August, 2007

Banking panics in a dynamic general equilibrium framework

Francesca Carapella*

University of Minnesota
and Federal Reserve Bank of Minneapolis

ABSTRACT

Historically banking panics have been associated with price declines in the United States. In his "Debt-Deflation theory of the Great Depression" Fisher hypothesized that banking panics were a consequence of mismatched balance sheets, with the mismatch arising from having liabilities at book value and asset at market value. This paper develops a dynamic general equilibrium model of banks that can be used to analyze Fishers hypothesis. The mechanism at work is a dynamic interaction between depositors and banks based off beliefs about other players' future strategies. If banks fear prices to fall in the future, they know they'll be unable to pay their nominal obligations back. Depositors may be aware of that and run on the bank when facing fears of a decrease in the price level. Hence a banking panic occurs.
The model is a version of Lucas-Stockey cash-credit economy with explicit characterization of banks/firms behavior. There exists a sunspot stationary equilibrium that features lower prices when banking panics and defaults occur relative to no panics and no defaults, and such fall in prices is self-fulfilling. Also, in this environment the Friedman and Schwartz insight that during the Great Depression a looser monetary policy would have resulted in a milder cycle applies. If a monetary authority injects money when there are fears of deflation then banks will not default and households will not panic. This framework is also suitable to being calibrated and tested to the data.

JEL: E53, E58, G21, N12. Keywords: banking panics, debt-deflation.

*I am very grateful to V.V Chari, Larry Jones, Chris Phelan, Antoine Martin, Jarek Nosal, Laurence Ales, Pricila Maziero, Roozbeh Hosseini, Facundo Piguillem and Anderson Schneider for very helpful comments. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System

1. Introduction

TO BE ADDED

2. Model

The economy is modeled as a dynamic game between a continuum [0,1] of identical households and a continuum [0,1] of identical banks. Banks are not anonymous in the sense that the history of their past actions is publicly observable.

Time is discrete and infinite.

A. Households

Households are modeled similarly to the Lucas-Stokey (1987) cash credit economy: their preferences are defined over two type of goods, cash goods (c_1) and credit goods (c_2) and are represented by a utility function $U : R_+^2 \to R_+$, such that $U_i > 0, i = 1, 2$ and $U_{ii} < 0, i = 1, 2$. In every period households receive an endowment y and they have access to a technology that allows them to transform y into either cash goods or credit goods. Households are also endowed with a non perishable good, called money, that they can use to transfer wealth intertemporally. They can also transfer wealth from one period to another by buying another asset, called deposits, from banks.

Each household is divided into a worker and a shopper: at the beginning of every period the asset market opens and they take their portfolio decisions together, as a household. So they decide how much money to carry into the period and how many deposits to purchase. Then the goods' market opens and the worker and the shopper are separated from each other: the shopper takes the money in his portfolio and go to other households' location to purchase consumption goods. The shopper is physically constrained to purchase cash goods paying right away using money, whereas he can purchase credit goods for current consumption but settle their payments in the next period using the money that he will have at the beginning of the next period.

At the same time as the shopper purchases consumption goods the worker stays at home and produces cash or credit goods using the endowment y.

At the end of the period the shopper returns home and consumption takes place. Unspent cash is brought into tomorrow together with the gross return on the deposits made in the previous period.

B. Banks

The financial and productive sectors in this economy are integrated and represented by banks. So banks are not just intermediaries between who supplies funds and who demands funds but they have access to the productive opportunities and carry them out. Therefore banks should be thought of as banks/firms.

They have a fixed endowment L in every period, and they have access to a productive technology $f : R_+^2 \rightarrow R_+, f_i > 0, i = 1, 2$. The inputs to the productive technology are an investment of cash goods and the fixed factor L.

Banks behave competitively and there is free entry into banking in the following sense: in every period there is a continuum [0,1] of banks actually operating but there is also a continuum of banks that can freely enter the banking sector. In order to become operating, a bank must purchase a banking license at a fixed cost K[1]

Once they are operating, banks offer deposit contracts to households and carry out production: the type of contract they can offer is such the rate of return on deposits is fixed in nominal terms[2]

The deposit contract between households and banks is one-to-one: each household

---

[1] This cost is going to equal the value of lifetime profits the bank will earn when in business.
Alternatively we can think of outside banks as renting their L to the banks in business who use the additional L for production and then pay the return for having rented L to outside banks. So that all the results and the equilibrium characterization that will be described are going to go through entirely.
Another alternative way of having new banks replacing the ones that go out of business would be to just assume that in every period there is just a continuum [0,1] of banks, and that for every bank that goes out of business and loses its L there is another one that is born and endowed with L until it stays in business

[2] This restriction is meant to capture one of the key features of deposit contracts in reality.

can choose which bank it wants to deposit at, but only one bank. So the household cannot differentiate his portfolio among different banks, if he wanted to. Also, each bank can take the deposits only of a single household.

C. Timing of players' moves

At the beginning of every period the outcome θ of a random variable, or sunspot, $\Theta \in \{0, 1\}$ is publicly observed. The realization of the sunspot is going to play the role of a correlation device among players' strategies, in order to construct a particular equilibrium of the game. θ follows a Markov chain with transition function represented by:

$$\Pi = \begin{array}{|c|c|} \hline \pi_{11} & \pi_{01} \\ \hline \pi_{10} & \pi_{00} \\ \hline \end{array}$$

At time t, after the realization of the sunspot, θ_t, banks simultaneously choose whether to default or not: if they don't default they sell the output from the productive technology on the goods' market, they pay households back for the deposits they made and the interest rate that it was promised to them as a return to the deposit.

