Optimal central-bank intervention: an example*

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Abstract

We numerically study an optimum in a matching model with four main ingredients: (i) some people have known histories and others are anonymous; (ii) idiosyncratic shocks that produce heterogeneous earning and spending realizations; (iii) central bank intervention in a "market" in claims in which the participants are those with known histories; (iv) a two-period cycle in aggregate productivity. The optimum is chosen from among periodic allocations that satisfy incentive constraints. There is a role for central bank intervention—which, by construction, consists of a quantity of zero-interest loans at one date with repayment at the next date.

1 Introduction

Economists working in the area of monetary theory and policy have had two goals. They have tried to formulate models of monetary policy in which money (by which we mean currency) has a well-understood role. And they have hoped to find that optimal policy is guided by simple principles and is not too dependent on the details of the model. Here, building on work done primarily in the last decade or so, we find optimal policy in an example that

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satisfies the first goal. But optimal central-bank policy in the example does not satisfy the second goal.

The general ingredients of the example are: (i) heterogeneity in the degree to which different people are monitored (have publicly known histories); (ii) idiosyncratic shocks that give rise to heterogeneity in earning and spending realizations; and (iii) central-bank intervention in a "market" in claims or credit in which the participants are those who are heavily monitored. Ingredient (i)—in particular, the existence of people who are hardly monitored—gives rise to a role for money (as opposed to credit): those who are hardly monitored have to use money. The combination of (i) and (ii) produces the economic problem: it is desirable but difficult to free current spending and earning from recent earning and spending realizations. Finally, (iii) implies that the connection between central-bank intervention and that problem is indirect because those who are hardly monitored are not participants in the market in claims. That accounts for why our example does not satisfy the second goal.

Although (i)-(iii) are not controversial, we know of no other analysis of optimal policy that rests on those ingredients. There are several reasons. First, only very recently have models been formulated that augment ingredient (ii) with ingredents (i) and (iii). Second, because most models of central-bank intervention do not satisfy the first goal, the idea that central-bank activity should be geared to solving the economic problem implied by ingredients (i) and (ii) is not widely accepted. But, perhaps, it ought to be. Central banks have a monopoly on money. If money has a role because some people are imperfectly monitored, then it is not far-fetched that management of that monopoly should be directed at something closely related to what makes money important. Moreover, recall that central banks in the U.K., the U.S., and several other countries emerged as legally mandated monopoly issuers of banknotes from systems in which there were many private banks issuing banknotes. Given such a monopoly, there is a role for two arrangements that partially substitute for private note issue: trade in claims among the would-be issuers of private banknotes (something like a credit market) and central-bank participation in such trade. Our model includes versions of those arrangements.

Borrowing from Cavalcanti and Wallace [2], we adopt an extreme version of heterogeneity in the degree to which different people are monitored (have publicly known histories): some people are perfectly monitored (have known histories) and the rest are not monitored at all (have private histories). At each date, there are two stages. At the first stage, there are pairwise meetings at random in which perishable goods can be produced and consumed; at the second stage, all the monitored people are together and money can be transferred among them, possibly with participation by the central bank. To give such transfers a role, there is randomness in the pairwise meetings that produces heterogeneous spending and earning realizations. To give central-bank participation a potential role in the simplest possible way, there is a two-period cycle (a seasonal) in aggregate productivity in pairwise meetings. We study the ex ante optimum in a class of symmetric, two-date periodic, and implementable allocations.

2 The model

The example we use is a variant of that described in Wallace [6], which, in turn, is closely related to Cavalcanti and Wallace [2], which, itself is a variant of Shi [4] and Trejos and Wright [5]. Time is discrete. There is a non-atomic and unit measure of each of $K \geq 3$ specialization types of infinitely lived people and there are K distinct, produced, and perishable goods at each date. A specialization-type k person, $k \in \{1, 2, ..., K\}$, produces only good k and consumes only good $k+1$ (modulo K). Each person maximizes expected discounted utility with discount factor $\beta \in (0, 1)$. For a specialization-type k person, period utility at date t is $u(y_{k+1}) - y_k/\delta_t$, where $y_{k+1} \in \mathbb{R}_+$ is consumption of good $k+1$ and $y_k \in \mathbb{R}_+$ is production of good k, and

$$\delta_t = \begin{cases} \delta_h > 0 \text{ if } t \text{ is even} \\ \delta_l > 0 \text{ if } t \text{ is odd} \end{cases} \tag{1}$$

is the periodic "productivity" parameter with $\delta_h > \delta_l$. (In what follows, we refer to an even date as a high (productivity) date and an odd date as a low (productivity) date.) The function $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly concave, strictly increasing, differentiable, and satisfies $u(0) = 0$ and $u'(\infty) = 0$. In addition, $u'(0)$ is sufficiently large.

