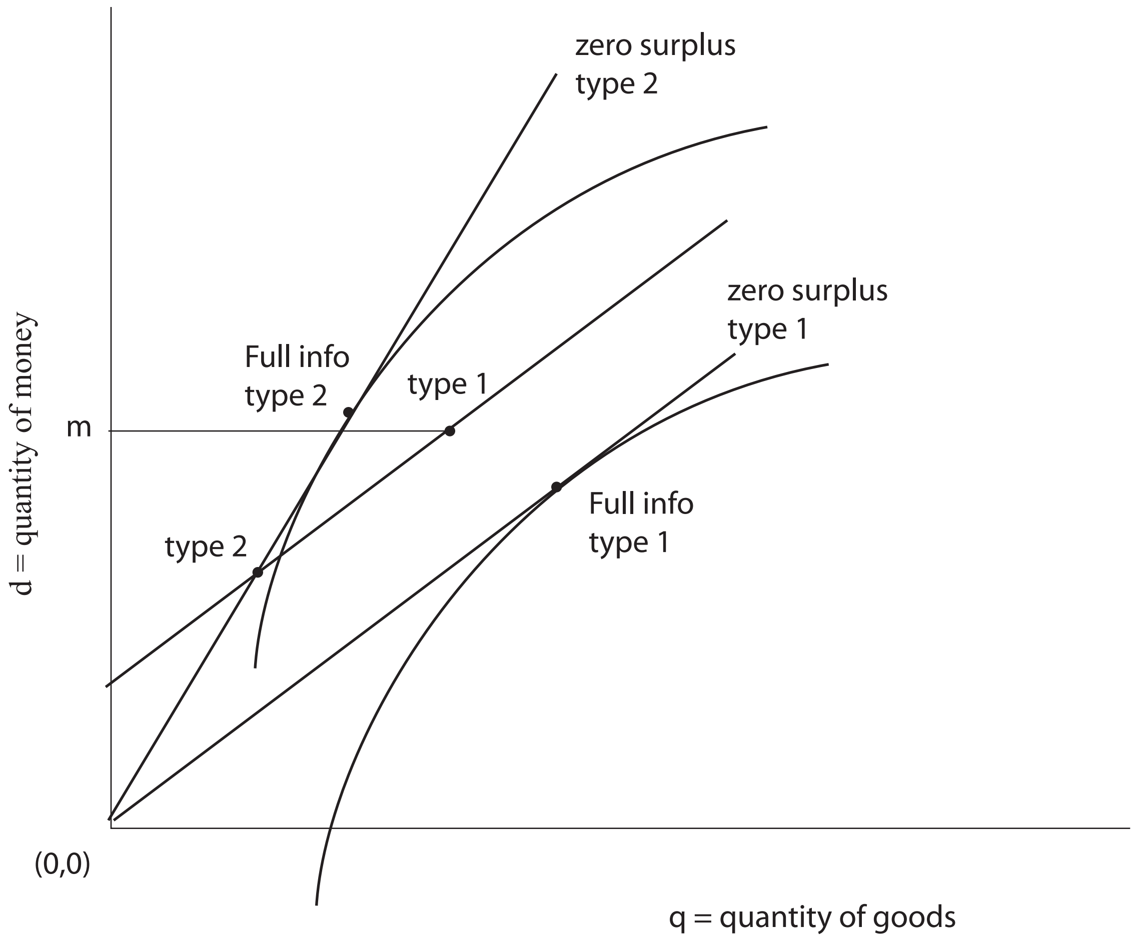

Figure 4: Case 3

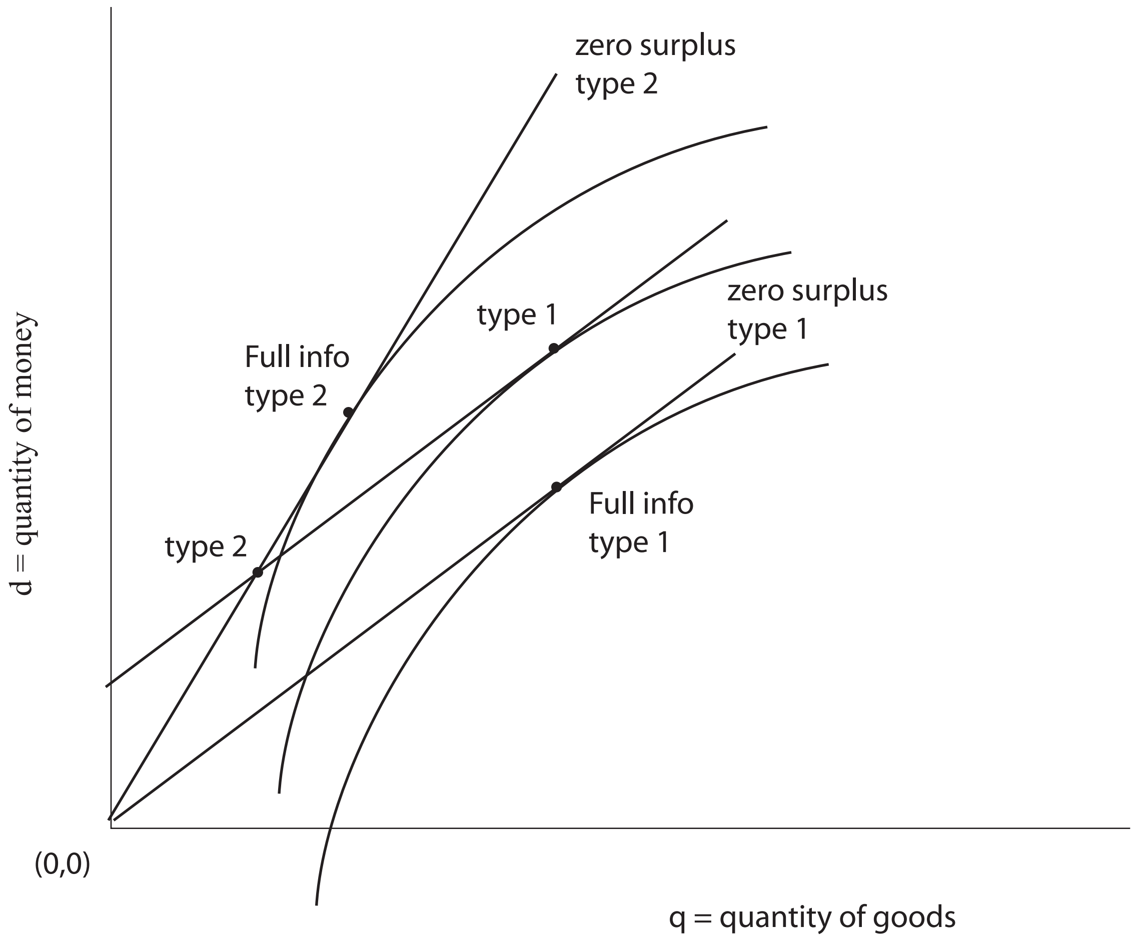

Figure 5

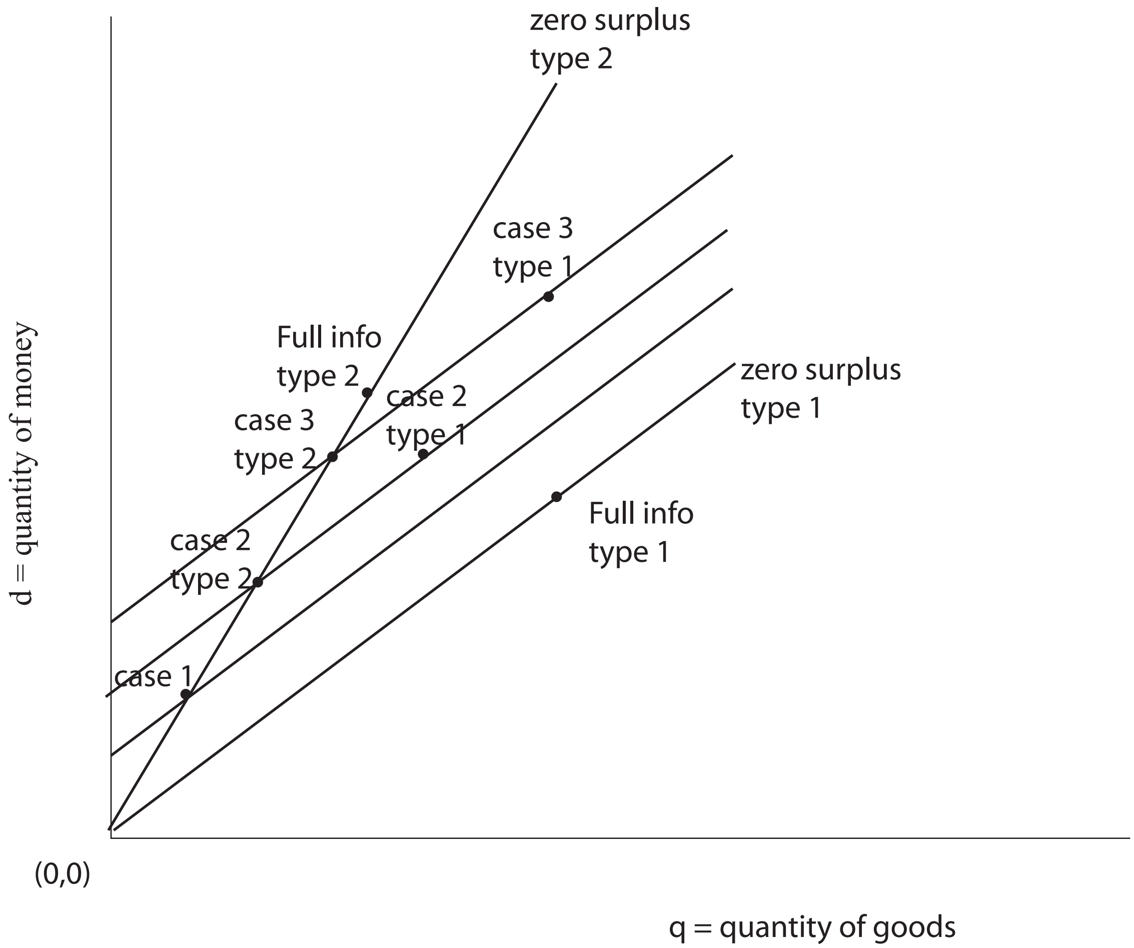
zero surplus
type 2
case 3
type 1
Full info
type 2
case 2
type 1
case 3
type 2
zero surplus
type 1
Full info
type 1
case 2
type 2
case 1
d = quantity of money
(0,0)
q = quantity of goods

Figure 6: Value Function