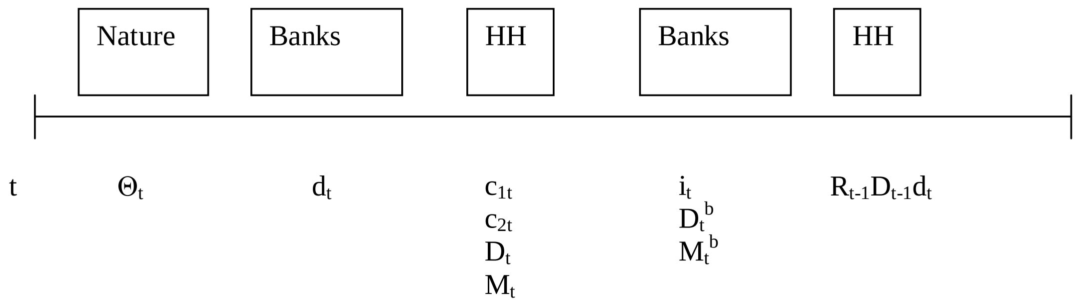
If they default they don't pay depositors back at all, not only for the promised return but also for the actual deposits that households made. So households who deposited, and therefore invested in the productive technology, lose not only the return on their investment but also the assets they invested if there is default.

After banks have decided whether to default or not, households move: they choose their consumption allocation (c_{1t}, c_{2t}) and their asset holdings (M_t, D_t).

Then banks move again: using the deposits they sold to households (D_t^b) they decide how much to invest in the productive technology and how much money to demand (M_t^b)[3].

[3]One factor of production is an investment of cash good, so banks need some money to be able to purchase

At the end of the period, if banks did not default, then depositors get the return promised in the previous period on the deposits made in the previous period ($R_{t-1}D_{t-1}$). Graphically the timing of players' moves is represented in Fig.1:

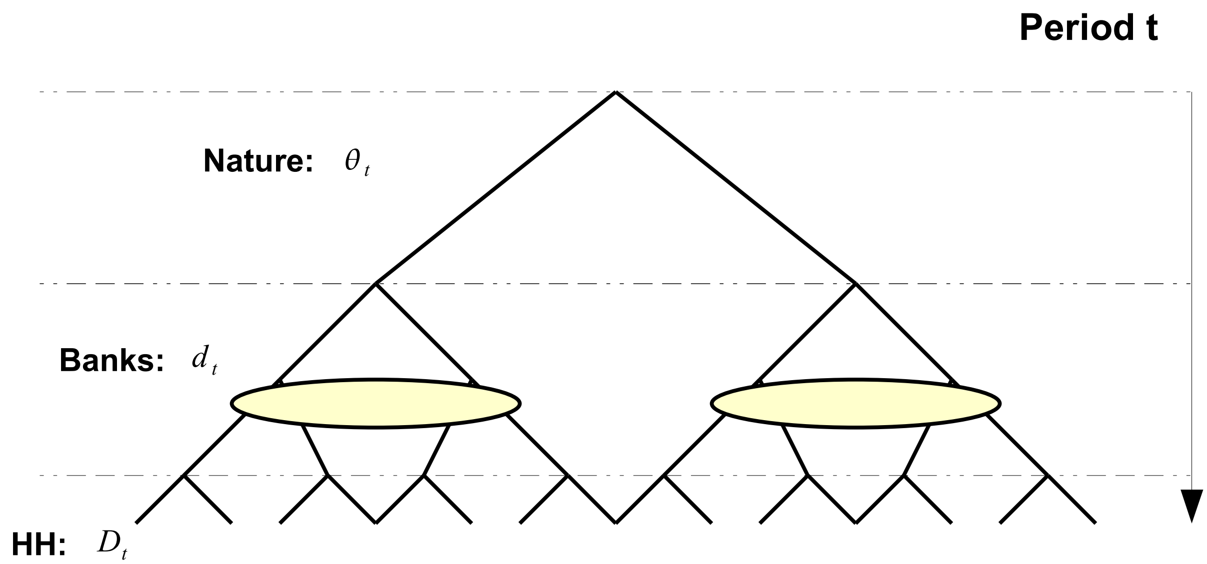


So for every period a stage game can be defined, where Nature first draw a realization of θ from Θ, then banks simultaneously take their default decision. After having observed banks default decision, households choose consumption allocation and asset holdings (in particular they choose whether to deposit or not). Then banks move again and choose how much to invest in the productive technology and how much money to demand, and at the very end of the period households get paid back for the deposits they made in the previous period if banks did not default on them at the beginning of the period.

The stage game is represented in extensive form in Fig.2:

---

it and carry out production



D. Players' actions and strategies

Let $h^t = (d_s, \theta_s / s \leq t)$ denote the history of the game at time t when the information set where households move is reached and H^t denote the set of possible histories at time t with $H^0 = \{\emptyset\}$, so that h^t is a typical element of H^t. Such history includes the sequence of current and past default decisions of every bank and current and past realizations of Θ.

Banks have two decisions problems at two different information sets: in the first one they decide whether to default or not. Let $\mathcal{A}_t^b = \{1, 0\}$ be the action space for banks' first move at every time t, and let $d_t \in \mathcal{A}_t^b$ be such that $d_t = 1$ denotes no default and $d_t = 0$ denotes default. The history of the game at this information set is h_{t-1}, so we can define the vector of relevant state variables for each bank at their first move by $x_t^b = (h^{t-1}, D_{t-1}^b, \theta_t)$, with D_{t-1}^b denoting the deposits that the bank sold to households in the previous period. Let X_t^b be the set of possible such vectors after history h^{t-1}.

