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This paper addresses the positive implications of indexing risky debt to observable aggregate conditions. These issues are pursued within the context of the celebrated financial accelerator model of Bernanke, Gertler, and Gilchrist (1999). The principal conclusions include: (1) the estimated level of indexation is significant, (2) the business cycle properties of the model are significantly affected by this degree of indexation.

JEL codes: E32 and E44.

Key Words: agency costs, financial accelerator, and business cycles.

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1. Introduction.

The fundamental function of credit markets is to channel funds from savers to entrepreneurs who have some valuable capital investment project. These efforts are hindered by agency costs arising from asymmetric information. A standard result in a subset of this literature, the costly state verification (CSV) framework, is that risky debt is the optimal contract between risk-neutral lenders and entrepreneurs. The modifier risky simply means that there is a non-zero chance of default. In the CSV model external parties can observe the realization of the entrepreneur's idiosyncratic production technology only by expending a monitoring cost. Townsend (1979) demonstrates that risky debt is optimal in this environment because it minimizes the need for verification of project outcomes. This verification is costly but necessary to align the incentives of the firm with the bank.

Aggregate conditions will also affect the ability of the borrower to repay the loan. But since aggregate variables are observed by both parties, it may be advantageous to have the loan contract indexed to the behavior of aggregate variables. That is, why should the loan contract call for costly monitoring when the event that leads to a poor return is observable by all parties?[2] Carlstrom, Fuerst, and Paustian (2012) examine questions of this type within the financial accelerator of Bernanke, Gertler, and Gilchrist (1999), hereafter BGG. Carlstrom et al. (2012) demonstrate that the privately optimal contract in the BGG model includes indexation of the loan repayment to the aggregate return to capital.[3]

In this paper we explore the business cycle implications of indexing the BGG loan contract to the aggregate return to capital. There are of course many other aggregate variables to which the contract could be indexed, and our particular choice is somewhat ad hoc. But there are several reasons to begin with indexation to the return on capital. First, the return on capital is a natural choice as it is fundamental to the outcome of the project in the BGG framework. Second, BGG implicitly impose a particular form of return-to-capital indexation. Third, since we are assuming that the CSV framework proxies for agency

[2]This is the logic behind Shiller and Weiss's (1999) suggestion of indexing home mortgages to movements in aggregate house prices.
[3] The privately optimal contract also includes indexation to household consumption but we do not pursue this possibility here.

cost effects in the entire US financial system, it seems reasonable to include some form of indexation to mimic the myriad ex post returns on external financing. For example, in contrast to the model assumption where entrepreneurs get zero in the event of bankruptcy, this is clearly not the implication of Chapter 11 bankruptcy. In any event, we do not model the choice of indexation, but instead assume that it is imposed by some mechanism outside the model. We use familiar Bayesian methods to estimate the degree of indexation.

To avoid misspecification problems in the estimation we need a complete model of the business cycle. We use the recent contribution of Justiniano, Primiceri, and Tambalotti (2011), hereafter JPT, as our benchmark. A novelty of the JPT model is that it includes two shocks to the capital accumulation technology. The first shock is a non-stationary shock to the relative cost of producing investment goods, the "investment specific technology shock" (IST). The second is a stationary shock to the transformation of investment goods into installed capital, the "marginal efficiency of investment shock" (MEI). For business cycle variability, JPT find that the IST shocks are irrelevant, while the MEI shocks account for a substantial portion of business cycle fluctuations. JPT note that the MEI shocks might be interpreted as shocks to agency costs. That is, in a model with financial frictions such as Carlstrom and Fuerst (1997), shocks to borrower net worth will look like MEI shocks.

Our principal results include the following. First, the estimated level of indexation significantly exceeds unity, much higher than the assumed BGG indexation of approximately zero. Further the indexation model is a significantly better fit to the data when compared to BGG. We also compare the financial models to JPT. The financial models make predictions on the behavior of the risk premium on which JPT is silent. We nest the JPT model by treating fluctuations in the risk premium as i.i.d. measurement error. This model horserace results in the indexation model dominating BGG, which in turn dominates JPT.

Second, under the estimated level of indexation, the financial model and JPT have remarkably similar business cycle properties, ie., the estimated level of indexation leads to movements in net worth in the financial model that accommodate real behavior quite similar to JPT. We see this in two ways. First, the economic response to an MEI shock is remarkably similar in JPT and the indexation model. Second, the variance decomposition of real variables is quite similar in JPT and in the indexation model. The only significant difference between the two models is that the indexation model makes predictions for the risk premium.

Third, we find that whether financial shocks or MEI shocks are more important drivers of the business cycle depends upon the level of indexation. Under BGG, financial shocks account for a significant part of the variance of investment spending. But under the estimated level of indexation, financial shocks become much less important and the MEI shocks rise in importance.

The paper proceeds as follows. Section 2 presents a simple example that illustrates the importance of contract indexation to the financial accelerator. Section 3 develops the DSGE model. Section 4 presents the estimation results. Section 5 concludes.

2. Why does indexation matter? A simple example.

This section presents a simple intuitive example that demonstrates the importance of indexation in determining the size of the financial accelerator. Consider a world with agency costs in which the portion of net worth owned by entrepreneurs (nw_t) has a positive effect on the value of capital (q_t):

$$q_t = p * nw_t + \epsilon_t^d \tag{1}$$

where the expression is in log deviations and ϵ_t^d is an exogenous shock to capital prices, eg., a shock to MEI in the general equilibrium model below. Equation (1) is a manifestation of agency costs in that the distribution of net worth across lenders and borrowers affects asset prices. The idea is that higher net worth in the hands of entrepreneurs makes it easier for them to access a loan with which to buy capital, so that higher levels of net worth act like a demand channel on asset prices. In the general equilibrium model below, the value of p is a function of the agency cost and (installed) capital adjustment cost parameters.

The entrepreneur accumulates net worth to mitigate the agency problems involved in direct lending. The agency problem arises from a CSV problem in the entrepreneur's production technology. The entrepreneur takes one unit of input and creates ω_t units of capital, where the unit-mean random variable ω_t is privately observed by the entrepreneur but can be verified by the lender only by paying a cost. This CSV problem makes equity finance problematic, so that the optimal contract is given by a risky debt contract with a promised repayment of r_t^p. The repayment r_t^p cannot be indexed to ω_t because it is privately observed. But it can be indexed to the aggregate price of capital:

$$r_t^p = \chi q_t. \tag{2}$$

Entrepreneurial net worth accumulates with the profit flow from the investment project, but decays via consumption of entrepreneurs (which is a constant fraction of net worth). Log-linearized this evolution is given by:

$$nw_t = \kappa\left(q_t - r_t^p\right) + nw_{t-1} + r_t^p + \epsilon_t^n \tag{3}$$

where $\kappa > 1$ denotes leverage (the ratio of project size to net worth) and ϵ_t^n is an exogenous shock to net worth. Using the indexation assumption (2), we can express (3) as

$$nw_t = q_t[\kappa - \chi(\kappa - 1)] + nw_{t-1} + \epsilon_t^n \tag{4}$$

Note that since $\kappa > 1$, the slope of the net worth equation is decreasing in the level of indexation.

Equations (1) and (4) are a simultaneous system in net worth and the price of capital. We can solve for the two endogenous variables as a function of the pre-determined and exogenous variables:

$$nw_t = \frac{nw_{t-1} + \epsilon_t^n + \epsilon_t^d[\kappa - \chi(\kappa - 1)]}{\{1 - p[\kappa - \chi(\kappa - 1)]\}} \tag{5}$$

$$q_t = \frac{p(nw_{t-1} + \epsilon_t^n) + \epsilon_t^d}{\{1 - p[\kappa - \chi(\kappa - 1)]\}} \tag{6}$$

The inverse of the denominator in (5)-(6) is the familiar "multiplier" arising from two endogenous variables with positive feedback. This then implies that exogenous shocks are "multiplied" or "financially accelerated", and that the degree of this multiplication depends upon the level of indexation. The effect of indexation on the financial multiplier is highly nonlinear. Figure 1 plots the multiplier for κ

= 2, and p = 0.45, both of these values roughly correspond to the general equilibrium analysis below. Note that moving from χ = 0 to χ = 1, has an enormous effect on the multiplier. But there are sharp diminishing returns so the multiplier is little changed as we move from χ = 1 to χ = 2.

Consider three special cases of indexation: $\chi = 0$, $\chi = 1$, and $\chi = {}^{\kappa}/_{(\kappa - 1)}$. The first is the implicit assumption in BGG; the second implies complete indexation; the third eliminates the financial accelerator altogether. In these cases, net worth and asset prices are given by:

Indexation	**Net worth**	**Capital price**	**Multiplier** (p=0.45, κ =2)
$\chi = 0$	$\dfrac{nw_{t-1} + \epsilon_t^n + \kappa\epsilon_t^d}{1 - p\kappa}$	$\dfrac{p(nw_{t-1} + \epsilon_t^n) + \epsilon_t^d}{1 - p\kappa}$	10
$\chi = 1$	$\dfrac{nw_{t-1} + \epsilon_t^n + \epsilon_t^d}{1 - p}$	$\dfrac{p(nw_{t-1} + \epsilon_t^n) + \epsilon_t^d}{1 - p}$	1.82
$\chi = {}^{\kappa}/_{(\kappa - 1)}$.	$nw_{t-1} + \epsilon_t^n$	$p(nw_{t-1} + \epsilon_t^n) + \epsilon_t^d$	1

For both $\chi = 0$ and $\chi = 1$, exogenous shocks to asset prices and net worth have multiple effects on the equilibrium levels of net worth and capital prices. Since $\kappa > 1$, this effect is much larger under BGG's assumption of no indexation ($\frac{1}{1-p\kappa} \gg \frac{1}{1-p}$). Further, under the BGG assumption, exogenous shocks to asset prices (ϵ_t^d) have an added effect as they are weighted by leverage. But for all levels of indexation, there are always agency cost effects in that the price of capital is affected by the level of entrepreneurial net worth. The financial multiplier effects are traced out in Figure 2: an exogenous shock to asset prices has a much larger effect on both net worth and asset prices in the BGG framework. Finally, since $\kappa \approx 2$ the financial accelerator largely disappears when $\chi = 2$.

Before proceeding, it is helpful to emphasize the two parameters that are crucial in our simple example as they will be manifested below in the richer general equilibrium environment. Our reduced form parameter p in equation (1) is the agency cost parameter. In a Modigliani-Miller world we would have p = 0, as the distribution of net worth would have no effect on asset prices or real activity. Second, the indexation parameter χ determines the size of the financial accelerator, ie., how do unexpected movements in asset prices feed in to net worth? These are two related but logically distinct ideas. That is, one can imagine a world with agency costs (p > 0), but with very modest accelerator effects ($\chi = {}^{\kappa}/_{(\kappa - 1)}$). To anticipate our empirical results, this is the parameter set that wins the model horse race. That is, the data is consistent with an agency cost model but with trivial accelerator effects. In such an environment, shocks to net worth will affect real activity, but other real shocks (eg., MEI shocks) will not be accelerated.

3. The Model.

The benchmark model follows the JPT framework closely. The model of agency costs comes from BGG with the addition of exogenous contract indexation. The BGG loan contract is between lenders and entrepreneurs, so we focus on these two agents first before turning to the familiar framework of JPT.

Lenders.

The representative lender accepts deposits from households (promising a sure real return R_t^d) and provides loans to the continuum of entrepreneurs. These loans are intertemporal, with the loans made at the end of time t being paid back in time t+1. The realized gross real return on these loans is denoted by R_{t+1}^L. Each individual loan is subject to idiosyncratic and aggregate risk, but since the lender holds an entire portfolio of loans, only the aggregate risk remains. The lender has no other source of funds, so the level of loans will equal the level of deposits. Hence, real dividends are given by $Div_{t+1} = (R_{t+1}^L D_t - R_t^d D_t)$. The intermediary seeks to maximize its equity value which is given by:

$$Q_t^L = E_t \sum_{j=1}^{\infty} \frac{\beta^j \Lambda_{t+j}}{\Lambda_t} Div_{t+j} \tag{9}$$

where Λ_t is the marginal utility of real income for the representative household that owns the lender.

