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Charles T. Carlstrom, Timothy S. Fuerst, and Matthias Paustian

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This paper addresses the positive and normative implications of indexing risky debt to observable aggregate conditions. These issues are pursued within the context of the celebrated financial accelerator model of Bernanke, Gertler, and Gilchrist (1999). The principal conclusions are that the optimal degree of indexation is significant, and that the business cycle properties of the model are altered under this level of indexation.

JEL Codes: E22, E32, E44.

Key words: Credit market frictions, Indexation.

Charles T. Carlstrom is at the Federal Reserve Bank of Cleveland, and he can be reached at charles.t.carlstrom@clev.frb.org. Timothy S. Fuerst is at Bowling Green State University and the Federal Reserve Bank of Cleveland. He can be reached at tfuerst@bgsu.edu. Matthias Paustian is at the Bank of England, and he can be reached at matthias.paustian@bankofengland.co.uk.

1. Introduction.

The fundamental function of credit markets is to channel funds from savers to entrepreneurs who have some valuable capital investment project. These efforts are hindered by agency costs arising from asymmetric information. A standard result in a subset of this literature, the costly state verification (CSV) framework, is that risky debt is the optimal contract between risk-neutral lenders and entrepreneurs. The modifier risky simply means that there is a non-zero chance of default. In the CSV model external parties can observe the realization of the entrepreneur's idiosyncratic production technology only by expending a monitoring cost. Townsend (1979) demonstrates that risky debt is optimal in this environment because it minimizes the need for verification of project outcomes. This verification is costly but necessary to align the incentives of the firm with the bank.

Aggregate conditions will also affect the ability of the borrower to repay the loan. But since aggregate variables are observed by both parties, it may be advantageous to have the loan contract indexed to the behavior of aggregate variables. Although Townsend's (1979) CSV structure did not include aggregate risk, we constrain our analysis to the class of debt contracts and explore the effect of indexing this debt to aggregate variables. How is the financial accelerator affected by the degree of indexation? What are the welfare consequences of alternative indexation schemes? What indexation scheme is optimal? We explore these issues in the celebrated financial accelerator model of Bernanke, Gertler, and Gilchrist (1999), hereafter BGG.

Such state-contingent contracts have recently received more attention. For instance, one prominent proposal for the reform of the financial system is to require large financial institutions to issue debt that is automatically converted to equity under certain aggregate conditions.

Similarly, Shiller and Weiss (1999) suggest indexing home mortgages to movements in aggregate house prices.

Our principle results include the following. First, the agency cost model is isomorphic to a real business cycle (RBC) model with an endogenous and time-varying distortion on total capital accumulation. Second, this agency cost distortion arises because entrepreneurs do not internalize the effect of their behavior on aggregate conditions generally and household consumption in particular. Third, for TFP shocks, the optimal level of indexation is typically close to the loan repayment being fully indexed to movements in aggregate conditions. This indexation rate implies that bankruptcy rates are largely pre-determined, ie., under optimal indexation, bankruptcy rates are largely unresponsive to innovations in aggregate shocks.

Our final result is that under optimal indexation the financial accelerator is significantly muted. This suggests that theoretical studies that find large accelerator effects do so because they restrict the degree of indexation. For example, the agency cost model of BGG can produce amplification of technology shocks, while the agency cost model of Carlstrom and Fuerst (1997) typically delivers a dampening of shocks. This difference does not arise from the underlying model of agency costs, but is instead a consequence of the fact that in BGG the intertemporal loan contract is suboptimal as it is not indexed to innovations in observable conditions. In Carlstrom and Fuerst (1997) the loan contract is intratemporal and is similar to assuming full indexation in BGG. Hence, the financial accelerator does not arise from agency costs per se, but from the nature of contract indexation.

The key variable in the analysis is leverage, defined to be the ratio of project size to the borrower's net worth. The underlying distortion is linked to leverage, suggesting that optimal debt-indexation will dampen movements in leverage. When debt contracts are not indexed, borrowers' net worth absorbs all of the innovation in the project's return so that leverage fluctuates too much. First-best behavior is achievable if the borrower's net worth and the first-best project size move together over the cycle so that leverage remains constant. But this first-best behavior is not achievable because entrepreneurs do not internalize the effect of their net worth levels on the future level of leverage.