The values for a bank to be in business at the beginning of time t if the states are x_t^b and the price level in the previous period was p_{t-1} is $v_t^i(x_t^b, p_{t-1})$. So banks' first decision problem is to choose whether to default or not according to which payoff is larger:

$$v_t^i(x_t^b, p_{t-1}) = \max_{\{d_t \in \{1,0\}\}} \Big(f(i_{t-1}, L) - \frac{R_{t-1} D_{t-1}^b}{p_t} + \delta E_{\theta_{t+1}/\theta_t} v_{t+1}^{i,d=1}, \quad (1)$$

$$f(i_{t-1}, L) + \delta E_{\theta_{t+1}/\theta_t} v_{t+1}^{i,d=0} \Big)$$

where $f(\frac{D_{t-1}^b}{p_{t-1}}, L) - \frac{R_{t-1} D_{t-1}^b}{p_t}$ is the current payoff from not defaulting and $\delta E_{\theta_{t+1}/\theta_t} v_{t+1}^{i,d=1}$ is the present expected continuation value of the bank given that it has not defaulted today ($d_t = 1$), and δ is banks' discount factor. And $f(\frac{D_{t-1}^b}{p_{t-1}}, L)$ is the current payoff from defaulting and $\delta E_{\theta_{t+1}/\theta_t} v_{t+1}^{i,d=0}$ is the present expected continuation value of the bank given that it has defaulted today ($d_t = 0$).

When banks get to the node where they face their second decision problem, households have already moved, so the history of the game is h^t. The action space for banks' second move at time t is denoted $\mathcal{A}_t^B(x_t^b) = \{D_t^b(x_t^b, d_t), i_t(x_t^b, d_t), M_t^b(x_t^b, d_t)) : (D_t^b, i_t, M_t^b) \in \mathbb{R}_+^3\}$. A strategy for a bank at time t is $\sigma_t^B : X_t^b \to \Delta(\mathcal{A}_t^B(x_t^b))$.

Banks' second decision problem is to maximize expected profits:

$$\max_{\{i_t, D_t^b, M_t^b\}} [Prob(d_{t+1} = 1)\Big(f(i_t, L) - \tfrac{R_t D_t^b}{p_{t+1}} \Big) + Prob(d_{t+1} = 0) f(i_t, L)] \quad (2)$$

$$s.t. \quad p_t i_t \leq M_t^b$$

$$M_t^b \leq D_t^b$$

When households move the history of the game includes the sequence of current and

past default decisions of every bank and current and past realizations of Θ, so it is h^t. Therefore the vector of relevant state variables for each household when they reach the information set where they move is denoted $x_t^H = (A_t, h^t, D_{t-1})$, with A_t being the assets that the household enters the period with, and D_{t-1} the deposits made in the previous period. X_t^H denotes the set of possible such vectors after history h^t. Let $\mathcal{A}_t^H(x_t^H) = \{(c_{1t}(x_t^H), c_{2t}(x_t^H), D_t(x_t^H), M_t(x_t^H), A_{t+1}(x_t^H)) \in \mathbb{R}_+^4\}$ denote the action space for households' move at every time and $\sigma_t^H : X_t^H \to \Delta(\mathcal{A}_t^H(x_t^H))$ households' strategy at time t.
Let $\mu_t^H : H^{t+1} \to [0,1]$ denote the conditional probability [1] that history $h^{t+1} \succ h^t$ will be realized if h_t is the realized history at time t. The value (or the payoff) of a household starting from states x_t^H and aggregate states x_t [2] is:

$$\begin{aligned} v_t(x_t^H)(\sigma_t) &= \max_{\{\sigma_t^H \in \Delta(\mathcal{A}_t^H(x_t^H))\}} (U(c_1(x_t^H)(\sigma_t^B), c_2(x_t^H)(\sigma_t^B)) + \\ &\quad \beta \sum_{h^{t+1}} \mu_t(h^{t+1}/h^t) v_{t+1}(x_{t+1}^H)(\sigma_{t+1})) \\ &s.t. \\ M_t(x_t^H)(\sigma_t^B) + D_t(x_t^H)(\sigma_t^B) &= A_t \\ p_t(\mathrm{x}_t) c_{1t}(x_t^H)(\sigma_t^B) &\leq M_t(x_t^H)(\sigma_t^B) \qquad \text{if } d_t = 1 \\ p_t(\mathrm{x}_t)(c_{1t}(x_t^H)(\sigma_t^B) + c_{2t}(x_t^H)(\sigma_t^B)) &\leq M_t(x_t^H)(\sigma_t^B) \qquad \text{if } d_t = 0 \\ A_{t+1}(x_t^H) &= M_t(x_t^H)(\sigma_t^B) - p_t(\mathrm{x}_t)(c_{1t}(x_t^H)(\sigma_t^B) + c_{2t}(x_t^H)(\sigma_t^B)) \\ &\quad + p_t(\mathrm{x}_t) y_t + d_t R_{t-1}(\mathrm{x}_{t-1}) D_{t-1}(x_{t-1}) \end{aligned} \tag{3}$$

where the first constraint is an asset market budget constraint: the household allocates his assets between money to carry within the period and deposits. The second constraint is a cash in advance constraint on a subset of the consumption goods, cash goods. This is the relevant cash in advance constraint if banks did not default at the beginning of the period ($d_t = 1$). The third constraint is the relevant cash in advance constraint when banks default at the beginning of the period ($d_t = 0$) and it involves both cash and credit goods. Therefore when there is default the payment system is broken: while in no default households would purchase credit goods for current consumption but settle their payments only at the beginning of the next period using also the return on deposits previously made, when banks default deposits will not be payed back at the end of the period, therefore there won't be enough resources in the next period to pay for current consumption of credit goods. So also credit goods must be purchased using cash. The last constraint is the law of motion for assets: assets at the beginning of the next period will be given by unspent cash, income from the sales of the endowment and the return on previously made deposits if banks do not default ($d_t = 1$).

E. Equilibrium

I am going to focus only on pure strategy equilibria throughout the paper and the notion of equilibrium I am going to use is subgame perfect Nash equilibrium.