As in Cavalcanti and Wallace [2], the set of each specialization type is partitioned permanently in an exogenous way into two sets: the fraction $\alpha < .5$ are monitored and the rest are not. That is, the history of each monitored person (each m person) is common knowledge, while that of each nonmonitored person (each n person) is private to the person. It is as if each m person wears a computer chip that transmits everything about the person to everyone else. In contrast, the only thing known about an n person is the person's type. In particular, an n person can hide money. The parameter α is best thought of as the economy's monitoring capacity.

Each date is divided into two stages. Production and consumption occur only at the first stage and in pairwise meetings. At that stage, each person meets at random one other person, except that m people do not meet each other.[1] (Each m person meets an n person with probability 1, while each n person meets an m person with probability $\alpha/(1-\alpha)$ and meets another n person with probability $(1-2\alpha)/(1-\alpha)$.) A meeting between specialization types k and $k+1$ is called a single-coincidence meeting. Other meetings are called no-coincidence meetings. At the second stage, all m people are together and engage in trade that resembles insurance—with all payments being in money. In addition, any central bank intervention occurs at the second stage. The set of possible individual holdings of money at the start of each date is $\{0,1\}$. The role of this restriction is to limit the number of unknowns.

The random meetings produce heterogeneous consumption (spending) and production (earning) opportunities. The source of such heterogeneity could, instead, be idiosyncratic taste shocks. In either case, as noted above, the problem in the economy is to free a person's current consumption and production from the person's past realizations. That is, from the point of view of the ex ante welfare criterion, it is desirable to have $\arg\max[u(y_t)-y_t/\delta_t] \equiv y_t^*$, the first-best output level, produced and consumed at every single-coincidence meeting. However, even if money holdings were unrestricted, n people who have experienced a string of consumption opportunities would have little money left to spend and those who have experienced a string of earning opportunities would have little incentive to produce much. The assumption that money holdings are in $\{0,1\}$ exacerbates this effect, but is not misleading in a qualitative sense. With money holdings in $\{0,1\}$, an earning opportunity that results in the aquisition of money makes an n peson so "rich" that the person cannot be induced to produce if the next opportunity is also an earning opportunity, while a spending opportunity that results in the expenditure of money makes an n person so "poor" that the person cannot spend if the next opportunity is another

[1] The only role of this assumption is to limit the number of unknowns. The assumption would have to be dropped to study a cashless limit in which the fraction of m people approaches unity. (This assumption will be dropped in subsequent drafts.)

spending opportunity.

3 Incentive feasible allocations

We consider only two-date periodic and symmetric allocations. Without central-bank intervention, the stock of money does not vary between high and low dates; with intervention, it can. Symmetry means that everyone in the same state at a given date makes the same (lottery) trade, a state that does not depend on the person's specialization type.[2] Thus, the role of lotteries here is to partially overcome the indivisibility of money; the lotteries do not serve to convexify the possibly nonconvex constraints.

Let $S = \{m, n\} \times \{0, 1\}$ be the set of individual states, where the set $\{0, 1\}$ is the amount of money held. Generic elements of S will be denoted s and s'. The following table contains the variables we choose to maximize ex ante utility.

Table 1. Variables

$y_\tau^{ss'}$ and $\lambda_\tau^{ss'}$	trades in single-coincidence meetings
η_τ	transfer of money in no-coincidence meetings
$\mu_{k\tau}^{mi}$	stage-2 prob of going from i to k units of money
$\theta_\tau^m, \theta_\tau^n$	distributions over $\{0, 1\}$

Here $y_\tau^{ss'}$ denotes output when the producer is in state s and the consumer is in state s' and τ denotes the date in the sense of high or low (productivity), while $\lambda_\tau^{ss'}$ denotes the probability that the n person with money surrenders it or that the n person without money receives a unit. The second row describes the probability that an m person transfers money in a no-coincidence meeting. The μ variable describes stage-2 transfers for the m people, while $\theta_\tau^m = (\theta_\tau^{m0}, \theta_\tau^{m1})$ and $\theta_\tau^n = (\theta_\tau^{n0}, \theta_\tau^{n1})$ are distributions over money at the start of date τ prior to pairwise meetings.