The paper most closely related to ours is Krishnamurty (2003). Krishnamurty introduces insurance markets into a three period model where borrowing is secured by collateral as in Kiyotaki and Moore (1997). These insurance markets allow for state contingent debt that is indexed to aggregate shocks as in our framework. Krishnamurty shows that such insurance eliminates any feedback from collateral values onto investment and thus reduces the collateral amplification to zero. While our main findings are consistent with Krishnamurty, there are important differences in the analysis. First, we study state contingent debt in a fully calibrated DSGE model. This allows us to examine how debt indexation schemes interact with the endogenous net worth accumulation of borrowers, an effect which is not present in the three-period setup of Krishnamurty. Endogenity of net worth is important in order to examine the welfare consequences of different indexation schemes. Second, we choose the CSV framework rather than collateral constraints for generating financial frictions. The BGG model is often the preferred model of financial frictions, because default occurs in equilibrium and credit spreads arise endogenously. Thus, our contribution is to show how to introduce indexation in this widely used model of credit spreads.

The paper proceeds as follows. Section 2 presents the basic model. Section 3 examines the efficiency gains of contract indexation. Section 4 provides a qualitative analysis of the link between contract indexation and the financial accelerator. Section 5 presents the quantitative analysis on the optimal level of indexation and the effect that this indexation has on the model's behavior. Section 6 concludes.

2. The Model.

Households.

The typical household consumes the final good (C_t) and sells labor input (N_t) to the firm at real wage w_t. Preferences are given by

$$U(C_t, L_t) \equiv \frac{C_t^{1-\sigma}}{1-\sigma} - B\frac{N_t^{1+\eta}}{1+\eta}.$$

The household budget constraint is given by

$$C_t + D_t + Q_t^L S_t \leq w_t N_t + R_{t-1}^D D_{t-1} + (Q_t^L + Div_t) S_{t-1}$$

The household chooses the level of deposits (D_t) which are then used by the lender to fund the entrepreneurs (more details below). The (gross) real rate R_t^d on these deposits is known at time-t. The household owns shares in the final goods firms, capital-producing firms, and in the lender. The former two are standard, so we simply focus on the shares of the lender. This share price is denoted by Q_t^L with Div_t denoting lender dividends, and S_t the number of shares held by the representative household (in equilibrium S_t = 1). The optimization conditions include:

$$-U_N(t)/U_c(t) = w_t \quad (1)$$

$$U_c(t) = E_t \beta U_c(t+1) R_t^d \quad (2)$$

Final goods firms.

Final goods are produced by competitive firms who hire labor and rent capital in competitive factor markets at real wage w_t and rental rate r_t. The production function is Cobb-Douglass where A_t is the random level of total factor productivity:

$$Y_t = K_t^{\alpha}(A_t N_t)^{1-\alpha} \tag{3}$$

The optimization conditions include:

$$MPN_t = w_t \tag{4}$$

$$MPK_t = r_t \tag{5}$$

New Capital Producers.

The production of new capital is subject to adjustment costs. In particular, investment firms take $I_t \phi(\frac{I_t}{\delta K_t})$ consumption goods and transform them into I_t investment goods that are sold at price Q_t. Their profits are thus given by $Q_t I_t - I_t \phi(\frac{I_t}{\delta K_t})$, where the function ϕ is convex with $\phi(1) = 1,\ \phi'(1) = 0$ and $\phi"(1) = \psi$. Variations in investment lead to variations in the price of capital. Variations in the price of capital are a key part of the aggregate uncertainty facing the entrepreneur.

Lenders.

The representative lender accepts deposits from households (promising sure return R_t^d) and provides loans to the continuum of entrepreneurs. These loans are intertemporal, with the loans made at the end of time t being paid back in time t+1. The gross real return on these loans is denoted by R_{t+1}^L. Each individual loan is subject to idiosyncratic and aggregate risk, but since the lender holds an entire portfolio of loans, only the aggregate risk remains. The lender has no other source of funds, so the level of loans will equal the level of deposits. Hence, dividends are given by $Div_{t+1} = R_{t+1}^L D_t - R_t^d D_t$. The intermediary seeks to maximize its equity value which is given by:

$$Q_t^L = E_t \sum_{j=1}^{\infty} \beta^j \frac{U_c(t+j)}{U_c(t)} Div_{t+j} \tag{6}$$

The FOC of the lender's problem is:

$$E_t \frac{\beta U_c(t+1)}{U_c(t)} [R_{t+1}^L - R_t^d] = 0 \tag{7}$$

The first-order condition shows that in expectation, the lender makes zero profits, but ex-post profits and losses can occur. We assume that losses are covered by households as negative dividends. This is similar to the standard assumption in the Dynamic New Keynesian (DNK) model, eg., Woodford (2003). That is, the sticky price firms are owned by the household and pay out profits to the household. These profits are typically always positive (for small shocks) because of the steady state mark-up over marginal cost. Similarly, one could introduce a steady-state wedge (eg., monopolistic competition among lenders) in the lender's problem so that dividends are always positive. But this assumption would have no effect on the model's dynamics so we dispense from it for simplicity.

Entrepreneurs and the Loan Contract.