The FOC of the lender's problem is:

$$E_t \frac{\Lambda_{t+1}}{\Lambda_t}\left[R_{t+1}^L - R_t^d\right] = 0 \tag{10}$$

The first-order condition shows that in expectation, the lender makes zero profits, but ex-post profits and losses can occur. We assume that losses are covered by households as negative dividends. This is similar to the standard assumption in the Dynamic New Keynesian (DNK) model, eg., Woodford (2003). That is, the sticky price firms are owned by the household and pay out profits to the household. These profits are typically always positive (for small shocks) because of the steady state mark-up over marginal cost. Similarly, one could introduce a steady-state wedge (eg., monopolistic competition among lenders) in the lender's problem so that dividends are always positive. But this assumption would have no effect on the model's dynamics so we dispense from it for simplicity.

Entrepreneurs and the Loan Contract.

Entrepreneurs are the sole accumulators of physical capital. At the beginning of period t, the entrepreneurs sell all of their accumulated capital to "capital agencies" at beginning-of-period capital price Q_t^{beg}. At the end of the period, the entrepreneurs purchase the entire capital stock $\overline{K}_t$, including any net additions to the stock, at end-of-period price Q_t. This re-purchase of capital is financed with entrepreneurial net worth (NW_t) and external financing from a lender. The external finance takes the form of a one period loan contract. The gross return to holding capital from time-t to time t+1 is given by:

$$R_{t+1}^k \equiv \frac{Q_{t+1}^{beg}}{Q_t}. \tag{11}$$

Below we will show that $Q_{t+1}^{beg} = Q_{t+1}(1-\delta) + [\rho_{t+1}u_{t+1} - a(u_{t+1})] \approx Q_{t+1}(1-\delta) + \rho_{t+1}$, the latter term coinciding with the expression in BGG. Variations in R_{t+1}^k are the source of aggregate risk in the loan contract. The external financing is subject to a costly-state-verification (CSV) problem because of idiosyncratic risk. In particular, one unit of capital purchased at time-t is transformed into ω_{t+1} units of capital in time t+1, where ω_{t+1} is a idiosyncratic random variable with density $\phi(\omega)$ and cumulative distribution $\Phi(\omega)$. The realization of ω_{t+1} is directly observed by the entrepreneur, but the lender can observe the realization only if a monitoring cost is paid. Assuming that the entrepreneur and lender are risk-neutral, Townsend (1979) demonstrates that the optimal contract between entrepreneur and intermediary is risky debt in which monitoring only occurs if the promised payoff is not forthcoming. Payoff does not occur for sufficiently low values of the idiosyncratic shock, $\omega_{t+1} < \varpi_{t+1}$. Let R^p_{t+1} denote the promised gross rate-of-return so that R^p_{t+1} is defined by

$$R^p_{t+1}(Q_t\bar{K}_t - NW_t) \equiv \varpi_{t+1}R^k_{t+1}Q_t\bar{K}_t \,. \tag{12}$$

We find it convenient to express this in terms of the leverage ratio $\bar{\kappa}_t \equiv \left(\frac{Q_t\bar{K}_t}{NW_t}\right)$ so that (4) becomes

$$R^p_{t+1} \equiv \varpi_{t+1}R^k_{t+1}\frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} \tag{13}$$

With $f(\varpi_{t+1})$ and $g(\varpi_{t+1})$ denoting the entrepreneur's share and lender's share of the project outcome, respectively, the lender's ex post realized t+1 return on the loan contract is defined as:

$$R^L_{t+1} \equiv \frac{R^k_{t+1}g(\varpi_{t+1})Q_t\bar{K}_t}{Q_t\bar{K}_t - NW_t} = R^k_{t+1}g(\varpi_{t+1})\frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} \tag{14}$$

where

$$f(\varpi) \equiv \int_{\varpi}^{\infty}\omega\phi(\omega)d\omega - [1 - \Phi(\varpi)]\varpi \tag{15}$$

$$g(\varpi) \equiv [1 - \Phi(\varpi)]\varpi + (1 - \mu)\int_0^{\varpi}\omega\phi(\omega)d\omega \tag{16}$$

Recall that the lender's return is linked to the return on deposits via (2):

$$E_tR^L_{t+1}\Lambda_{t+1} = R^d_tE_t\Lambda_{t+1} \tag{17}$$

Since R^k_{t+1} is publicly observed, we consider contract indexation schemes in which R^p_{t+1} and ϖ_{t+1} are functions of a pre-determined portion (Ω_t) and the time t+1 realization of R^k_{t+1}:

$$\varpi_{t+1} \equiv \Omega_t\left(R^k_{t+1}\right)^{\chi - 1} \tag{18}$$

$$R^p_{t+1} = \Omega_t\left(R^k_{t+1}\right)^{\chi}\frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} \tag{19}$$

Since we linearize the model, this assumed functional form is with no loss of generality. As we vary the indexation parameter χ, we trace out a variety of possible indexation schemes. Different indexation schemes then imply different behavior for the lender's return. For example, for $\chi = 1$, the bankruptcy rate is predetermined, while the loan repayment R_{t+1}^{p} and lender's return R_{t+1}^{L} are perfectly indexed to innovations in R_{t+1}^{k}.

There are of course many other t+1 variables to which the contract could be indexed, and our particular choice is somewhat ad hoc. But as noted earlier, there are several reasons to begin with indexation to the return on capital. First, the return on capital is a natural choice as it is fundamental to the outcome of the project. Second, the influential BGG model implicitly imposes a particular form of R^k indexation. That is, BGG assume that the lender's return is predetermined, $R_{t+1}^{L} = R_t^d$. This implies $\chi \approx$ -0.01, so that the loan repayment is largely independent of innovations in R_{t+1}^{k}. Third, since we are assuming that the CSV structure proxies for the agency cost effects in the entire US financial system, it seems reasonable to include some form of indexation to mimic the myriad ex post returns on external financing. In any event, we do not model the choice of χ, but instead assume that it is imposed by some mechanism outside the model.

The choice variables for the contracting problem are $\overline{K}_t$ and the pre-determined part of the repayment Ω_t. For a given indexation parameter χ, the end-of-time-t contracting problem is thus given by:

$$max_{\overline{K}_t,\Omega_t}\{E_t R_{t+1}^{k} Q_t \overline{K}_t f(\varpi_{t+1})\} \tag{20}$$

subject to

$$\varpi_{t+1} \equiv \Omega_t \left(R_{t+1}^{k}\right)^{\chi-1} \tag{21}$$

$$E_t R_{t+1}^{k} Q_t \overline{K}_t \Lambda_{t+1} g(\varpi_{t+1}) \geq R_t^d E_t \Lambda_{t+1}[Q_t \overline{K}_t - NW_t] \tag{22}$$

After some re-arrangement, the optimization conditions include:

$$(\bar{\kappa}_t - 1) E_t R_{t+1}^{k} f(\varpi_{t+1}) = \frac{-E_t \varpi_{t+1} f'(\varpi_{t+1})}{E_t \varpi_{t+1} \Lambda_{t+1} g'(\varpi_{t+1})} E_t \Lambda_{t+1} R_{t+1}^{k} g(\varpi_{t+1}) \tag{23}$$

$$E_t \Lambda_{t+1} R_{t+1}^{k} \frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} g(\varpi_{t+1}) = R_t^d E_t \Lambda_{t+1} \tag{24}$$

The contract is defined by Ω_t and leverage ratio $\bar{\kappa}_t$ that satisfy (15)-(16).

Entrepreneurs have linear preferences and discount the future at rate β. Given the high return to internal funds, they will postpone consumption indefinitely. To limit net worth accumulation and ensure that there is a need for external finance in the long run, we assume that fraction (1-γ) of the entrepreneurs die each period. Their accumulated assets are sold and the proceeds transferred to households as consumption. Given the exogenous death rate, aggregate net worth accumulation is described by

$$\mathrm{NW_t} = \gamma NW_{t-1}\bar{\kappa}_{t-1}R_t^k f(\varpi_t)\eta_{nw,t} \quad (25)$$

where $\eta_{nw,t}$ is an exogenous disturbance to the distribution of net worth. We assume it follows the stochastic process

$$log\,\eta_{nw,t} = \rho_{nw} log\,\eta_{nw,t-1} + \varepsilon_{nw,t}, \quad (26)$$

with $\varepsilon_{nw,t}$ *i.i.d.* N(0,σ_{nw}^2). Equation (25) implies that $\mathrm{NW_t}$ is determined by the realization of R_t^k and the response of ϖ_t to these realizations. NW_t then enters the contracting problem in time t so that the realization of R_t^k is propagated forward.

As in Christiano, Motto, and Rostagno (2010), and Gilchrist, Ortiz and Zakrajšek (2009), we also consider time variation in the variance of the idiosyncratic shock ω_t. The variance of ω_t is denoted by σ_t and follows the exogenous stochastic process given by

$$log\,\sigma_t = \rho_\sigma log\sigma_{t-1} + \varepsilon_{\sigma,t},$$

Shocks to this variance will alter the risk premium in the model.

Final good producers.

Perfectly competitive firms produce the final consumption good Y_t combining a continuum of intermediate goods according to the CES technology:

$$Y_t = \left[\int_0^1 Y_t(i)^{1/(1+\lambda_{p,t})}\,di\right]^{1+\lambda_{p,t}} \quad (27)$$

The elasticity $\lambda_{p,t}$ follows the exogenous stochastic process

$$log\,\lambda_{p,t} = \left(1-\rho_p\right) log\,\lambda_p + \rho_p\, log\,\lambda_{p,t-1} + \varepsilon_{p,t} - \theta_p\varepsilon_{p,t-1}, \quad (28)$$

where $\varepsilon_{p,t}$ is *i.i.d.* $N(0,\sigma_p^2)$. Fluctuations in this elasticity are price markup shocks. Profit maximization and the zero profit condition imply that the price of the final good, P_t, is the familiar CES aggregate of the prices of the intermediate goods.

Intermediate goods producers.

A monopolist produces the intermediate good *i* according to the production function

$$Y_t(i) = max\{A_t^{1-\alpha}K_t(i)^{\alpha}L_t(i)^{1-\alpha} - A_t\Upsilon_t^{\frac{\alpha}{1-\alpha}}F; 0\}, \tag{29}$$

where $K_t(i)$ and $L_t(i)$ denote the amounts of capital and labor employed by firm *i*. *F* is a fixed cost of production, chosen so that profits are zero in steady state. The variable A_t is the exogenous non-stationary level of TFP progress. Its growth rate ($z_t \equiv \Delta \ln A_t$) is given by

$$z_t = (1-\rho_z)\gamma_z + \rho_z z_{t-1} + \varepsilon_{z,t}, \tag{30}$$

with $\varepsilon_{z,t}$ *i.i.d.*$N(0,\sigma_z^2)$. The other non-stationary process Υ_t is linked to the investment sector and is discussed below.

Every period a fraction ξ_p of intermediate firms cannot choose its price optimally, but resets it according to the indexation rule

$$P_t(i) = P_{t-1}(i)\pi_{t-1}^{\iota_p}\pi^{1-\iota_p}, \tag{31}$$

where $\pi_t \equiv P_t/P_{t-1}$ is gross inflation and π is its steady state. The remaining fraction of firms chooses its price $P_t(i)$ optimally, by maximizing the present discounted value of future profits

$$E_t\left\{\sum_{s=0}^{\infty}\xi_p^s \frac{\beta^s \Lambda_{t+s}/P_{t+s}}{\Lambda_t/P_t}\left[P_t(i)\left(\prod_{k=1}^{s}\pi_{t+k-1}^{\iota_p}\pi^{1-\iota_p}\right)Y_{t+s}(i) - W_{t+s}L_{t+s}(i) - P_{t+s}\rho_{t+s}K_{t+s}(i)\right]\right\} \tag{32}$$

where the demand function comes from the final goods producers, Λ_t/P_t is the marginal utility of nominal income for the representative household, and W_t is the nominal wage.