Definition 1. *A stationary symmetric equilibrium is:*

1. *a symmetric strategy profile for households* $\sigma^H = \{\sigma_t^H\}_{t=0}^{\infty}$
2. *a symmetric strategy profile for banks* $\sigma^B = \{\sigma_t^B\}_{t=0}^{\infty}$
3. *pricing functions* $p_t(\mathrm{x}_t), R_t(\mathrm{x}_t)$

such that for any $t, x_t^H \in X_t^H$*, households maximize; for any* $t, x_t^b \in X_t^b$*, banks maximize and prices clear the markets:*

$$c_t^b(\mathrm{x}_t) + c_{1t}(\mathrm{x}_t) + c_{2t}(\mathrm{x}_t) + i_t(\mathrm{x}_t) = y_t + f(i_{t-1}(\mathrm{x}_t), L)$$
$$M_t(\mathrm{x}_t) + D_t(\mathrm{x}_t) = \overline{M}_t$$

where in the resource constraint $c_t^b(\mathrm{x}_t)$ stands for bank's consumption when aggregate states are x_t, $c_{1t}(\mathrm{x}_t)$ for households' consumption of cash good, $c_{2t}(\mathrm{x}_t)$ for households' consumption of credit good, i_t for the investment in the productive technology, $f(i_{t-1}(\mathrm{x}_t), L)$ for the output of the technology that is realized at t using inputs from $t-1$. In the money market constraint $M_t(\mathrm{x}_t)$ stands for the money the households wants to carry within the period, $D_t(\mathrm{x}_t)$ for the deposits the household wants to purchase, and $\overline{M}$ for the stock of money supply. Let γ_t denote the rate of growth of money supply at time t.

3. Equilibrium characterization

This economy has several equilibria. I am going to construct one where fears of a fall in prices drive banks to default and forgo future profits. I am going to do that by using the correlation device θ in an easy fashion:

· At every t, if $\theta_t = 1$ banks don't default and households deposit

· At every t, if a bank defaults then households won't deposit in that bank forever after

· at t if $\theta_t = 0$ then banks default and households don't deposit

The conditional distribution over possible histories $h^{t+1} \succ h^t$ induced by the distribution of Θ and by players' strategies is then such that the probability of banks defaulting equals $\pi(\theta_{t+1} = 0/\theta_t)$.

For the remainder of this section maintain the following assumptions:

Assumption 1. $U(c_1, c_2) = \log(c_1) + \log(c_2)$.

Assumption 2. $f(i_t, L) = \begin{cases} \kappa i_t + L & \text{if} \quad i_t > 0, \ \text{with } \kappa > 1 \\ 0 & \text{if} \quad i_t = 0 \end{cases}$

Assumption 3.

$$\begin{aligned} 1 + \pi(0/1) &> \frac{1}{\kappa\beta^2} > 1 - \pi(0/1) \\ \beta &> \kappa[\beta^2(1+\pi(0/1)) - 1][\frac{1}{\pi(1/1)} + \frac{1}{\frac{1}{\kappa\beta^2} - \pi(1/1)}] \end{aligned}$$

The goal of this section is to prove existence of a stationary equilibrium where fears of deflation drive banks to default, and where deflation is self-fulfilling. In order to obtain an analytical characterization of the existence result I am going to modify the environment slightly: let there be a a fraction λ of banks that are "robots" in the sense that they will pay depositor back if they can, they have no strategic choice. The remaining fraction $(1-\lambda)$ of banks behaves instead strategically. Whether a bank is a λ bank or a $(1-\lambda)$ bank is not observable to households and banks cannot reveal it to households.

A λ bank can pay depositors back if current period profits are non negative ($\pi_t = f_t - \frac{R_{t-1}D_{t-1}}{p_t} \geq 0$). Define:

1. $\phi = \frac{(1+\pi(0/1)-\frac{1}{\kappa\beta^2})}{2\pi(0/1)}$
2. $\Lambda = 1 + \frac{1}{2\beta\pi(0/1)+\frac{\beta\pi(1/1)}{(1-\phi)}} - \frac{\kappa\phi}{\pi(1/1)}$
3. $\Gamma = 1 + \beta\kappa(1-\phi)(1-\lambda) - (1-\lambda)\frac{\kappa\phi}{\pi(1/1)}$

Proposition 1. *If assumptions (1.)-(3.) are satisfied then:*
$\forall\lambda : \frac{\kappa\phi y}{\pi(1/1)\Gamma} > \max(\delta L(1+\frac{\delta\pi(0/1)}{1-\delta\pi(1/1)}), L + \frac{\kappa\phi y}{\Lambda})$ *there exists a symmetric stationary equilibrium with constant money supply such that prices in default are lower than prices in no default.*

Proof. The proof of Proposition 1. is in three steps: first I construct strategies for all players; then taking as given that banks will play the constructed strategy I check that households constructed strategy is a maximizer of problem (3); then taking as given that households will play the constructed strategy I check that banks constructed strategy maximizes both problems (1) and (2).

Let $\sigma_t^B = (d_t = 1, D_t^b = D_t, i_t = \frac{D_t}{p_t}, M_t^b = D_t^b)$ when $x_t^b = (h^{t-1}, D_{t-1}^b, \theta = 1)$.

Let $\sigma_t^B = (d_t = 0, D_t^b = 0, i_t = 0, M_t^b = 0)$ when $x_t^b = (h^{t-1}, D_{t-1}^b, \theta = 0)$.

Let $\sigma_t^H = (c_{1t} = \frac{\overline{M}-D_t}{p_t}, c_{2t}, D_t, M_t = \overline{M} - D_t)$ when $h^t = (h^{t-1}, d_t = 1, \theta_t = 1)$.

Let $\sigma_t^H = (c_{1t} = \frac{\overline{M}}{2}, c_{2t} = \frac{\overline{M}}{2}, D_t = 0, M_t = \overline{M})$ when $h^t = (h^{t-1}, d_t = 0, \theta_t = 0)$.

So if households take as given that banks' strategies are σ_t^B then they know that banks will default when $\theta_t = 0$ and will not default when $\theta_t = 1$.