Entrepreneurs are the sole accumulators of physical capital. The time t+1 rental rate and capital price are given by r_{t+1} and Q_{t+1}, respectively, implying that the gross return to holding capital from time-t to time t+1 is given by:

$$R^k_{t+1} \equiv \frac{r_{t+1}+(1-\delta)Q_{t+1}}{Q_t}. \tag{8}$$

Agency costs imply a steady-state distortion on capital accumulation. Below we will find it convenient to eliminate this steady-state distortion by adding a constant subsidy to capital accumulation so that the return to capital will be given by $R^k_{t+1}(1+sub)$.

At the end of period t, the entrepreneurs sell all of their accumulated capital, and then re-purchase it along with any net additions to the capital stock. This purchase is financed with entrepreneurial net worth (NW_t) and external financing from a lender. The external financing is subject to a costly-state-verification (CSV) problem. In particular, one unit of capital purchased at time-t is transformed into ω_{t+1} units of capital in time t+1, where ω_{t+1} is a idiosyncratic random variable with density $\phi(\omega)$ and cumulative distribution $\Phi(\omega)$. The realization of ω_{t+1} is directly observed by the entrepreneur, but the lender can observe the realization only if a monitoring cost is paid. Assuming that the entrepreneur and lender are risk-neutral, Townsend (1979) demonstrates that the optimal contract between entrepreneur and intermediary is risky debt in which monitoring only occurs if the promised payoff is not forthcoming. Payoff does not occur for sufficiently low values of the idiosyncratic shock, $\omega_{t+1} < \varpi_{t+1}$. Let Z_{t+1} denote the promised gross rate-of-return so that Z_{t+1} is defined by

$$Z_{t+1} \equiv \varpi_{t+1} R^k_{t+1}(1+sub)\frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} \tag{9}$$

where we have found it convenient to define $\bar{\kappa}_t$ to be the leverage ratio:

$$\bar{\kappa}_t \equiv \left(\frac{Q_t K_{t+1}}{NW_t}\right) \tag{10}$$

The CSV problem takes as exogenous the return on capital (R^k_{t+1}) and the opportunity cost of the lender. With $f(\varpi_{t+1})$ and $g(\varpi_{t+1})$ denoting the entrepreneur's share and lender's share of the project outcome, respectively, the lender's ex post realized t+1 return on the loan contract is defined as:

$$R^L_{t+1} \equiv R^k_{t+1}(1+sub)g(\varpi_{t+1})\frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} \tag{11}$$

where

$$f(\varpi) \equiv \int_{\varpi}^{\infty} \omega\phi(\omega)d\omega - [1-\Phi(\varpi)]\varpi \tag{12}$$

$$g(\varpi) \equiv [1-\Phi(\varpi)]\varpi + (1-\mu)\int_0^{\varpi} \omega\phi(\omega)d\omega \tag{13}$$

Recall that the lender's return is linked to the return on deposits via (7):

$$E_t R^L_{t+1} U_c(t+1) = R^d_t E_t U_c(t+1) \tag{14}$$

The end-of-time-t contracting problem is thus given by[1]:

$$\text{Max } E_t R^k_{t+1}(1+sub)Q_t K_{t+1} f(\varpi_{t+1}) \tag{15}$$

subject to

$$E_t R^k_{t+1}(1+sub)Q_t K_{t+1} U_c(t+1) g(\varpi_{t+1}) \geq R^d_t E_t U_c(t+1)[Q_t K_{t+1} - NW_t] \tag{16}$$

[1] The participation constraint for the lender (16) differs from BGG in that the lender internalizes the marginal utility of consumption for households. Since the contracting problem takes $U_c(t+1)$ as given, the lender is still risk neutral. This internalization follows directly from the previous section.

The optimization conditions include:

$$E_t R_{t+1}^k (1+sub)[f'(\varpi_{t+1}) + \lambda_t U_c(t+1) g'(\varpi_{t+1})] = 0 \tag{17}$$

$$E_t R_{t+1}^k (1+sub)[f(\varpi_{t+1}) + \lambda_t U_c(t+1) g(\varpi_{t+1})] = \lambda_t R_t^d E_t U_c(t+1) \tag{18}$$

$$E_t U_c(t+1) R_{t+1}^k (1+sub) \frac{\bar{\kappa}_t}{\bar{\kappa}_{t-1}} g(\varpi_{t+1}) = R_t^d E_t U_c(t+1) \tag{19}$$

The contracting problem takes as given the deposit rate R_t^d and the random variables $U_c(t+1)$ and R_{t+1}^k. Equation (17) can be solved for the multiplier so that (18) becomes

$$E_t R_{t+1}^k (1+sub) \left[f(\varpi_{t+1}) + U_c(t+1) g(\varpi_{t+1}) \left\{ \frac{-E_t R_{t+1}^k f'(\varpi_{t+1})}{E_t R_{t+1}^k U_c(t+1) g'(\varpi_{t+1})} \right\} \right] =$$

$$\left\{ \frac{-E_t R_{t+1}^k f'(\varpi_{t+1})}{E_t R_{t+1}^k U_c(t+1) g'(\varpi_{t+1})} \right\} R_t^d E_t U_c(t+1) \tag{20}$$