Employment agencies

Firms are owned by a continuum of households, indexed by $j \epsilon [0,1]$. Each household is a monopolistic supplier of specialized labor, $L_t(j)$, as in Erceg et al. (2000). A large number of competitive employment agencies combine this specialized labor into a homogenous labor input sold to intermediate firms, according to

$$L_t = \left[\int_0^1 L_t(j)^{1/(1+\lambda_{w,t})}\, dj\right]^{1+\lambda_{w,t}} \tag{33}$$

As in the case of the final good, the desired markup of wages over the household's marginal rate of substitution, $\lambda_{w,t}$, follows the exogenous stochastic process

$$log\, \lambda_{w,t} = (1-\rho_w) log\lambda_w + \rho_w\, log\lambda_{w,t-1} + \varepsilon_{w,t} - \theta_w \varepsilon_{w,t-1}, \tag{34}$$

with $\varepsilon_{w,t}$ *i.i.d.*N (0, σ_w^2). This is the wage markup shock. Profit maximization by the perfectly competitive employment agencies implies that the wage paid by intermediate firms for their homogenous labor input is

$$W_t = \left[\int_0^1 W_t\,(j)^{-1/\lambda_{w,t}} dj\right]^{-\lambda_{w,t}} \tag{35}$$

<u>Capital agencies.</u>

The capital stock is managed by a collection of perfectly competitive capital agencies. These firms are owned by households and discount cash flows with Λ_t, the marginal utility of real income for the representative household. At the beginning of period t, these agencies purchase the capital stock $\overline{K}_{t-1}$ from the entrepreneurs at beginning-of-period price Q_t^{beg}. The agencies produce capital services by varying the utilization rate u_t which transforms physical capital into effective capital according to

$$K_t = u_t\,\overline{K}_{t-1}. \tag{36}$$

Effective capital is then rented to firms at the real rental rate ρ_t. The cost of capital utilization is $a(u_t)$ per unit of physical capital. The capital agency then re-sells the capital to entrepreneurs at the end of the period at price Q_t. The profit flow is thus given by:

$$Q_t(1-\delta)\,\overline{K}_{t-1} + [\rho_t u_t - a(u_t)]\,\overline{K}_{t-1} - Q_t^{beg}\,\overline{K}_{t-1} \tag{37}$$

Profit maximization implies

$$Q_t^{beg} = Q_t(1-\delta) + [\rho_t u_t - a(u_t)] \tag{38}$$

$$\rho_t = a'(u_t) \tag{39}$$

In steady state, u = 1, a(1) = 0 and $\vartheta \equiv$ a'' (1)/a'(1). Hence, in the neighbourhood of the steady state

$$Q_t^{beg} \approx Q_t(1-\delta) + \rho_t \tag{40}$$

which is consistent with BGG's definition of the intertemporal return to holding capital $R_t^k \equiv \frac{\rho_t + (1-\delta)Q_t}{Q_{t-1}}$.

New Capital Producers.

New capital is produced according to the production technology that takes $\mathrm{I_t}$ investment goods and transforms them into $\mu_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$ new capital goods. The time-t profit flow is thus given by

$$Q_t\mu_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t - P_t^I I_t \tag{41}$$

where P_t^I is the relative price of the investment good. The function S captures the presence of adjustment costs in investment, as in Christiano et al. (2005). The function has the following convenient steady state properties: $S = S' = 0$ *and* $S'' > 0$. These firms are owned by households and discount future cash flows with Λ_t, the marginal utility of real income for the representative household. JPT refer to the investment shock μ_t as a shock to the marginal efficiency of investment (MEI) as it alters the transformation between investment and installed capital. JPT conclude that this shock is the primary driver of output and investment at business cycle frequencies. The investment shock follows the stochastic process

$$log\mu_t = \rho_\mu log\mu_{t-1} + \varepsilon_{\mu,t}, \tag{42}$$

where $\varepsilon_{\mu,t}$ is *i.i.d.*N $(0, \sigma_\mu^2)$.

Investment Producers.

A competitive sector of firms produce investment goods using a linear technology that transforms one consumption good into Υ_t investment goods. The exogenous level of productivity Υ_t is non-stationary with a growth rate ($\upsilon_t \equiv \Delta\log\Upsilon_t$) given by

$$\upsilon_t = (1-\rho_\upsilon)\gamma_\upsilon + \rho_\upsilon \upsilon_{t-1} + \varepsilon_{\upsilon,t}.$$

The constant returns production function implies that the price of investment goods (in consumption units) is equal to $\frac{1}{\Upsilon_t}$.

Households.

Each household maximizes the utility function

$$E_t\left\{\sum_{s=0}^{\infty}\beta^s\ b_{t+s}\left[\ln(C_{t+s}-hC_{t+s-1})-\varphi\ \frac{L_{t+s}(j)^{1+\psi}}{1+\psi}\right]\right\}, \tag{43}$$

where C_t is consumption, h is the degree of habit formation and b_t is a shock to the discount factor. This intertemporal preference shock follows the stochastic process

$$logb_t = \rho_b logb_{t-1}+\varepsilon_{b,t}, \tag{44}$$

with $\varepsilon_{b,t}$ ~*i.i.d.*N (0, σ_b^2). Since technological progress is nonstationary, utility is logarithmic to ensure the existence of a balanced growth path. The existence of state contingent securities ensures that household consumption is the same across all households. The household's flow budget constraint is

$$C_t+T_t+D_t+\frac{B_t}{P_t}\leq\frac{R_{t-1}B_{t-1}}{P_t}+\frac{W_t(j)}{P_t}L_t(j)+D_{t-1}R_{t-1}^d+profits_t, \tag{45}$$

where D_t denotes real deposit at the lender, T_t is lump-sum taxes, and B_t is holdings of nominal government bonds that pay gross nominal rate R_t . The term $profits_t$ denotes the combined profit flow of all the firms owned by the representative agent including lenders, intermediate goods producers, capital agencies, and new capital producers. Every period a fraction ξ_w of households cannot freely set its wage, but follows the indexation rule

$$W_t(j) = W_{t-1}(j)(\pi_{t-1}e^{z_{t-1}+\frac{\alpha}{1-\alpha}\upsilon_t})^{\iota_w}(\pi e^{\gamma_z+\frac{\alpha}{1-\alpha}\gamma_\upsilon})^{1-\iota_w}, \tag{46}$$

The remaining fraction of households chooses instead an optimal wage $W_t(j)$ by maximizing

$$E_t\left\{\sum_{s=0}^{\infty}\xi_w^s\beta^s\left[-b_{t+s}\varphi\frac{L_{t+s}(j)^{1+\psi}}{1+\psi}+\frac{\Lambda_{t+s}}{P_{t+s}}W_t(j)L_{t+s}(j)\right]\right\} \tag{47}$$

subject to the labor demand function coming from the firm.

The government

A monetary policy authority sets the nominal interest rate following a feedback rule of the form

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\emptyset_\pi} \left(\frac{X_t}{X_t^*}\right)^{\phi_x}\right]^{1-\rho_R} \left[\frac{X_t/X_{t-1}}{X_t^*/X_{t-1}^*}\right]^{\phi_{dx}} \eta_{mp,t}, \tag{48}$$

where R is the steady state of the gross nominal interest rate. The interest rates responds to deviations of inflation from its steady state, as well as to the level and the growth rate of the GDP gap (X_t/X_t^*). The monetary policy rule is also perturbed by a monetary policy shock, $\eta_{mp,t}$, which evolves according to

$$log\, \eta_{mp,t} = \rho_{mp} log\, \eta_{mp,t-1} + \varepsilon_{mp,t}, \tag{49}$$

where $\varepsilon_{mp,t}$ is *i.i.d.*N $(0, \sigma_{mp}^2)$.

Fiscal policy is fully Ricardian. The government finances its budget deficit by issuing short term bonds. Public spending is determined exogenously as a time-varying fraction of output.

$$G_t = \left(1 - \frac{1}{g_t}\right) Y_t, \tag{50}$$

where the government spending shock g_t follows the stochastic process

$$log\, g_t = \left(1 - \rho_g\right) log\, g + \rho_g log g_{t-1} + \varepsilon_{g,t}, \tag{51}$$

with $\varepsilon_{g,t} \sim i.i.d. N\left(0, \sigma_g^2\right)$.

Market clearing.

The aggregate resource constraints are given by:

$$C_t + \frac{I_t}{Y_t} + G_t + a(u_t)\, \overline{K}_{t-1} = Y_t \tag{52}$$

$$\overline{K}_t = (1 - \delta)\, \overline{K}_{t-1} + \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{53}$$

This completes the description of the model. We now turn to the estimation of the linearized model.

4. Estimation.

The linearized version of the model equations are collected in the appendix. Two key agency cost parameters are the entrepreneurial survival rate (γ) and the elasticity of the risk premium to leverage (ν). These parameters are calibrated to match the long run or steady state level of the risk premium and leverage ratio. In particular, for a 200 bp annual risk premium (BAA-Treasury spread), and a leverage ratio of $\bar{\kappa}$ = 1.95, the model implies an entrepreneurial survival rate of γ = 0.98, and a risk premium elasticity of ν = 0.041. Similarly, steady state behaviour implies that we calibrate δ = 0.025, and (1-1/g) = 0.22. The remaining parameters are estimated using familiar Bayesian techniques. For the non-financial related parameters of the model we use the same priors as in JPT.

We treat as observables the growth rates of real GDP, consumption, investment, the real wage, and the relative price of investment. The other observables include employment, inflation, the nominal rate, and the risk premium. Employment is measured as the log of per capita hours. Inflation is the consumption deflator, and the nominal rate is the federal funds rate. The risk premium is the spread between the BAA and ten year Treasury. The time period for the estimation is 1954:3-2009:1. We choose the end of the sample period to avoid the observed zero bound on the nominal rate.

We estimate three versions of the model. Along with all the exogenous shocks outlined in the paper, we also include i.i.d. measurement error between the model's risk premium and the observed risk premium. The first model we label JPT as it corresponds to the model without agency costs (ν = 0). Note that to match the observed risk premium the JPT model will assign all risk premium variation to the i.i.d. measurement error. The remaining two models have operative agency costs (ν = 0.041). In the model labeled BGG we impose the level of indexation implicitly assumed by BGG, χ = - 0.01. For the model labeled Indexation, we estimate the value of χ, using a diffuse prior with a uniform distribution centered at 0 and with a standard deviation of 2. The two agency cost models also include two financial shocks: (i) time-varying movements in idiosyncratic risk, and (ii) exogenous redistributions of net worth. Both of these shocks are irrelevant in the JPT model in which lending is not subject to the CSV problem. We posit priors for the standard deviation and autocorrelation of these financial shocks in a manner symmetric with the non-financial exogenous processes in JPT.

The estimation results are summarized in Table 1. The two agency cost models dominate the JPT model as the JPT model cannot capture the forecastability of the risk premium.[4] Comparing the two agency models, the data rejects the BGG level of indexation preferring a level of contract indexation that is economically significant: χ = 1.84 with a 90% confidence interval between 1.56 and 2.13. As suggested by the example in section 2, this level of indexation will imply significantly different responses to shocks compared to the BGG assumption. We will see this manifested in the IRF below.

Two other differences in parameter estimates are worth some comment. First, the BGG model estimates a significantly smaller size for investment adjustment costs (S'') in the table: S'' = 1.51 for BGG, but 2.10 for Indexation, and 2.80 for JPT. The level of adjustment costs has two contrasting effects. First, lower adjustment costs will increase the response of investment to aggregate shocks. Second, lower adjustment costs imply smaller movements in the price of installed capital (Q_t) and thus smaller financial accelerator effects.

A second important difference in parameter estimates is in the standard deviation of the shocks. Compared to JPT, the BGG model estimates a significantly smaller volatility in the MEI shocks, and instead shifts this variance on to net worth shocks. Recall that the principal conclusion of JPT is the importance of the MEI shocks in the business cycle. An interesting question we take up below is why the BGG model downplays these shocks so significantly.