Therefore households' problem becomes:

$$
\begin{aligned}
v_t^{d=1}(x_t^H) &= \max\{U(c_1(x_t^H), c_2(x_t^H)) + \\
&\quad \beta\pi(1/1)v_{t+1}^{d=1}(x_{t+1}^H)) + \beta\pi(0/1)v_{t+1}^{d=0}(x_{t+1}^H)\} \\
&\quad s.t. \\
M_t(x_t^H) + D_t(x_t^H) &= A_t \\
p_t(\mathrm{x}_t)c_{1t}(x_t^H) &\leq M_t(x_t^H) \\
A_{t+1}(x_t^H) &= M_t(x_t^H) - p_t(\mathrm{x}_t)(c_{1t}(x_t^H) + c_{2t}(x_t^H)) \\
&\quad + p_t(\mathrm{x}_t)y_t + R_{t-1}(\mathrm{x}_{t-1})D_{t-1}(x_{t-1})
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
v_t^{d=0}(x_t^H) &= \max\{U(c_1(x_t^H), c_2(x_t^H)) + \\
&\quad \beta\pi(1/0)v_{t+1}^{d=1}(x_{t+1}^H)) + \beta\pi(0/0)v_{t+1}^{d=0}(x_{t+1}^H)\} \\
&\quad s.t. \\
M_t(x_t^H) + D_t(x_t^H) &= A_t \\
p_t(\mathrm{x}_t)(c_{1t}(x_t^H) + c_{2t}(x_t^H)) &\leq M_t(x_t^H) \\
A_{t+1}(x_t^H) &= M_t(x_t^H) - p_t(\mathrm{x}_t)(c_{1t}(x_t^H) + c_{2t}(x_t^H)) \\
&\quad + p_t(\mathrm{x}_t)y_t
\end{aligned} \tag{5}
$$

Necessary optimality conditions are:

$$
\begin{aligned}
U_{1t}^{d=1} - p_t^{d=1}(\mu_t^{d=1} + \beta\pi(1/1))v_{t+1_1}^{d=1}(A_{t+1}, h^{t+1}, D_t))) + \beta\pi(0/1))v_{t+1_1}^{d=0}(A_{t+1}, h^{t+1}, D_t))) &= 0 \\
U_{2t}^{d=1} - p_t^{d=1}(\beta\pi(1/1))v_{t+1_1}^{d=1}(A_{t+1}, h^{t+1}, D_t))) + \beta\pi(0/1))v_{t+1_1}^{d=0}(A_{t+1}, h^{t+1}, D_t))) &= 0 \\
-\lambda_t^{d=1} + \mu_t^{d=1} + \beta\pi(1/1)v_{t+1_1}^{d=1}(A_{t+1}, h^{t+1}, D_t)) + \beta\pi(0/1)v_{t+1_1}^{d=0}(A_{t+1}, h^{t+1}, D_t)) &= 0 \\
-\lambda_t^{d=1} + \beta\pi(1/1)v_{t+1_3}^{d=1}(A_{t+1}, h^{t+1}, D_t)) &\leq 0 \\
v_{t+1_1}^{d=1}(A_{t+1}, D_t) - \lambda_{t+1}^{d=1} &= 0 \\
v_{t+1_3}^{d=1}(A_{t+1}, D_t) - \beta\pi(1/1)v_{t+2_1}^{d=1}(A_{t+1}, h^{t+1}, D_t))R_t &= 0
\end{aligned}
$$

in no default and:

$$
\begin{aligned}
U_{1t}^{d=0} - p_t^{d=0}(\mu_t^{d=0} + \beta\pi(0/0)v_{t+1_1}^{d=0}(A_{t+1}, h^{t+1}, D_t) + \beta\pi(1/0)v_{t+1_1}^{d=1}(A_{t+1}, h^{t+1}, D_t)) &= 0 \\
U_{2t}^{d=0} - p_t^{d=0}(\mu_t^{d=0} + \beta\pi(0/0)v_{t+1_1}^{d=0}(A_{t+1}, h^{t+1}, D_t) + \beta\pi(1/0)v_{t+1_1}^{d=1}(A_{t+1}, h^{t+1}, D_t)) &= 0 \\
-\lambda_t^{d=0} + \mu_t^{d=0} + \beta\pi(1/0)v_{t+1_1}^{d=1}(A_{t+1}, h^{t+1}, D_t)) + \beta\pi(0/0)v_{t+1_1}^{d=0}(A_{t+1}, h^{t+1}, D_t)) &= 0 \\
-\lambda_t^{d=0} + \beta\pi(1/0)v_{t+1_3}(A_{t+1}, h^{t+1}, D_t)) &\leq 0 \\
v_{t+1_1}^{d=0}(A_{t+1}, D_t) - \lambda_{t+1}^{d=0} &= 0
\end{aligned}
$$

The conjectured solution to the household problem, that satisfy all the necessary optimality conditions:

$$
\begin{aligned}
p_t c_{1t}^{nd} &= M_t^{nd} \\
p_t(c_{1t}^d + c_{2t}^d) &= M_t^d = \overline{M}
\end{aligned}
$$

Intertemporal optimality conditions also require:

$$
\frac{U_{1t}^{nd}}{p_t^{nd}} \geq \beta R_t \pi(1/1)\frac{U_{2t+1}^{nd}}{p_{t+1}^{nd}} \quad \text{with " = " if } D_t > 0 \tag{6}
$$

$$
\frac{U_{2t}^{nd}}{p_t^{nd}} = \beta(\pi(0/1)\frac{U_{1t+1}^{d}}{p_{t+1}^{d}} + \pi(1/1)\frac{U_{1t+1}^{nd}}{p_{t+1}^{nd}}) \tag{7}
$$

$$
\frac{U_{1t}^{nd}}{p_t^{nd}} = \beta\pi(1/1)R_t(\beta\pi(0/1)\frac{U_{1t+2}^{d}}{p_{t+2}^{d}} + \beta\pi(1/1)\frac{U_{1t+2}^{nd}}{p_{t+2}^{nd}}) \tag{8}
$$

(9)