As regards constraints, the distributions of money and the transfers of money at both stages must be consistent with each other. This implies that the net stage-2 transfers at the high date and those at the low date

[2]Absent such symmetry, we could have randomization over the trades made by people in a given state. Such randomization is useful for some purposes; for example, it can be used to easily prove that ex ante welfare is weakly increasing in α. For us, it introduces too many unknowns.

sum to zero. Put differently, the central bank's intervention satisfies a zero budget constraint over the two dates so that if positive net transfers at one date are interpreted as a central bank loan to the insurance scheme, then that loan is repaid at zero interest at the next date by way of offsetting negative net transfers. To allow the central bank to make a positive profit on its intervention over two dates, which could be interpreted as lending at a positive interest rate, we would need to include a way to disburse the implied profit. One possibility is a probabilistic transfer scheme to the n people who after stage 1 have no money. We suspect that such additional scope for policy would not add much, because it turns out that the related transfer scheme in no-coincidence meetings, η, is not used. Also excluded is tax financed deflation. The $\{0, 1\}$ money holdings and the stationarity that we impose preclude the use of taxes to finance a deflation.[3] However,our specification does allow for a variant of explicit interest, positive or negative, on the money held by n people through what happens in pairwise trades with m people. If n people on average over high and low dates consume more per unit of money transferred in meetings with m people than they produce per unit of money received in meetings with m people, then n people on average earn positive interest on their money; if the reverse occurs, then their money is taxed.

Next, we turn to incentive constraints. In pairwise meetings, we permit both individual defection and cooperative defection by the pair in a meeting. Defection by an n person has no future consequences for the person.[4] Defection by an m person implies that the person becomes an n person at the next stage or date. (In other words, there is free exit from the set of m people, but not free entry into that set.) For reasons described below, in the

[3] With a larger set of individual holdings, we could mimic deflation in the following way. Suppose the set of holdings is $\{0, 1, ..., B\}$, with $B > 1$. Then, ignoring for a moment, the periodicity, an approximation to a constant deflation rate could be accomplished as follows. Let net transfers at stage 2 be negative and constant. Then offset the decrease by the following probabilistic addition to money holdings. If a person starts a period prior to pairwise meetings with $0 < j < B$ units of money, then augment that holding by one unit with probability γ, where $\gamma/j = \rho \leq 1/(B-1)$. This scheme approximates having the value of money grow through deflation. It is a deflation analogue of the approximate inflation scheme in Deviatov [3]. However, it is redistributive because the net taxes at stage 2 are collected from the m people, while the transfers that mimic deflation go to everyone.

[4] One approach would have the entire economy shut down in response to a known defection. We rule out such punishments.

second stage we require only that m people not want to defect individually. Finally, we have to insure that n people with money do not want to hide it. The incentive constraints by kind of meeting appear in the following table.

Table 2. Incentive constraints

meeting $(prod)(con)$	Nature of the constraints
$(n0)(n1)$	pairwise core
$(n0)(m1)$	'outside' the $(n0)(n1)$ payoff frontier
$(m0)(n0)$	individual defection by $(m0)$
$(m1)(n0)$	individual defection by $(m1)$
$(m0)(n1)$	'outside' the $(n0)(n1)$ frontier and truth-telling by $(n1)$
$(m1)(n1)$	individual defection and truth-telling by $(n1)$
η	individual defection by $(m1)$
stage 2	individual defection

The first two rows pertain to meetings in which the producer is in state $(n0)$. (An n person with money cannot be induced to produce.) And, in order for production to occur, the consumer must have money. For such a meeting between two n people, we require that the lottery trade be in the pairwise core for the meeting—the pairwise core defined taking continuation values of money as given. Because trade involves a switch of money holdings (with some probability), any such trade satisfies the following condition: either output is the first-best level, y_τ^*, or money is surrendered with probability 1 and output is bounded above by y_τ^* (see [1]). Now consider the second row. If there is a cooperative defection from this trade, then that constitutes a defection by the m consumer who necessarily bypasses stage 2 and enters the next date as an n person with the money retained after the trade. Therefore, the pairwise core requirement for this meeting is that the prescribed trade be (weakly) outside the payoff frontier implied by pairwise core trades for the first row meeting. In addition, the prescribed trade must satisfy the obvious individual rationality (IR) constraints.