As a form of sensitivity analysis, Table 1 also includes results when the agency cost elasticity ν is estimated. For the Indexation model, the estimate is ν = 0.034, with confidence bands that include the calibrated value of ν = 0.041. The other parameter estimates are quite close to the values obtained when calibrating ν. Further, the model with ν calibrated is slightly preferred to the model where ν is estimated. We thus conclude that imposing the agency cost elasticity is with little loss of generality.

[4] If we allow for serially correlated measurement error, then the JPT model and the Indexation model fit similarly.

Table 2 reports the variance decomposition of four key variables: GDP, investment, net worth, and the risk premium. The JPT results are replicated here: the MEI shocks account for a substantial amount of business cycle variability in GDP (71% at the 8-quarter horizon) and investment (83% at the 8-quarter horizon). This conclusion is largely unchanged in the Indexation model. Evidently the estimated level of indexation results in real behaviour similar to a model without agency costs. This is particularly clear in the IRFs presented in Figure 3 that we discuss below.

In contrast to the Indexation model, BGG places much less weight on the MEI shocks and instead shifts this variance to the financial shocks (the idiosyncratic variance and net worth shocks) and the monetary policy shock. For the case of investment at the 8-quarter horizon, the BGG model places 29% of the variance on the MEI shocks (compared to 77% for Indexation and 83% for JPT). The importance of the two financial shocks increases from 12% under Indexation, to 33% for BGG. Under BGG, the monetary policy shock increases to 12% of the 8-quarter investment variance compared to 2% for Indexation and 4% for JPT.

Why does the BGG model downplay the MEI shocks? The answer is quite apparent from Figure 3a. The figure sets all parameter values to those estimated in the Indexation model, except for the level of indexation ($\chi \approx 0$ for BGG), and the level of agency cost effects ($\nu = 0$ for JPT).[5] A positive innovation in MEI leads to a fall in the price of capital. Since the BGG contract is not indexed to the return to capital, the shock leads to a sharp decline in entrepreneurial net worth, and thus a sharp increase in the risk premium. This procyclical movement in the risk premium is in sharp contrast to the data. Hence, the Bayesian estimation in the BGG model estimates only a small amount of variability coming from these shocks. Notice that in the Indexation model net worth is almost unchanged in response to an MEI shock, so that the impact effect on the risk premium is countercyclical. The Indexation model is thus consistent with MEI shocks driving the cycle, and the risk premium being countercyclical. The similarity of the

[5]Alternatively we could have considered the IRFs for each model at each model's parameter estimates. These IRFs are similar to those reported here, but we find Figure 3 more intuitive as it is holding all other parameters fixed except for the degree of indexation and the presence of agency costs.

Indexation and JPT model is also apparent: the two IRFs to an MEI shock largely lie on top of one another.

Since the BGG model downplays the importance of MEI shocks, it must shift this variance to other shocks. Figures 3b-3c plot the IRFs to the two financial shocks. The good news is that the spread is now countercyclical. But the difficulty with the financial shocks is that they result in countercyclical consumption. This is the familiar co-movement puzzle that arises when a positive shock in one sector (eg., higher net worth mitigates agency costs in capital accumulation) leads to a downward production movement in the other sector. As an aside, note that a shock to net worth has a larger effect on net worth and capital prices in the BGG model. This is just a manifestation of the multiplier intuition outlined in section 2.

Figure 3d plots the IRF to a monetary shock. In the case of BGG, the IRFs exhibit plausible co-movement and countercyclical spreads. The BGG estimation does not put more weight on these policy shocks because the funds rate is an observable, and thus limits possible interest rate variability. In contrast, it is quite clear why the Indexation model puts so little weight on monetary policy shocks. In the case of Indexation, the spread is procyclical, a clear counterfactual prediction.

As a form of sensitivity analysis, we also consider a proxy for the risk premium developed by Gilchrist, Ortiz, and Zakrajšek (2009). This spread measure (hereafter labeled GOZ) is available from 1973:1 to the present. Table 3 presents results estimation results analogous to Table 1: a horse race between JPT, BGG, and the indexation model.[6] The basic message of the GOZ results is similar to those with the BAA spread: (i) the indexation model dominates BGG, which in turn dominates JPT, and (ii) the estimated level of indexation is significant, $\chi = 1.32$.

But there are two interesting differences between the two estimations. First, this estimated level of indexation is smaller with GOZ (1.32) than with the BAA spread (1.84). Second, with the GOZ spread the estimated level of capital adjustment costs (S") and the variance of the MEI shocks are both much

[6] We also estimated the model using the BAA spread for this shorter time period. But since the BAA results for the shorter period are quite similar to those for the longer time period, we omit presenting these results here.

larger. To explore the economic impact of these differences we computed and compared the variance decompositions and the impulse response functions across the two estimations (GOZ and BAA). To briefly summarize, neither of these differences had any significant economic impact: both the variance decompositions and the impulse response functions were quite similar across the two estimations. This result suggests that an indexation of 1.84 (under BAA) and an indexation of 1.32 (under GOZ) are not that far apart economically. This is apparently a manifestation of the nonlinear multiplier effects exhibited in Figure 1. That is, there is a large difference between $\chi = 0$ and $\chi = 1$, but very modest differences between $\chi = 1.32$ and $\chi = 1.84$. As for the adjustment costs, the model's business cycle dynamics are largely driven by MEI shocks. The impact effect of an MEI shock is weighted by the size of adjustment costs (S").[7] Hence, a larger S" largely cancels out a larger MEI variance.

5. Conclusion.

This paper began as an empirical investigation of the importance of agency costs and contract indexation in the business cycle. We are left with a curious conclusion. The agency cost model with indexation dominates JPT's no-agency-cost framework, but only because it provides predictions on the risk premium for which JPT is silent. But in terms of the other aggregate variables, the JPT and indexation model fit similarly. That is, the estimated level of indexation largely eliminates agency costs from the business cycle. The two exogenous shocks that are unique to the agency cost model (shocks to idiosyncratic variance and shocks to net worth) are unimportant in the financial models because they yield negative co-movement between consumption and output.

[7] See equation (A9) in the appendix.

APPENDIX.

1. Linearized System of Equations:

$$\hat{y}_t = \frac{y+F}{y}\left[\alpha \hat{k}_t + (1-\alpha)\hat{L}_t\right] \quad \text{(A1)}$$

$$\hat{\rho}_t = \hat{w}_t + \hat{L}_t - \hat{k}_t \quad \text{(A2)}$$

$$\hat{s}_t = \alpha\hat{\rho}_t + (1-\alpha)\hat{w}_t \quad \text{(A3)}$$

$$\hat{\pi}_t = \frac{\beta}{1+\beta\iota_p}E_t\hat{\pi}_{t+1} + \frac{\iota_p}{1+\beta\iota_p}\hat{\pi}_{t-1} + \frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta\iota_p)\xi_p}\hat{s}_t + \hat{\lambda}_{p,t} \quad \text{(A4)}$$

$$\hat{\lambda}_t = \frac{h\beta e^{\gamma_z}}{(e^{\gamma_z}-h\beta)(e^{\gamma_z}-h)}E_t\hat{c}_{t+1} - \frac{e^{2\gamma_z}+h^2\beta}{(e^{\gamma_z}-h\beta)(e^{\gamma_z}-h)}\hat{c}_t + \frac{he^{\gamma_z}}{(e^{\gamma_z}-h\beta)(e^{\gamma_z}-h)}\hat{c}_{t-1} + \frac{h\beta e^{\gamma_z}\rho_z - he^{\gamma_z}}{(e^{\gamma_z}-h\beta)(e^{\gamma_z}-h)}\hat{z}_t + \frac{e^{\gamma_z}-h\beta\rho_b}{(e^{\gamma_z}-h\beta)}\hat{b}_t + \left[\frac{h\beta e^{\gamma_z}\rho_\upsilon - he^{\gamma_z}}{(e^{\gamma_z}-h\beta)(e^{\gamma_z}-h)}\right]\left(\frac{\alpha}{1-\alpha}\right)\hat{\upsilon}_t \quad \text{(A5)}$$

$$\hat{\lambda}_t = \hat{R}_t + E_t\left(\hat{\lambda}_{t+1} - \hat{z}_{t+1} - \hat{\pi}_{t+1} - \left(\frac{\alpha}{1-\alpha}\right)\hat{\upsilon}_{t+1}\right) \quad \text{(A6)}$$

$$\hat{\rho}_t = \vartheta\hat{u}_t \quad \text{(A7)}$$

$$E_t\hat{r}^k_{t+1} = \hat{\lambda}_t - E_t\hat{\lambda}_{t+1} + E_t\hat{z}_{t+1} + \left(\frac{\alpha}{1-\alpha}\right)E_t\hat{\upsilon}_{t+1} \quad \text{(A8)}$$

$$\hat{q}_t = -\hat{\mu}_t + e^{2(\gamma_z+\gamma_\upsilon)}S''\left(\hat{\imath}_t - \hat{\imath}_{t-1} + \hat{z}_t + \left(\frac{1}{1-\alpha}\right)\hat{\upsilon}_t\right) - \beta e^{2(\gamma_z+\gamma_\upsilon)}S''E_t\left(\hat{\imath}_{t+1} - \hat{\imath}_t + \hat{z}_{t+1} + \left(\frac{1}{1-\alpha}\right)\hat{\upsilon}_{t+1}\right) \quad \text{(A9)}$$

$$\hat{k}_t = \hat{u}_t + \hat{\bar{k}}_{t-1} - \hat{z}_t - \left(\frac{1}{1-\alpha}\right)\hat{\upsilon}_t \quad \text{(A10)}$$

$$\hat{\bar{k}}_t = (1-\delta)e^{-(\gamma_z+\gamma_\upsilon)}\left(\hat{\bar{k}}_{t-1} - \hat{z}_t - \left(\frac{1}{1-\alpha}\right)\hat{\upsilon}_t\right) + \left[1-(1-\delta)e^{-(\gamma_z+\gamma_\upsilon)}\right](\hat{\mu}_t + \hat{\imath}_t) \quad \text{(A11)}$$

$$\hat{w}_t = \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{w}_{t+1} - \frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\beta\iota_w)\xi_w}\hat{g}_{w,t} + \frac{\iota_w}{1+\beta}\hat{\pi}_{t-1} - \frac{1+\beta\iota_w}{1+\beta}\hat{\pi}_t + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} + \frac{\iota_w}{1+\beta}\left(\hat{z}_{t-1} + \left(\frac{\alpha}{1-\alpha}\right)\hat{\upsilon}_{t-1}\right) - \frac{1+\beta\iota_w-\beta\rho_z}{1+\beta}\hat{z}_t - \frac{1+\beta\iota_w-\beta\rho_\upsilon}{1+\beta}\left(\frac{\alpha}{1-\alpha}\right)\hat{\upsilon}_t + \hat{\lambda}_{w,t} \quad \text{(A12)}$$

$$\hat{g}_{w,t} = \hat{w}_t - (\psi\hat{L}_t + \hat{b}_t - \hat{\lambda}_t) \quad \text{(A13)}$$

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)[\phi_\pi\hat{\pi}_t + \phi_x(\hat{x}_t - \hat{x}^*_t)] + \phi_{dx}[(\hat{x}_t - \hat{x}_{t-1}) - (\hat{x}^*_t - \hat{x}^*_{t-1})] + \hat{\eta}_{mp,t} \quad \text{(A14)}$$

$$\hat{x}_t = \hat{y}_t - \frac{\rho k}{y}\hat{u}_t \quad \text{(A15)}$$

$$\frac{1}{g}\hat{y}_t = \frac{1}{g}\hat{g}_t + \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{\imath}_t + \frac{\rho k}{y}\hat{u}_t \quad \text{(A16)}$$

$$\hat{r}^d_t = \hat{R}_t - E_t\hat{\pi}_{t+1} \quad \text{(A17)}$$

$$\hat{r}^k_t = \beta e^{-(\gamma_z+\gamma_\upsilon)}(1-\delta)\hat{q}_t + \left[1-\beta e^{-(\gamma_z+\gamma_\upsilon)}(1-\delta)\right]\hat{\rho}_t - \hat{q}_{t-1} \quad \text{(A18)}$$