From the banks profit maximizing condition we also have that: $R_t = \frac{p_{t+1}^{nd}}{p_t^{nd}} \frac{1}{\pi(1/1)} f_1(\frac{D_t}{p_t^{nd}}, L)$ which with a linear technology as specified in ASSUMPTION 2. is:

$$R_t = \frac{p_{t+1}^{nd}}{p_t^{nd}} \frac{1}{\pi(1/1)} \kappa \tag{10}$$

With constant money supply, a stationary equilibrium where consumption allocation and assets holdings are unchanged over time conditional on θ and where the optimal level of deposits chosen in the current period is the same as the deposits carried from the previous period (i.e. $D_{t+1} = D_t = D$) will have constant prices: $p_{t+1}^{d=1} = p_t^{d=1}$ and $p_{t+1}^{d=0} = p_t^{d=0}$. So in a stationary equilibrium if $\theta = 1$ one Euler equation implies:

$$\kappa\beta^2[\pi(0/1)\frac{2}{\bar{M}} + \pi(1/1)\frac{1}{(\bar{M} - D)}] = \frac{1}{(\bar{M} - D)} \tag{11}$$

From which D can be pinned down:

$$D = \bar{M}[\frac{(1 + \pi(0/1) - \frac{1}{\kappa\beta^2})}{2\pi(0/1)}] \tag{12}$$

Under the first part of ASSUMPTION 3. $D \in (0, \overline{M})$:

$$D = \phi\overline{M} \qquad \text{with } \phi = \frac{(1 + \pi(0/1) - \frac{1}{\kappa\beta^2})}{2\pi(0/1)} \in (0, 1) \tag{13}$$

Also when $\theta = 1$ consumption of cash good is pinned down by the cash-in-advance constraint and consumption of credit good from the Euler equation (7), so that:

$$
\begin{aligned}
c_1 &= \frac{1}{p}(\overline{M} - D) \\
c_2 &= \frac{1}{p}\frac{1}{\beta(\frac{2\pi(0/1)}{\overline{M}} + \frac{\pi(1/1)}{\overline{M}-D})}
\end{aligned}
$$

From the resource constraint:

$$
f(\frac{D}{p}, L) - \frac{RD}{p} + \frac{\overline{M}}{p} + \frac{1}{p}\frac{1}{\beta(\frac{2\pi(0/1)}{\overline{M}} + \frac{\pi(1/1)}{\overline{M}(1-\phi)})} + \frac{D}{p} = y + f(\frac{D}{p}, L) \tag{14}
$$

That implies:

$$
p = \frac{\overline{M}}{y}\Lambda \qquad \text{with } \Lambda = [1 + \frac{1}{\beta(2\pi(0/1) + \frac{\pi(1/1)}{(1-\phi)})} - \frac{\kappa\phi}{\pi(1/1)}] \tag{15}
$$

When $\theta = 0$ a solution to the households' problem is such that:

$$
\begin{aligned}
c_1^{d=0} &= \frac{\overline{M}}{2p^{d=0}} \\
c_2^{d=0} &= \frac{\overline{M}}{2p^{d=0}}
\end{aligned}
$$

The resource constraint is then

$$
f(\frac{D}{p}, L) + \frac{\overline{M}}{p^{d=0}} = y + f(\frac{D}{p}, L) \tag{16}
$$

So that if $\theta = 0$:

$$p^{d=0} = \frac{\overline{M}}{y} \tag{17}$$

Under ASSUMPTION 3. $\Lambda > 1$ so that in a stationary equilibrium prices in no default are higher than prices in default.

Now taking households' strategies as given I check that banks' best response is actually to play the strategies σ_t^B as previously constructed. I want to show that when $\theta_{t+1} = 0$ the λ banks that behave mechanically make negative profits ($\pi_(t + 1 < 0$) and that the $(1 - \lambda)$ strategic banks want to default. I also want to show that when $\theta_{t+1} = 1$ the λ banks that behave mechanically make positive profits ($\pi_{t+1} < 0$) and that the $(1 - \lambda)$ strategic banks don't want to default.

Now let $\theta_{t+1} = 0$. Since households' strategy calls for not depositing in a bank that has defaulted then the payoff from defaulting and not defaulting ($d_{t+1} = 0$ and $d_{t+1} = 1$) are:

$d_{t+1} = 0 \;\; f_{t+1}(\frac{D_t}{p_t}, L)$

$d_{t+1} = 1 \;\; f_{t+1}(\frac{D_t}{p_t}, L) - \frac{R_t D_t}{p_{t+1}} + W_{t+1}^{d_{t+1}=1}$

where $W_{t+1}^{d_{t+1}=1}$ denotes the present continuation value of not defaulting at time t.

Let the strategic banks believe that the λ banks that behave mechanically will make negative profits. I want to show that if the strategic banks don't default then $\frac{R_t D_t}{p_{t+1}} > W_{t+1}^{d_{t+1}=1}$.

Suppose the $(1 - \lambda)$ banks don't default:

Then the resource constraint at $t + 1$ is:

$$\lambda c_{1t+1}^d + \lambda c_{2t+1}^d + (1 - \lambda) c_{1t+1}^{nd} + (1 - \lambda) c_{2t+1}^{nd} + (1 - \lambda)\frac{D_{t+1}}{p_{t+1}} = y + (1 - \lambda)\frac{R_t D_t}{p_{t+1}}$$

$$\lambda\frac{\bar{M}}{p_{t+1}} + (1-\lambda)\frac{\bar{M}}{p_{t+1}} + (1-\lambda)c^{nd}_{2t+1} = y + (1-\lambda)\frac{R_t D_t}{p_{t+1}}$$

The idea is now to solve for the price that would result in equlibrium as a function of λ and then check for what values of λ the $(1-\lambda)$ banks will prefer to default and then check that actually for the λ banks it is the case that $\pi_{t+1} < 0$