The next 4 rows describe meetings in which the producer is an m person. In the first two of these rows, the only relevant constraints are IR constraints on the m producer who can defect and become an n person at the next date. Cooperative defection is impossible because the n consumer has nothing to offer. In the third and forth of these rows, the n consumer may want to hide money. And in the third, the n consumer has something to offer. Therefore, we again require that the trade be outside the payoff frontier for the $(n0)(n1)$ meeting. In the fourth of these rows, the n consumer has nothing to offer, because a defection that involves a transfer of money to the m person is useless because at most one unit can be carried into the next date. (We permit the m person to carry 2 units of money between stages.[5])

The penultimate row describes the constraint for transfers in no-coincidence meetings. Of course, the only meeting to which this pertains is when the m person has money and the n person does not. The last row pertains to stage 2. We consider only individual defection at stage 2 for the following reasons. At stage 2, there are no static gains from trade; in a defecting group, some people necessarily enter the next date with less money than they would have by not defecting. Thus, for group defection to occur, some members of the group would have to compensate other members in the future. But it is unclear how that can be accomplished. They cannot commit and they no longer participate in stage 2. Moreover, unless the defection is by a positive measure of people, they must cope with aggregate risk for the pairwise meeting outcomes of the group. (One seemingly unfortunate consequence of the unit bound on money holdings is that it eliminates the constraint for an m person who enters stage 2 with 2 units of money and is asked to surrender 1 unit. However, the absence of that constraint is offset by a stricter constraint on an m producer with money in pairwise meetings. Such an m producer is asked to produce despite the fact that any money received is surrendered.)

3.1 The optimum problem

The objective in our optimum problem, denoted W, is ex ante expected utility before people are assigned to type, m or n, before they are assigned initial money holdings, and at the start of a high (productivity) date prior to pairwise meetings. (The choice of high as opposed to low for the initial date is arbitrary.) That is,

$$W = \alpha \sum_{i=0,1} \theta_h^{mi} v_h^{mi} + (1-\alpha) \sum_{i=0,1} \theta_h^{ni} v_h^{ni}, \tag{2}$$

[5]This assumption is innocuous. What matters is that their action is observed. If they disposed of money holdings in excess of one unit at the end of stage 1, such disposal could be offset by new money supplied at stage 2.

where v_τ^s denotes the expected discounted value of being in state s at the start of date τ, prior to pairwise meetings. An alternative expression for W is a weighted average of $u(y_\tau^{ss'}) - y_\tau^{ss'}/\delta_\tau \equiv g(y_\tau^{ss'})$, the surplus, where the weight assigned to $g(y_h^{ss'})$ is proportional to the measure of (s, s') meetings at a high date, the weight assigned to $\beta g(y_l^{ss'})$ is proportional to the measure of (s, s') meetings at a low date, and where the factor of proportionality is $1/[K(1-\beta)]$. The measures depend on θ^m and θ^n.

For the optimum, we maximize W over all two-date periodic allocations that satisfy all the constraints. To isolate the role of policy, we also present the optimum subject to no policy—subject to zero net stage 2 transfers at each date. Policy is allowed simply by dropping that constraint.

We have set out the model ignoring lotteries over output. That is without loss of generality. Suppose, by way of contradiction, that an incentive-feasible allocation has a lottery over output in a meeting. By the pairwise core requirement, the lottery cannot be for a meeting between n people. Therefore, suppose the lottery is for a meeting between an m person and an n person. Consider a new allocation that is identical except that it replaces the lottery over output by a deterministic output that keeps the period payoff to the n person unchanged. (If the n person is the producer, then the deterministic output is the mean of the lottery; otherwise, it is less than the mean.) It follows that such replacement leaves discounted utility of n people unchanged and increases that of m people. It follows that the new allocation is incentive feasible (because defection is always to n status) and has higher welfare.