For the **agency cost model**, we replace (A8) with

$$E_t\hat{r}^k_{t+1} = \hat{\lambda}_t - E_t\hat{\lambda}_{t+1} + E_t\hat{z}_{t+1} + \left(\frac{\alpha}{1-\alpha}\right)E_t\hat{\upsilon}_{t+1} + \nu\left(\hat{q}_t + \hat{k}_t - \hat{n}_t\right) + \hat{\sigma}_t \quad \text{(A8')}$$

And add the following equations:

$$\hat{n}_t = \kappa\frac{\gamma}{\beta}(\hat{r}^k_t - \hat{r}^l_t) + \frac{\gamma}{\beta}(\hat{r}^l_t + \hat{n}_{t-1}) + \gamma\kappa\frac{rp}{\beta}\left(\hat{k}_{t-1} + \hat{q}_{t-1} + \hat{r}^k_t\right) - \hat{z}_t - \left(\frac{1}{1-\alpha}\right)\hat{\upsilon}_t + \hat{\eta}_{nw,t} \quad \text{(A19)}$$

$$\hat{r}^l_t = \hat{r}^d_{t-1} + [1+\Theta_g(\chi-1)](\hat{r}^k_t - E_{t-1}\hat{r}^k_t) \quad \text{(A20)}$$

2. The Derivation of A8' and A21.

The optimal contract (15)-(16) can be expressed as

$$\bar{\kappa}_t E_t R^k_{t+1} f(\varpi_{t+1}) = \frac{-E_t \varpi_{t+1} f'(\varpi_{t+1})}{E_t \varpi_{t+1} \Lambda_{t+1} g'(\varpi_{t+1})} R^d_t E_t \Lambda_{t+1} \tag{A21}$$

$$E_t \Lambda_{t+1} R^k_{t+1} \frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} g(\varpi_{t+1}) = R^d_t E_t \Lambda_{t+1} \tag{A22}$$

It is convenient to define $F(\varpi_{t+1}) \equiv \frac{-f'(\varpi_{t+1})}{g'(\varpi_{t+1})}$, where $\Psi \equiv \frac{\varpi_{ss} F'(\varpi_{ss})}{F(\varpi_{ss})} > 0$, by the second order condition.

Linearizing (A21)-(A22) we have

$$E_t(\hat{r}^k_{t+1} - \hat{r}^d_t) + \kappa_t = (\Psi - \Theta_f) E_t \varpi_{t+1} \tag{A23}$$

$$E_t(\hat{r}^k_{t+1} - \hat{r}^d_t) = \left(\frac{1}{\kappa - 1}\right) \kappa_t - \Theta_g E_t \varpi_{t+1} \tag{A24}$$

where $\Theta_g \equiv \frac{\varpi_{ss} g'(\varpi_{ss})}{g(\varpi_{ss})}$, with $0 < \Theta_g < 1$, and $\Theta_f \equiv \frac{\varpi_{ss} f'(\varpi_{ss})}{f(\varpi_{ss})} < 0$. Solving (A23)-(A24) we have:

$$E_t \varpi_{t+1} = \frac{\kappa}{\kappa - 1} \frac{1}{(\Psi - \theta_f + \theta_g)} \kappa_t \tag{A25}$$

$$E_t(\hat{r}^k_{t+1} - \hat{r}^d_t) = \left[\frac{(\Psi - \theta_f + \Theta) - \kappa \theta_g}{(\kappa - 1)(\Psi - \theta_f + \Theta)}\right] \kappa_t \equiv \nu \kappa_t \tag{A26}$$

Using the definition of leverage and the deposit rate, (A26) is the same as (A8'). The linearized lender return (6) and bankruptcy cut-off (10) are given by

$$\hat{r}^l_{t+1} = \hat{r}^k_{t+1} + \Theta_g \varpi_{t+1} - \left(\frac{1}{\kappa - 1}\right) \kappa_t \tag{A27}$$

$$\varpi_{t+1} = \Omega_t + (\chi - 1) \hat{r}^k_{t+1} \tag{A28}$$

From (A25), we then have that

$$\Omega_t = \frac{\kappa}{\kappa - 1} \frac{1}{(\Psi - \theta_f + \theta_g)} \kappa_t + (1 - \chi) E_t \hat{r}^k_{t+1} \tag{A29}$$

so that (A28) is given by

$$\varpi_{t+1} = \frac{\kappa}{\kappa - 1} \frac{1}{(\Psi - \theta_f + \theta_g)} \kappa_t + (\chi - 1)(\hat{r}^k_{t+1} - E_t \hat{r}^k_{t+1}) \tag{A30}$$

Taking the expectation of (A27), and subtracting this from the original (A27), we have

$$\hat{r}^l_t = \hat{r}^d_{t-1} + [1 + \Theta_g(\chi - 1)](\hat{r}^k_t - E_{t-1} \hat{r}^k_t) \tag{A31}$$

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Figure 1: the multiplier as a function of indexation.

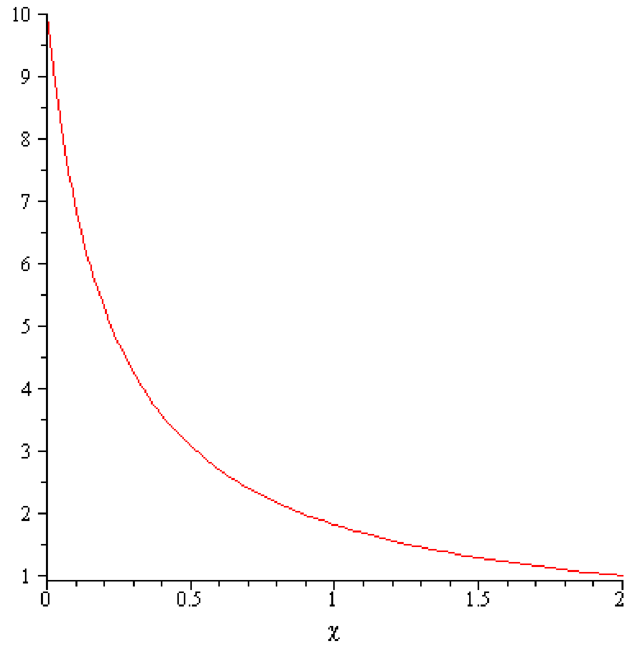
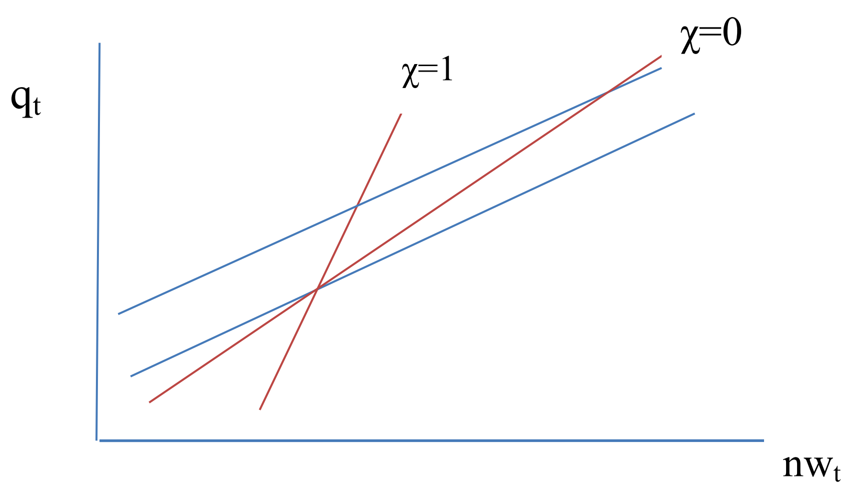


Figure 2: a shock to asset demand.



(Asset price is blue line. Net worth evolution is red line.) Demand shock shifts up asset price. The new equilibrium in (n,q) space depends upon the level of indexation. Lower levels of indexation amplify these effects.

Figure 3.a. Impulse Response Functions to a One Standard Deviation Marginal Efficiency of Investment Shock
Keeping parameters constant to the Indexation Model

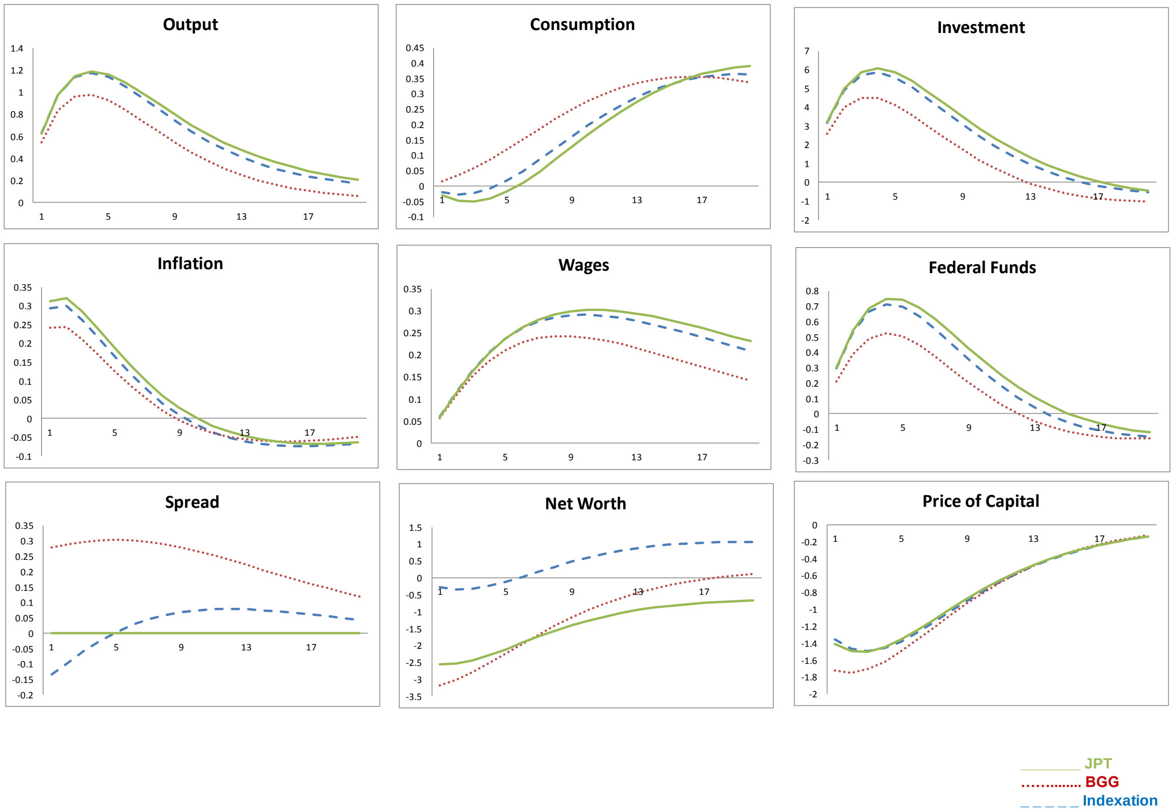


Figure 3.b. Impulse Response Functions to a One Standard Deviation Net Worth Shock