Using also one Euler equation:

$$\begin{aligned}
\bar{M} + (1-\lambda)p_{t+1}c^{nd}_{2t+1} &= p_{t+1}y + (1-\lambda)R_t D_t \\
\frac{U_{1t}}{p_t} &= \beta\pi(1/1)R_t\frac{U_{2t+1}}{p_{t+1}}
\end{aligned}$$

$$\begin{aligned}
\bar{M} + (1-\lambda)p_{t+1}c^{nd}_{2t+1} &= p_{t+1}y + (1-\lambda)R_t D_t \\
\frac{1}{p_t c^{nd}_{1t}} &= \beta\pi(1/1)R_t\frac{1}{p_{t+1}c^{nd}_{2t+1}}
\end{aligned}$$

$$\begin{aligned}
\bar{M} + (1-\lambda)p_{t+1}c^{nd}_{2t+1} &= p_{t+1}y + (1-\lambda)R_t D_t \\
\frac{1}{\bar{M} - D_t} &= \frac{\beta\kappa}{p_{t+1}c^{nd}_{2t+1}}
\end{aligned}$$

$$\begin{aligned} (1-\lambda)p_{t+1}c_{2t+1}^{nd} &= p_{t+1}y + (1-\lambda)R_tD_t - \bar{M} \\ p_{t+1}c_{2t+1}^{nd} &= \beta\kappa(\bar{M} - D_t) \end{aligned}$$

So that:

$$(1-\lambda)[\beta\kappa\bar{M} - \beta\kappa D_t] = p_{t+1}y + (1-\lambda)\frac{\kappa}{\pi(1/1)}D_t - \bar{M}$$

Suppose we were starting from a stationary equilibrium at time t and $D_t = \phi\bar{M}$

Then:

$$p_{t+1} = \frac{(1-\lambda)[\beta\kappa\bar{M} - \beta\kappa D_t] + \bar{M} - (1-\lambda)\frac{\kappa}{\pi(1/1)}\phi\bar{M}}{y}$$

$$p_{t+1} = \frac{\bar{M}[\beta\kappa(1-\phi)(1-\lambda) + 1 - (1-\lambda)\frac{\kappa\phi}{\pi(1/1)}]}{y}$$

Now I want to find a λ such that also the $(1-\lambda)$ strategic banks will want to default: that is to say such that $\frac{R_tD_t}{p_{t+1}} > W_{t+1}^{d_{t+1}=1}$

Suppose that at time $t+1$ the realization of the sunspot (that still coordinates $(1-\lambda)$ strategic banks beliefs about what the other λ banks will do) is $\theta_{t+1} = 0$

Then the discounted continuation value from non defaulting at $t+1$ is:

$$\begin{aligned} W_{t+1}^{d_{t+1}=1} &= \delta[\pi(0/0)f_{t+2} + \pi(1/0)f_{t+2} - \pi(1/0)\frac{R_{t+1}D_{t+1}}{p_{t+2}}] + \\ &\quad \delta^2\pi(1/0)[\pi(0/1)f_{t+3} + \pi(1/1)f_{t+3} - \pi(1/1)\frac{R_{t+2}D_{t+2}}{p_{t+3}}] + \\ &\quad \delta^3\pi(1/0)\pi(1/1)[\pi(0/1)f_{t+4} + \pi(1/1)f_{t+4} - \pi(1/1)\frac{R_{t+3}D_{t+3}}{p_{t+4}}] + ... \\ &= \delta L + \delta^2\pi(1/0)L + \delta^3\pi(1/0)\pi(1/1)L + \delta^4\pi(1/0)\pi(1/1)^2L + ... \\ &= \delta L[1 + \delta\pi(1/0)(1 + \delta\pi(1/1) + \delta^2\pi(1/1)^2 + ...)] \\ &= \delta L[1 + \frac{\delta\pi(1/0)}{1-\delta\pi(1/1)}] \end{aligned}$$

At time $t+1$ the nominal value of paying depositors back is: $R_tD_t = \frac{\kappa}{\pi(1/1)}\phi\bar{M}$

So $(1-\lambda)$ strategic banks will default if:

$$\frac{\kappa}{\pi(1/1)}\phi\bar{M}\frac{y}{\bar{M}[\beta\kappa(1-\phi)(1-\lambda) + 1 - (1-\lambda)\frac{\kappa\phi}{\pi(1/1)}]} > \delta L[1 + \frac{\delta\pi(1/0)}{1-\delta\pi(1/1)}]$$

$$\frac{\kappa\phi y}{\pi(1/1)[\beta\kappa(1-\phi)(1-\lambda) + 1 - (1-\lambda)\frac{\kappa\phi}{\pi(1/1)}]} > \delta L[1 + \frac{\delta\pi(1/0)}{1-\delta\pi(1/1)}] \tag{18}$$

For $\lambda = 1$ I can just make y big enough with respect to L and done.

So for λ close to 1 (18) must also hold.

However for the $(1-\lambda)$ strategic banks to be willing to default I also need $\pi_{t+1} < 0$

That is:

$$\frac{\kappa\phi\bar{M}}{p_t} + L - \frac{\kappa\phi\bar{M}}{\pi(1/1)p_{t+1}} < 0$$

with $p_t = p^{nd} = \frac{\bar{M}}{y}\Lambda$

Let $\Gamma = [\beta\kappa(1-\phi)(1-\lambda) + 1 - (1-\lambda)\frac{\kappa\phi}{\pi(1/1)}]$

Then $p_{t+1} = \frac{\bar{M}}{y}\Gamma$

So that

$$\frac{\kappa\phi\bar{M}}{\bar{M}}\frac{y}{\Lambda} + L < \frac{\kappa\phi\bar{M}}{\bar{M}}\frac{y}{\Gamma}\frac{1}{\pi(1/1)}$$

$$L < \kappa\phi y[\frac{1}{\pi(1/1)\Gamma} - \frac{1}{\Lambda}]$$

$$L + \frac{\kappa\phi y}{\Lambda} < \frac{\kappa\phi y}{\pi(1/1)\Gamma} \tag{19}$$