4 An example

We assume $u(x) = 2\sqrt{x}$, which implies that the first-best level of output at date τ is $(\delta_\tau)^2$. We set $K = 3$, so that each person has probability 1/3 of being a producer or a consumer. And, quite arbitrarily, we set $\alpha = 0.25$, and $(\delta_h, \delta_l) = (1.25, 0.8)$. Finally, we set $\beta = .95$.

One incentive-feasible allocation treats m people like n people at both stages and, therefore, has no stage 2. For the above parameters, the best such allocation is very simple: it has half the people with money and has the first-best output level produced whenever the potential producer does not have money and the potential consumer does, which happens in one-quarter of all the single-coincidence meetings. Because this allocation is incentive-feasible and has a constant money supply, the optima with or without policy cannot do worse.

In fact, they do better—both for those who become n people and for those who become m people. They do better by having trade occur in a larger fraction of the single-coincidence meetings. Welfare, its components as expressed in (2), and the distributions of money appear in the following table.

Table 3. Welfare and money holdings

	no policy		optimal policy	
	high	low	high	low
θ^{m1}	1	1	1	1
$\theta_h^m v_h^m$	4.467		4.468	
θ^{n1}	0.399	0.399	0.392	0.401
$\theta_h^n v_h^n$	2.635		2.644	
W	3.093		3.100	
cons equiv	1.000		1.022	

With or without policy, the distributions of money are very similar and permit trade to occur in many more meetings than when m people are treated like n people. The fraction of n producer-m consumer meetings in which production can occur goes from .25 to about .6, while the faction of m producer-n consumer meetings in which production can occur goes from .25 to 1 (because m producers can be threatened with expulsion from the set of m people). The only offset is a small decline in the fraction of single-coincidence meetings involving only n people in which trade can occur: from .25 to about .24. With or without policy, m people have substantially higher welfare than n people. **{We should compute a consumption equivalent of the difference.}**.

Welfare, of course, increases with policy. In addition, both n and m people benefit from the policy intervention. The consumption equivalent of the welfare improvement is about 2%. (As is standard, it is computed holding the distribution of meetings what it is under no-policy.) To better understand the source of the welfare improvement under policy and some other aspects of both optima, we describe the trades that occur in the various meetings.

Table 4. Trades in meetings

	no policy		optimal policy	
meeting	high	low	high	low
$(n0)(n1)$	1.000 (0.988)	1.000 (0.966)	0.927 (1.000)	0.981 (1.000)
$(n0)(m1)$	0.695^* (0.663)	0.941^* (0.540)	0.779^* (0.781)	0.830^* (0.500)
$(m1)(n0)$	0.119 (0.000)	0.103 (0.000)	0.138 (0.000)	0.081 (0.000)
$(m1)(n1)$	$0.755^{*\dagger}$ (1.000)	$1.062^{\dagger}$ (0.812)	$0.756^{*\dagger}$ (1.000)	$1.157^{*\dagger}$ (0.955)

In this table, the rows are ordered as they were in the table describing the constraints, and, as above, the state of the producer appears first. As we just saw, there is no one in state $(m, 0)$, so the corresponding rows are dropped. Also, at the optimum, there is no transfer of money from m people to n people in no-coincidence meetings. Each entry describes the trade: the top number is output relative to the first best and the bottom number, in parentheses, is the probability that money is transferred. A star (*) denotes a binding producer IR constraint and a dagger (†) denotes a binding truth-telling constraint.

Some features are common to both optima. Because m people always start a period with money, production by them is from their point of view always a gift (the third and fourth rows). But output is larger in the fourth row than in the third row. The $(m1)(n1)$ meetings are the only source for an outflow of money from n people. Without any such outflow, there could not be any inflow (spending by m people in the $(n0)(m1)$ meetings, the second row). But the requirement that n people surrender money in the $(m1)(n1)$ meeting gives rise to binding truth-telling constraints in all the $(m1)(n1)$ meetings. That is why output in the $(m1)(n0)$ meetings, which is a gift from the points of view of both participants, is lower than in the $(m1)(n1)$ meetings. And the sizable outputs in the $(m1)(n1)$ meetings necessitate substantially higher welfare for m people than for n people. (At the low date, the IR constraint for the m producer is $\beta[v_h^{m1} - v_h^{n1}] \geq$ output.) Also, it turns out that interest is *not* paid on money held by n people. A comparison of the second and fourth rows reveals that on average n people pay more goods from m people (the fourth row) than they receive when selling goods to them (the second row). In that sense, the optimum taxes n people.