Keeping parameters constant to the Indexation Model

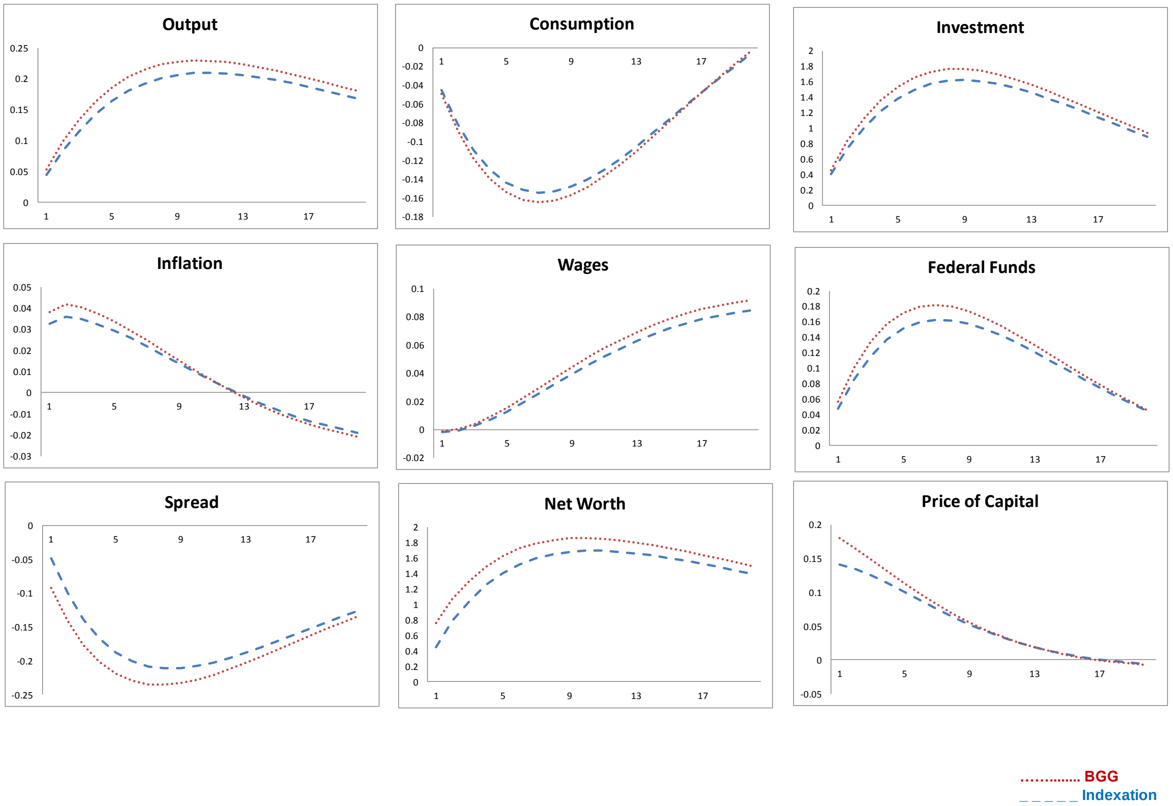


Figure 3.c. Impulse Response Functions to a One Standard Deviation Idiosyncratic Variance Shock

Keeping parameters constant to the Indexation Model

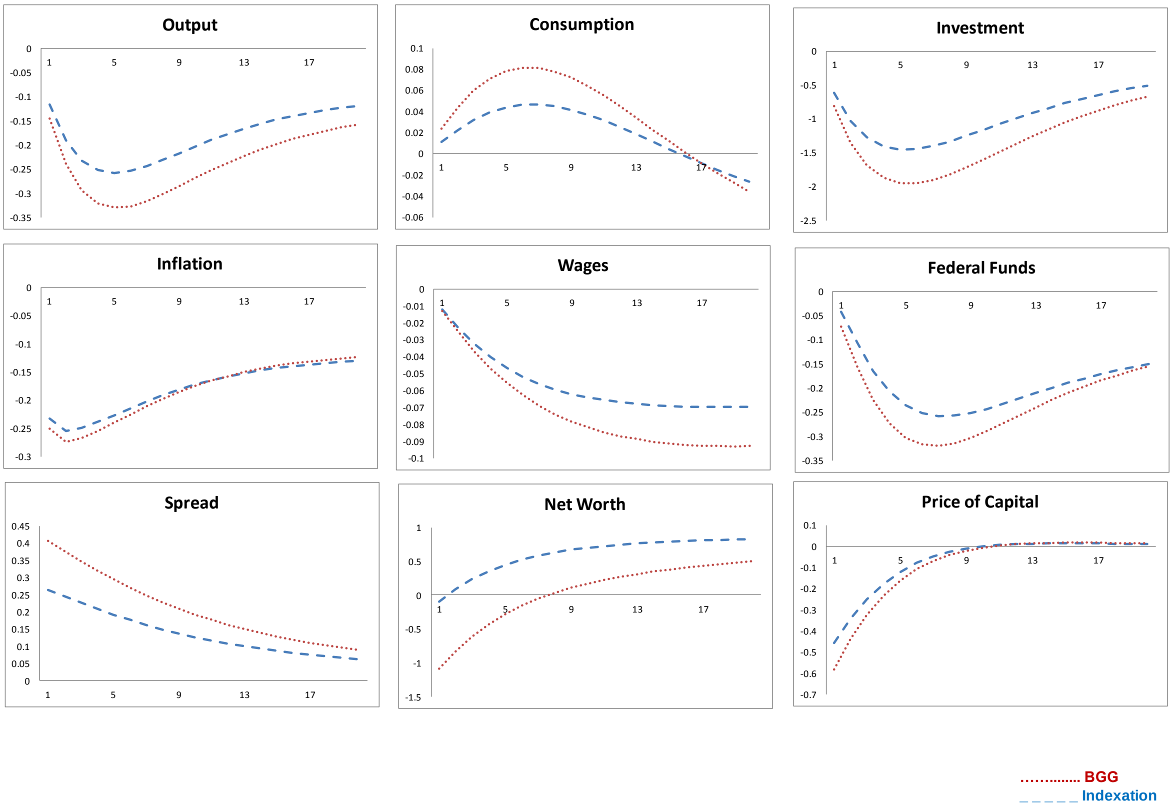


Figure 3.d. Impulse Response Functions to a One Standard Deviation Monetary Policy Shock