At the same time we can rewrite (18) as:

$$\frac{\kappa\phi y}{\pi(1/1)\Gamma} > \delta L[1 + \frac{\delta\pi(1/0)}{1-\delta\pi(1/1)}]$$

So that $(1-\lambda)$ strategic banks to be willing to default if:

$$\frac{\kappa\phi y}{\pi(1/1)\Gamma} > \max(\delta L[1+\frac{\delta\pi(1/0)}{1-\delta\pi(1/1)}], L+\frac{\kappa\phi y}{\Lambda}) \tag{20}$$

And at previous nodes when $\theta = 1$ I want no default so I want also:

$$\frac{\delta L}{1-\delta\pi(1/1)} > \frac{\kappa D}{\pi(1/1)p} = \frac{\kappa\phi\bar{M}y}{\pi(1/1)\bar{M}\Lambda}$$

$$\frac{\delta L}{1-\delta\pi(1/1)} > \frac{\kappa\phi y}{\pi(1/1)\Lambda} \tag{21}$$

TO BE COMPLETED

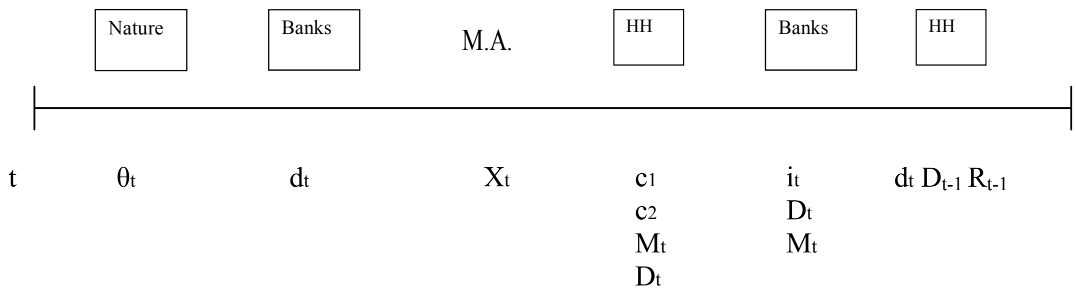
■

4. The game with a Monetary authority

Let the game be modified so that there is another player, called Monetary Authority endowed with a printing money technology. I am going to make two different assumptions about the information set where the Monetary Authority is moving in each period and I will characterize the equilibria of each resulting game.

A. Monetary authority moving after banks' default

Let the Monetary Authority move right after banks took their default decisions. So the relevant history of the game includes past and current realizations of θ and default decisions, therefore the Monetary Authority action can be contingent on both since both are publicly observable. The timing of Monetary Authority' s move is represented on a timeline in the following figure:



Let the Monetary Authority behave mechanically, as follows: if the current realization of Θ is $\theta_t = 0$ and a positive measure of banks default then inject money on the asset market to households in the amount X_t, otherwise don't take any action. The amount of the money injection X_t is just enough to keep current prices constant with respect to the previous period, and it is going to depend on the measure of banks that defaulted in the current period: if all banks defaulted then $X_t = p^{d=1}y - \overline{M}$ so that the new stock of money supply is $\overline{M}' = p^{d=1}y$, if a smaller measure of banks is defaulting then X_t is going to be smaller, but always just enough to keep prices at the stationary non default level $p^{d=1}$. As far as the next two proposition are concerned let the Monetary Authority be able to print money only the first time t that a $\theta_t = 0$ is realized: the Monetary Authority first announces if it is going to print money soon after the realization of θ_t and it injects the amount necessary to keep prices constant after banks decided whether to default or not. Let players be unable to observe if the Monetary Authority is active or not, that is to say unless the Monetary Authority announces that is going to inject money if needed, players move as if the Monetary Authority was not a player in the game. With this specification the game has no longer the default and no deposit equilibrium when $\theta = 0$, as the following proposition states.

Proposition 2. *Let assumptions (1.)-(3.) be satisfied and* $\lambda : \frac{\kappa\phi y}{\pi(1/1)\Gamma} > \max(\delta L(1+\frac{\delta\pi(0/1)}{1-\delta\pi(1/1)}), L+\frac{\kappa\phi y}{\Lambda})$*. Let the economy be in a stationary equilibrium where the initial level of deposits is the same as the stationary amount of deposits* $D_t = D$*. Let* $\theta_T = 0$ *with* $\theta_s = 1 \forall s < T$*. Let Monetary Authority announce that if a positive measure of banks default it is going to inject money in the amount necessary to keep prices constant. Then default is not an equilibrium strategy for banks.*

Proof. When $\theta = 0$ the $(1-\lambda)$ strategic banks' best response to other banks defaulting is to default if $\frac{R_t D_t}{p_{t+1}} > W_{t+1}^{d_{t+1}=1}$. However the Monetary Authority is going to inject money on the asset market to households in the exact amount needed to have prices maintained at $p^{d=1}$. At these prices banks' payoff from non defaulting is larger than the payoff from defaulting by (21), therefore default is not a best response to other banks defaulting. ■

The next step is to show that non default is indeed the equilibrium strategy of the modified game:

Proposition 3. *Maintain all the assumptions of Proposition 2. Let* $\theta_T = 0$ *with* $\theta_s = 1 \forall s < T$*. Let Monetary Authority announce that if a positive measure of banks default it is going to inject money in the amount necessary to keep prices constant. Then no default is an equilibrium strategy for banks.*

Proof. Same argument as in Proposition 2. ■

5. References

Fisher,Irving The Debt Deflation Theory of the Great Depression, Econometrica, Vol.1, (1933), pp.337-57

Friedman, Milton and A. J. Schwartz, Monetary History of the United States, 1867-1960, Princeton University Press, (1963) Lucas,Robert E. and Nancy L. Stokey, Money and Interest in a Cash-in-Advance Economy , Econometrica, Vol.55, No.3. (May 1987), pp.491-513