The main source of the welfare gain under policy relative to no policy seems to be the trades in the $(n0)(m1)$ meetings (the second row), when the producer is an n person. Under policy, output is smoothed relative to the first best—which, by itself, would tend to raise welfare. Policy is needed to allow that to happen. In all the $(n0)(m1)$ meetings, there is a binding producer participation constraint. Therefore, to raise output at the high date, the m person would have to turn over money with a higher probability. But under no policy and with the money holding of m people constant between high and low dates, the inflow into holdings by n people must equal the outflow from their holdings at each date. The only outflow occurs in the $(m1)(n1)$ meetings and it is maximal at the high date under no policy. Hence, without reducing the stock of money held by m people or increasing the stock held by n people, there cannot be higher spending by m people in the $(n0)(m1)$ meetings. Policy allows such higher spending because the above inflow and outflow must be equated only over both dates. Hence, under policy higher spending by m people in the $(n0)(m1)$ meeting at the high date is possible if offset by lower spending at the low date, which is what happens.

The next table reports aggregates as they are usually computed.

Table 5. Aggregates

	no policy		optimal policy	
	high	low	high	low
money supply	0.549	0.549	0.544	0.551
output	0.165	0.082	0.169	0.079
price level	1.930	3.399	1.991	3.682
income velocity	0.580	0.508	0.619	0.528

The stock of money and output are per capita aggregates. The money stock is measured at the start of a date prior to pairwise meetings. Under optimal policy, it is higher at the beginning of a low date reflecting the higher spending by m consumers at high dates. Output here is the weighted sum of output in the different single-coincidence meetings. Relative to no-policy, optimal policy gives rise to greater output dispersion. The price level is the output deflator—namely, nominal output as implied by the transfers of money in single-coincidence meetings divided by total output. Relative to no-policy, it is higher at both dates at the optimum, as is income velocity. And policy does not smooth the price level between high and low dates.

5 Concluding remarks

The main ingredients of the theory we have set out are: (i) heterogeneity in the degree to which different people are monitored; (ii) idiosyncratic shocks that give rise to heterogeneity in earning and spending realizations; and (iii) central-bank intervention in a "market" in claims in which the participants are those who are heavily monitored. The combination of (i) and (ii) produces the economic problem in the model: it is desirable to free current spending and earning from recent earning and spending realizations. While the theory is conceptually simple, its implications for policy are not easy to summarize. Despite that, we do not think any of the ingredients should be sacrificed. Nor do we see a way to produce an attractive and simpler model that includes them.

{I will add some remarks on related literature: limited participation models and a recent paper by Bullard et al.}

6 Appendix 2: The algorithm

We solve our optimum problem numerically using the General Algebraic Modeling System (GAMS), which is specifically designed for the solution of large linear, nonlinear, and mixed integer optimization problems. It consists of a language compiler and a large menu of stable integrated high-performance solvers. The solvers are divided into two groups: local solvers (which are fast, but do not guarantee that the global solution is located) and global solvers (which are slow, but are very likely to find the global optimum). As a global solver, we use a Branch-And-Reduce Optimization Navigator (BARON) solver. BARON implements a deterministic algorithm of the branch-and-bound type, which is guaranteed to find the global optimum under very general conditions. These conditions include bounds on variables and the functions of them that appear in the nonlinear programming problem to be solved. **{Should we cite something?}**.

References

[1] A. Berentsen, M. Molico and R. Wright, Indivisibilities, lotteries, and monetary exchange, *Journal of Economic Theory* **107** (2002), 70-94.

[2] R. Cavalcanti and N. Wallace, Inside and outside money as alternative media of exchange. *J. of Money, Banking, and Credit,* ***31*** (1999, part 2), 443-57.

[3] A. Deviatov, Money creation in a random matching model, *Topics in Macroeconomics*, forthcoming.

[4] S. Shi, Money and prices: a model of search and bargaining. *J. of Econ. Theory* **67** (1995), 467-98.

[5] A. Trejos and R. Wright, Search, bargaining, money and prices. *J. of Political Economy* **103** (1995), 118-41.

[6] N. Wallace, From private banking to central banking: ingredients of a welfare analysis, *International Econ Rev.*, **46** (2005), 619-36.