Keeping parameters constant to the Indexation Model

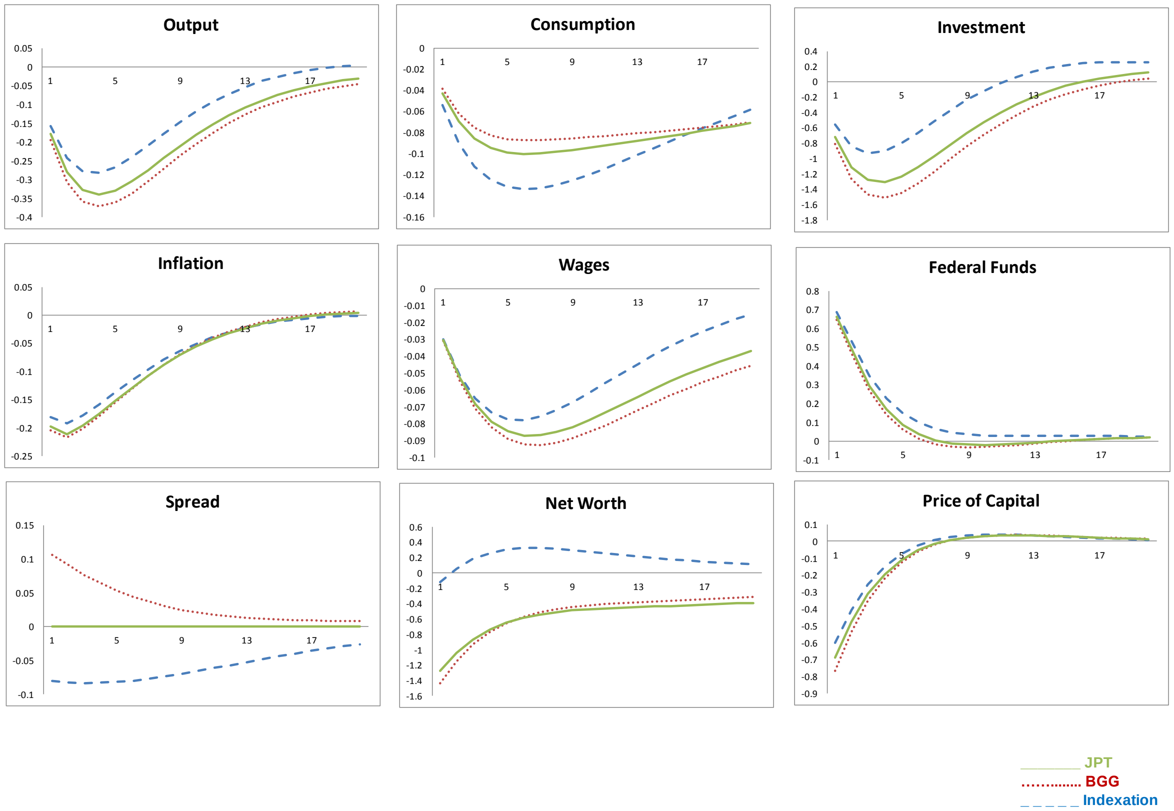


Table 1: Models Estimations and Models Comparisons

					JPT Model [a]			BGG Model [b]			Indexation Model [c]			Indexation Model (ν estimated) [d]		
Log data density					-1549.1			-1476.8			-1412.0			-1417.0		
Log Marginal density					-1627.4			-1546.0			-1503.8			-1506.2		
Posterior Model Probability					0%			0%			92%			8%		
Coefficient	**Description**	**Prior**			**Posteriors [f]**			**Posteriors**			**Posteriors**			**Posteriors**		
		Prior density [e]	**prior mean**	**pstdev**	**post. mean**	**5%**	**95%**	**post. mean**	**5%**	**95%**	**post. mean**	**5%**	**95%**	**post. mean**	**5%**	**95%**
α	**Capital share**	**N**	0.30	0.05	0.16	0.15	0.17	0.17	0.16	0.17	0.16	0.16	0.17	0.16	0.15	0.17
ι_p	**Price indexation**	**B**	0.50	0.15	0.14	0.07	0.21	0.12	0.04	0.22	0.12	0.05	0.18	0.12	0.05	0.19
ι_w	**Wage indexation**	**B**	0.50	0.15	0.10	0.05	0.16	0.10	0.06	0.15	0.11	0.06	0.16	0.10	0.05	0.15
γ_z	**SS technology growth rate**	**N**	0.50	0.03	0.46	0.42	0.50	0.48	0.45	0.51	0.46	0.41	0.51	0.49	0.46	0.52
γ_υ	**SS IST growth rate**	**N**	0.50	0.03	0.49	0.46	0.52	0.46	0.44	0.48	0.47	0.42	0.51	0.49	0.45	0.52
h	**Consumption habit**	**B**	0.50	0.10	0.88	0.84	0.91	0.85	0.82	0.89	0.87	0.84	0.90	0.88	0.85	0.91
λ_p	**SS mark-up goods prices**	**N**	0.15	0.05	0.24	0.17	0.29	0.27	0.24	0.32	0.23	0.17	0.29	0.26	0.22	0.31
λ_w	**SS mark-up wages**	**N**	0.15	0.05	0.13	0.08	0.18	0.14	0.10	0.18	0.16	0.08	0.24	0.18	0.14	0.21
$\log L^{ss}$	**SS hours**	**N**	0.00	0.50	0.28	-0.60	1.11	0.05	-0.55	0.59	0.22	-0.61	1.00	-0.10	-0.52	0.46
$100(\pi - 1)$	**SS quarterly inflation**	**N**	0.50	0.10	0.66	0.49	0.83	0.73	0.67	0.80	0.72	0.62	0.83	0.67	0.54	0.84
$100(\beta^{-1} - 1)$	**Discount factor**	**G**	0.25	0.10	0.12	0.05	0.18	0.14	0.06	0.20	0.18	0.11	0.25	0.12	0.06	0.17
Ψ	**Inverse frisch elasticity**	**G**	2.00	0.75	3.79	2.53	5.10	4.03	3.01	4.95	3.36	2.55	4.20	3.54	3.07	4.04
ξ_p	**Calvo prices**	**B**	0.66	0.10	0.80	0.76	0.84	0.82	0.78	0.86	0.80	0.77	0.83	0.81	0.77	0.84
ξ_w	**Calvo wages**	**B**	0.66	0.10	0.67	0.59	0.74	0.73	0.66	0.80	0.65	0.55	0.75	0.67	0.59	0.73
ϑ	**Elasticity capital utilization costs**	**G**	5.00	1.00	4.92	3.30	6.26	5.29	3.90	7.11	4.16	2.29	5.60	5.42	3.86	6.90
S''	**Investment adjustment costs**	**G**	4.00	1.00	2.80	1.81	3.74	1.51	1.24	1.77	2.10	1.45	2.75	2.78	1.85	3.74
ϕ_p	**Taylor rule inflation**	**N**	1.70	0.30	1.74	1.51	2.00	1.69	1.43	1.92	2.14	1.88	2.40	2.27	2.10	2.45
ϕ_y	**Taylor rule output**	**N**	0.13	0.05	0.05	0.03	0.07	0.06	0.04	0.09	0.09	0.06	0.12	0.11	0.07	0.14
ϕ_{dy}	**Taylor rule output growth**	**N**	0.13	0.05	0.20	0.16	0.25	0.21	0.19	0.24	0.19	0.15	0.23	0.20	0.17	0.23
ρ_R	**Taylor rule smoothing**	**B**	0.60	0.20	0.83	0.80	0.86	0.83	0.80	0.87	0.85	0.82	0.88	0.87	0.84	0.89
ρ_{mp}	**Monetary policy**	**B**	0.40	0.20	0.13	0.05	0.22	0.08	0.01	0.15	0.08	0.02	0.14	0.09	0.02	0.16
ρ_z	**Neutral technology growth**	**B**	0.60	0.20	0.35	0.26	0.44	0.28	0.18	0.39	0.34	0.25	0.42	0.36	0.28	0.44
ρ_g	**Government spending**	**B**	0.60	0.20	1.00	0.99	1.00	0.99	0.99	1.00	0.99	0.99	1.00	0.99	0.99	1.00
ρ_υ	**IST growth**	**B**	0.60	0.20	0.29	0.18	0.39	0.35	0.25	0.45	0.27	0.18	0.36	0.28	0.19	0.36
ρ_p	**Price mark-up**	**B**	0.60	0.20	0.96	0.94	0.99	0.96	0.93	0.99	0.97	0.95	1.00	0.97	0.95	0.99
ρ_w	**Wage mark-up**	**B**	0.60	0.20	0.98	0.96	0.99	0.98	0.97	1.00	0.98	0.97	0.99	0.98	0.97	0.99
ρ_b	**Intertemporal preference**	**B**	0.60	0.20	0.52	0.42	0.63	0.56	0.47	0.67	0.56	0.47	0.65	0.56	0.45	0.67
θ_p	**Price mark-up MA**	**B**	0.50	0.20	0.78	0.70	0.87	0.77	0.65	0.87	0.79	0.71	0.88	0.79	0.71	0.89
θ_w	**Wage mark-up MA**	**B**	0.50	0.20	0.94	0.90	0.97	0.96	0.94	0.98	0.93	0.90	0.97	0.93	0.90	0.96
ρ_σ	**Idiosyncratic variance**	**B**	0.60	0.20	**-**	-	-	1.00	0.99	1.00	0.99	0.98	1.00	0.99	0.98	1.00
ρ_{nw}	**Net worth**	**B**	0.60	0.20	-	-	-	0.78	0.67	0.88	0.86	0.74	0.97	0.89	0.83	0.99
ρ_μ	**Marginal efficiency of investment**	**B**	0.60	0.20	0.81	0.75	0.87	0.61	0.52	0.69	0.81	0.75	0.87	0.80	0.75	0.85
ν	**Elasticity risk premium**	**N**	0.05	0.02	**0**	**-**	**-**	**0.041**	-	-	**0.041**	-	-	0.034	0.02	0.05
χ	**Indexation**	**U**	0.00	2.00	**0**	-	-	**BGG**	-	-	1.84	1.56	2.13	2.06	1.44	2.73
standard deviation of shocks																
		Prior density	**prior mean**	**pstdev**	**post. mean**	**5%**	**95%**	**post. mean**	**5%**	**95%**	**post. mean**	**5%**	**95%**	**post. mean**	**5%**	**95%**
σ_{mp}	**Monetary policy**	**I**	0.20	1.00	0.22	0.20	0.24	0.23	0.21	0.25	0.22	0.20	0.24	0.22	0.20	0.24
σ_z	**Neutral technology growth**	**I**	0.50	1.00	0.97	0.89	1.06	0.98	0.90	1.07	0.97	0.89	1.04	0.98	0.90	1.06
σ_g	**Government spending**	**I**	0.50	1.00	0.37	0.34	0.39	0.37	0.34	0.40	0.37	0.34	0.40	0.37	0.34	0.39
σ_υ	**IST growth**	**I**	0.50	1.00	0.66	0.61	0.72	0.67	0.61	0.72	0.66	0.61	0.71	0.67	0.61	0.72
σ_p	**Price mark-up**	**I**	0.10	1.00	0.22	0.19	0.25	0.21	0.17	0.24	0.22	0.19	0.25	0.22	0.19	0.25
σ_w	**Wage mark-up**	**I**	0.10	1.00	0.25	0.22	0.28	0.26	0.23	0.28	0.26	0.23	0.28	0.25	0.22	0.28
σ_b	**Intertemporal preference**	**I**	0.10	1.00	0.04	0.03	0.06	0.04	0.03	0.05	0.04	0.03	0.05	0.04	0.03	0.05
σ_σ	**Idiosyncratic variance**	**I**	0.50	1.00	**-**	-	-	0.08	0.07	0.09	0.08	0.07	0.09	0.08	0.07	0.09
σ_{nw}	**Net worth**	**I**	0.50	1.00	-	-	-	0.62	0.31	0.92	0.42	0.19	0.65	0.43	0.18	0.68
σ_μ	**Marginal efficiency of investment**	**I**	0.50	1.00	5.34	4.04	6.82	3.80	3.15	4.41	4.39	3.31	5.58	5.41	4.19	6.59
σ_{me}	**Risk premium measurement error**	**I**	0.50	1.00	0.20	0.18	0.21	0.07	0.06	0.07	0.07	0.06	0.07	0.07	0.06	0.07

Note: calibrated coefficients: $\delta = 0.025$, g implies a SS government share of 0.22.

For the agency cost models (BGG and Indexation) the following parameters are also calibrated: entrepreneurial survival rate $\gamma = 0.98$, a SS risk premium $rp = 0.02/4$, and a SS leverage ratio $\kappa = 1.95$.

[a] In JPT model there are not financial (risk premium and net worth) shocks. The elasticity of risk premium, ν, is set to 0 and the indexation parameter, χ , is irrelevant and set to 0.

[b] In BGG model there are financial shocks and the elasticity of risk premium, ν, is calibrated to 0.041, while the indexation parameter, χ , is set to the implied in BGG, $\chi = (\Theta_g - 1)/\Theta_g$ where $\Theta_g = 0.985$.

[c] In the Indexation model there are financial shocks and the elasticity of risk premium, ν, is calibrated to 0.041, while the indexation parameter, χ ,is estimated.

[d] In the alternative Indexation model there are financial shocks and both the elasticity of risk premium, ν, and the indexation parameter, χ , are estimated.

[e] N stands for Norman, B-Beta, G-Gamma, U-Uniform, I-Inverted-Gamma distribution.

[f] Posterior percentiles are from 2 chains of 1,000,000 draws generated using a Random Walk Metropolis algorithm. We discard the initial 500,000 and retain one every 5 subsequent draws.

Table 2: Variance Decomposition at Different Horizons in the JPT, BGG, and Indexation Models

Output

	Monetary policy	Neutral technology	Government	Investment specific technology	Price mark-up	Wage mark-up	Intertemporal preference	Marginal efficiency of investment	Net Worth	Idiosyncratic variance	Measurement error of risk premium
4 quarters											
JPT	5.1	15.3	3.4	1.8	3.3	1.2	5.1	64.8	-	-	0.0
BGG	13.5	16.1	4.5	1.9	7.2	1.6	4.7	39.2	2.4	8.9	0.0
Indexation	5.4	12.9	3.2	2.3	3.7	1.7	6.1	61.3	0.6	2.9	0.0
8 quarters											
JPT	5.5	7.4	2.1	1.1	5.7	4.1	3.3	70.7	-	-	0.0
BGG	15.4	9.4	3.3	1.4	13.6	7.1	3.4	29.7	4.5	12.2	0.0
Indexation	5.5	6.6	2.0	1.6	6.3	5.5	4.4	62.9	1.2	3.8	0.0
16 quarters											
JPT	4.9	5.5	1.6	0.8	8.3	12.3	2.1	64.5	-	-	0.0
BGG	12.3	6.9	2.5	1.1	17.5	19.9	2.1	19.0	6.0	12.6	0.0
Indexation	4.6	4.8	1.6	1.3	8.8	14.7	3.1	54.1	2.3	4.7	0.0
1000 quarters											
JPT	3.0	3.6	10.0	0.4	7.5	34.2	1.2	40.1	-	-	0.0
BGG	5.3	3.0	3.0	0.6	12.0	46.5	0.9	8.0	3.6	17.0	0.0
Indexation	2.9	3.0	2.7	1.0	8.8	35.3	1.9	34.9	2.6	6.9	0.0

Investment

	Monetary policy	Neutral technology	Government	Investment specific technology	Price mark-up	Wage mark-up	Intertemporal preference	Marginal efficiency of investment	Net Worth	Idiosyncratic variance	Measurement error of risk premium
4 quarters											
JPT	3.9	2.3	0.0	0.2	3.2	0.1	1.1	89.2	-	-	0.0
BGG	12.5	4.7	0.1	0.2	6.9	0.1	3.9	47.2	7.6	16.8	0.0
Indexation	2.7	1.0	0.0	0.3	3.0	0.1	1.1	84.1	2.7	5.1	0.0
8 quarters											
JPT	3.6	6.2	0.0	0.2	4.9	0.6	1.2	83.2	-	-	0.0
BGG	11.7	10.2	0.1	0.8	11.0	0.4	3.8	28.6	12.1	21.3	0.0
Indexation	2.3	2.8	0.0	0.2	4.8	0.6	1.0	76.6	4.8	6.7	0.0
16 quarters											
JPT	3.2	10.6	0.0	0.7	7.4	2.6	1.1	74.3	-	-	0.0
BGG	9.1	11.6	0.1	1.5	13.6	1.8	2.9	20.7	15.3	23.5	0.0
Indexation	1.9	4.1	0.0	0.4	7.2	2.3	0.8	65.8	8.3	9.1	0.0
1000 quarters											
JPT	2.9	10.8	0.4	1.1	7.9	6.2	1.0	69.5	-	-	0.0
BGG	6.9	8.9	0.1	1.4	12.1	4.3	2.3	16.7	12.5	34.9	0.0
Indexation	1.8	3.8	0.0	0.5	8.0	4.5	0.8	59.7	8.8	12.1	0.0

Observed Risk Premium

	Monetary policy	Neutral technology	Government	Investment specific technology	Price mark-up	Wage mark-up	Intertemporal preference	Marginal efficiency of investment	Net Worth	Idiosyncratic variance	Measurement error of risk premium
4 quarters											
JPT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-	100.0
BGG	2.2	0.1	0.0	0.4	0.1	0.5	0.2	12.5	23.8	53.0	7.2
Indexation	5.0	1.5	0.1	0.5	0.4	0.4	1.3	5.8	14.1	55.4	15.6
8 quarters											
JPT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-	100.0
BGG	1.4	0.4	0.0	0.3	0.1	0.5	0.2	14.1	33.1	45.9	4.1
Indexation	5.6	2.8	0.1	0.5	0.8	0.3	1.9	4.2	27.5	47.7	8.7
16 quarters											
JPT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-	100.0
BGG	1.0	1.5	0.0	0.2	0.3	0.7	0.3	14.6	37.1	41.5	2.9
Indexation	5.5	4.8	0.1	0.6	1.4	0.3	2.4	6.4	35.5	37.7	5.4
1000 quarters											
JPT	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-	100.0
BGG	0.8	2.2	0.1	0.3	0.5	1.3	0.2	11.3	30.6	50.6	2.1
Indexation	4.6	6.1	0.1	1.2	1.8	1.2	2.1	6.3	34.1	38.5	4.1

Net Worth

	Monetary policy	Neutral technology	Government	Investment specific technology	Price mark-up	Wage mark-up	Intertemporal preference	Marginal efficiency of investment	Net Worth	Idiosyncratic variance	Measurement error of risk premium
4 quarters											
JPT	11.2	4.5	0.0	3.4	1.5	1.4	1.4	76.6	-	-	0.0
BGG	11.9	7.8	0.0	6.4	1.2	1.5	2.2	25.8	37.0	6.2	0.0
Indexation	0.7	34.0	0.0	32.5	0.2	0.2	0.1	2.8	28.1	1.4	0.0
8 quarters											
JPT	9.3	7.9	0.0	4.9	1.7	1.1	1.1	74.0	-	-	0.0
BGG	8.0	10.2	0.0	8.4	1.2	0.9	1.3	16.9	49.6	3.4	0.0
Indexation	1.0	27.9	0.0	26.7	0.2	0.4	0.3	2.0	37.4	3.9	0.0
16 quarters											
JPT	8.9	13.6	0.0	6.5	2.3	0.9	0.8	67.0	-	-	0.0
BGG	6.2	11.7	0.0	9.6	1.8	0.6	0.9	10.7	55.8	2.8	0.0
Indexation	0.8	21.6	0.0	20.3	0.5	0.4	0.4	8.5	40.2	7.3	0.0
1000 quarters											
JPT	7.5	19.6	1.0	6.3	4.7	0.9	0.5	59.5	-	-	0.0
BGG	1.9	3.8	0.0	3.5	3.1	1.3	0.3	2.6	19.4	64.0	0.0
Indexation	0.3	9.2	0.1	9.3	3.6	2.2	0.2	11.0	22.4	41.7	0.0

Table 3: Models Estimations and Models Comparisons with GOZ spread

					JPT Model [a]			BGG Model [b]			Indexation Model [c]			Indexation Model (ν estimated) [d]		
Log data density					-1055.3			-1027.7			-1001.6			-1000.4		
Log Marginal density					-1120.5			-1088.0			-1075.1			-1075.6		
Posterior Model Probability					0%			0%			61%			39%		
Coefficient	**Description**	**Prior**			**Posteriors [f]**			**Posteriors**			**Posteriors**			**Posteriors**		
		Prior density [e]	**prior mean**	**pstdev**	**post. mean**	**5%**	**95%**	**post. mean**	**5%**	**95%**	**post. mean**	**5%**	**95%**	**post. mean**	**5%**	**95%**
α	**Capital share**	**N**	0.30	0.05	0.16	0.14	0.17	0.16	0.15	0.18	0.16	0.14	0.17	0.16	0.14	0.17
ι_p	**Price indexation**	**B**	0.50	0.15	0.23	0.06	0.39	0.11	0.05	0.17	0.17	0.06	0.27	0.12	0.05	0.20
ι_w	**Wage indexation**	**B**	0.50	0.15	0.19	0.10	0.28	0.15	0.07	0.24	0.17	0.09	0.25	0.15	0.07	0.22
γ_z	**SS technology growth rate**	**N**	0.50	0.03	0.47	0.43	0.51	0.45	0.43	0.48	0.47	0.44	0.51	0.47	0.44	0.51
γ_υ	**SS IST growth rate**	**N**	0.50	0.03	0.49	0.45	0.52	0.52	0.50	0.55	0.49	0.46	0.52	0.50	0.46	0.55
h	**Consumption habit**	**B**	0.50	0.10	0.86	0.82	0.90	0.85	0.81	0.90	0.86	0.82	0.89	0.84	0.79	0.90
λ_p	**SS mark-up goods prices**	**N**	0.15	0.05	0.16	0.12	0.21	0.15	0.09	0.21	0.15	0.07	0.22	0.14	0.08	0.18
λ_w	**SS mark-up wages**	**N**	0.15	0.05	0.14	0.08	0.19	0.14	0.11	0.17	0.13	0.04	0.22	0.09	0.03	0.15
$\log L^{ss}$	**SS hours**	**N**	0.00	0.50	0.27	-0.63	1.15	0.15	-0.31	0.59	0.06	-0.47	0.64	0.08	-0.69	0.89
$100(\pi - 1)$	**SS quarterly inflation**	**N**	0.50	0.10	0.64	0.48	0.77	0.47	0.40	0.54	0.57	0.45	0.70	0.63	0.49	0.73
$100(\beta^{-1} - 1)$	**Discount factor**	**G**	0.25	0.10	0.17	0.09	0.25	0.18	0.08	0.27	0.16	0.09	0.22	0.14	0.07	0.21
Ψ	**Inverse frisch elasticity**	**G**	2.00	0.75	3.74	2.68	4.73	3.09	2.63	3.56	3.50	2.73	4.34	3.13	2.67	3.70
ξ_p	**Calvo prices**	**B**	0.66	0.10	0.82	0.77	0.86	0.81	0.76	0.86	0.80	0.76	0.85	0.77	0.72	0.83
ξ_w	**Calvo wages**	**B**	0.66	0.10	0.71	0.61	0.81	0.79	0.74	0.85	0.76	0.66	0.86	0.74	0.65	0.84
ϑ	**Elasticity capital utilization costs**	**G**	5.00	1.00	5.52	4.31	6.89	4.85	3.08	7.31	4.82	3.36	5.98	5.46	4.59	6.43
S''	**Investment adjustment costs**	**G**	4.00	1.00	3.68	2.82	4.54	2.61	2.03	3.12	3.04	2.43	3.59	3.00	2.32	3.57
ϕ_p	**Taylor rule inflation**	**N**	1.70	0.30	1.67	1.28	2.09	1.74	1.49	2.00	1.61	1.43	1.87	1.61	1.41	1.81
ϕ_y	**Taylor rule output**	**N**	0.13	0.05	0.06	0.02	0.10	0.09	0.06	0.13	0.08	0.04	0.12	0.07	0.04	0.10
ϕ_{dy}	**Taylor rule output growth**	**N**	0.13	0.05	0.26	0.22	0.31	0.28	0.24	0.33	0.25	0.20	0.28	0.27	0.23	0.31
ρ_R	**Taylor rule smoothing**	**B**	0.60	0.20	0.84	0.80	0.88	0.85	0.81	0.89	0.84	0.81	0.88	0.84	0.80	0.87
ρ_{mp}	**Monetary policy**	**B**	0.40	0.20	0.13	0.04	0.21	0.06	0.01	0.11	0.08	0.02	0.15	0.09	0.02	0.17
ρ_z	**Neutral technology growth**	**B**	0.60	0.20	0.40	0.29	0.49	0.32	0.21	0.44	0.36	0.26	0.46	0.31	0.21	0.41
ρ_g	**Government spending**	**B**	0.60	0.20	0.99	0.99	1.00	0.99	0.99	1.00	0.99	0.99	1.00	0.99	0.99	1.00
ρ_υ	**IST growth**	**B**	0.60	0.20	0.39	0.28	0.50	0.36	0.23	0.46	0.36	0.24	0.46	0.30	0.17	0.43
ρ_p	**Price mark-up**	**B**	0.60	0.20	0.92	0.87	0.97	0.94	0.89	0.98	0.93	0.87	0.98	0.93	0.88	0.98
ρ_w	**Wage mark-up**	**B**	0.60	0.20	0.98	0.96	1.00	0.97	0.95	0.99	0.97	0.95	0.99	0.96	0.94	0.99
ρ_b	**Intertemporal preference**	**B**	0.60	0.20	0.61	0.53	0.69	0.60	0.49	0.70	0.62	0.54	0.71	0.63	0.51	0.75
θ_p	**Price mark-up MA**	**B**	0.50	0.20	0.77	0.69	0.87	0.75	0.65	0.85	0.74	0.62	0.85	0.66	0.48	0.85
θ_w	**Wage mark-up MA**	**B**	0.50	0.20	0.95	0.92	0.98	0.99	0.98	1.00	0.98	0.96	1.00	0.98	0.97	1.00
ρ_σ	**Risk premium**	**B**	0.60	0.20	**-**	-	-	0.99	0.98	1.00	0.99	0.97	1.00	0.99	0.97	1.00
ρ_{nw}	**Net worth**	**B**	0.60	0.20	-	-	-	0.85	0.75	0.95	0.82	0.68	0.95	0.77	0.62	0.92
ρ_μ	**Marginal efficiency of investment**	**B**	0.60	0.20	0.77	0.69	0.84	0.59	0.51	0.68	0.73	0.66	0.81	0.74	0.66	0.82
ν	**Elasticity risk premium**	**N**	0.05	0.02	**0**	**-**	**-**	**0.041**	-	-	**0.041**	-	-	0.046	0.03	0.06
χ	**Indexation**	**U**	0.00	2.00	**0**	-	-	**BGG**	-	-	1.32	0.98	1.62	1.29	0.97	1.59
standard deviation of shocks																
		Prior density	**prior mean**	**pstdev**	**post. mean**	**5%**	**95%**	**post. mean**	**5%**	**95%**	**post. mean**	**5%**	**95%**	**post. mean**	**5%**	**95%**
σ_{mp}	**Monetary policy**	**I**	0.20	1.00	0.25	0.23	0.28	0.27	0.24	0.29	0.25	0.22	0.27	0.25	0.22	0.28
σ_z	**Neutral technology growth**	**I**	0.50	1.00	0.88	0.79	0.97	0.86	0.77	0.95	0.86	0.77	0.95	0.85	0.76	0.94
σ_g	**Government spending**	**I**	0.50	1.00	0.38	0.34	0.41	0.38	0.34	0.41	0.38	0.34	0.41	0.38	0.34	0.41
σ_υ	**IST growth**	**I**	0.50	1.00	0.58	0.52	0.64	0.58	0.52	0.63	0.58	0.52	0.63	0.58	0.52	0.64
σ_p	**Price mark-up**	**I**	0.10	1.00	0.25	0.21	0.28	0.23	0.19	0.26	0.23	0.19	0.26	0.23	0.19	0.27
σ_w	**Wage mark-up**	**I**	0.10	1.00	0.29	0.25	0.32	0.34	0.31	0.38	0.33	0.27	0.38	0.34	0.31	0.38
σ_b	**Intertemporal preference**	**I**	0.10	1.00	0.03	0.02	0.04	0.03	0.02	0.04	0.03	0.02	0.04	0.03	0.02	0.04
σ_σ	**Risk premium**	**I**	0.50	1.00	**-**	-	-	0.09	0.08	0.10	0.10	0.08	0.11	0.10	0.08	0.11
σ_{nw}	**Net worth**	**I**	0.50	1.00	-	-	-	0.34	0.16	0.51	0.34	0.15	0.53	0.36	0.16	0.56
σ_μ	**Marginal efficiency of investment**	**I**	0.50	1.00	6.37	5.15	7.53	5.82	4.85	6.65	5.66	4.80	6.60	5.65	4.90	6.46
σ_{me}	**Risk premium measurement error**	**I**	0.50	1.00	0.19	0.17	0.21	0.07	0.06	0.08	0.07	0.06	0.08	0.07	0.06	0.08

Note: calibrated coefficients: $\delta = 0.025$, g implies a SS government share of 0.22.

For the agency cost models (BGG and Indexation) the following parameters are also calibrated: entrepreneurial survival rate $\gamma = 0.98$, a SS risk premium $rp = 0.02/4$, and a SS leverage ratio $\kappa = 1.95$.

[a] In JPT model there are not financial (risk premium and net worth) shocks. The elasticity of risk premium, ν, is set to 0 and the indexation parameter, χ, is irrelevant and set to 0.

[b] In BGG model there are financial shocks and the elasticity of risk premium, ν, is calibrated to 0.041, while the indexation parameter, χ, is set to the implied in BGG, $\chi = (\Theta_g - 1)/\Theta_g$ where $\Theta_g = 0.985$.

[c] In the Indexation model there are financial shocks and the elasticity of risk premium, ν, is calibrated to 0.041, while the indexation parameter, χ, is estimated.

[d] In the alternative Indexation model there are financial shocks and both the elasticity of risk premium, ν, and the indexation parameter, χ, are estimated.

[e] N stands for Norman, B-Beta, G-Gamma, U-Uniform, I-Inverted-Gamma distribution.

[f] Posterior percentiles are from 2 chains of 1,000,000 draws generated using a Random Walk Metropolis algorithm. We discard the initial 500,000 and retain one every 5 subsequent draws.