The contract is defined by ϖ_{t+1} and leverage ratio $\frac{\bar{\kappa}_t}{\bar{\kappa}_{t-1}}$ that satisfy (19)-(20). If there were no aggregate uncertainty, then (20) implies that ϖ_{t+1} would be a function only of the spread $\frac{R_{t+1}^k (1+sub)}{R_t^d}$. For this reason below we consider indexation schemes in which ϖ_{t+1} (and thus Z_{t+1}) responds to innovations in R_{t+1}^k and a set of pre-determined variables[2]:

$$\varpi_{t+1} = \mathrm{P}_t \frac{\left(R_{t+1}^k\right)^{\chi-1}}{E_t\left[\left(R_{t+1}^k\right)^{\chi-1}\right]}. \tag{21}$$

$$Z_{t+1} = P_t \frac{\left(R_{t+1}^k\right)^{\chi}}{E_t\left[\left(R_{t+1}^k\right)^{\chi-1}\right]} (1+sub) \left(\frac{\bar{\kappa}_t}{\bar{\kappa}_{t-1}}\right). \tag{22}$$

[2] The indexation scheme (21) is a modest restriction on the contract space.

where P_t is time-t variable chosen to satisfy (20). As we vary χ, we trace out a variety of possible indexation schemes. From (11), different indexation schemes then imply different behavior for the lender's return. For example, for χ = 1, the bankruptcy rate is predetermined, while the loan repayment Z_{t+1} and lender's return R^L_{t+1} are perfectly indexed to innovations in R^k_{t+1}. In sharp contrast, BGG assumed that the lender's return is predetermined, $R^L_{t+1} = R^d_t$. This implies χ < 0, so that the loan repayment varies inversely with innovations in R^k_{t+1}.[3] Below we will find the value of χ that maximizes household utility.

Entrepreneurs have linear preferences and discount the future at rate β. Given the high return to internal funds, they will postpone consumption indefinitely. To limit net worth accumulation and ensure that there is a need for external finance in the long run, we assume that fraction (1-γ) of the entrepreneurs die each period. Their accumulated assets are sold and the proceeds transferred to households as consumption. As in Carlstom, Fuerst, and Paustian (2010), the assumption that dying entrepreneurs' assets are transferred to households implies that the welfare criterion can be taken to be household utility. Given the exogenous death rate, aggregate net worth accumulation is described by

$$\mathrm{NW_t} = \gamma NW_{t-1}\bar{\kappa}_{t-1}R^k_t(1+sub)f(\varpi_t) \tag{23}$$

Equivalently, we can use the definition of $f(\varpi_t)$ and $\bar{\kappa}_{t-1}$ and express net worth as:

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[3] BGG assumed that the lender's return did not vary with innovations in R^k_{t+1}. Differentiating (11) yields

$$\frac{R^L_{t+1}}{dR^k_{t+1}} = (1+sub)g(\varpi_{t+1})\frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1} + R^k_{t+1}(1+sub)g'(\varpi_{t+1})\frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1}\frac{d\varpi_{t+1}}{dR^k_{t+1}}$$

Evaluating this at the efficient steady-state and imposing the BGG assumption that $\frac{d\,R^L_{t+1}}{dR^k_{t+1}} = 0$, we have:

$$\frac{dln\varpi_{t+1}}{dlnR^k_{t+1}} = \frac{-1}{\left(\frac{g\prime(\varpi_{SS})\varpi_{SS}}{g(\varpi_{SS})}\right)} < -1$$

From (21) this implies that χ < 0.

$$\mathrm{NW_t} = \gamma NW_{t-1}\{R_t^L(1-\bar{\kappa}_{t-1}) + \bar{\kappa}_{t-1}R_t^k(1+sub)[1-\mu\int_0^{\varpi_t}\omega\phi(\omega)d\omega]\} \qquad (24)$$

Equation (22) implies that $\mathrm{NW_t}$ is determined by the realization of R_t^k and the contracted response of R_t^L and ϖ_t to these realizations. NW_t then enters the contracting problem in time t so that the realization of R_t^k is propagated forward.

<u>Market Clearing and Equilibrium.</u>

In equilibrium the household holds the shares of the lender, and the lender funds the entrepreneurs' projects: $S_t = 1$ and $D_t = Q_tK_{t+1} - NW_t$. For a given indexation parameter χ, the equilibrium is then defined by the variables $\{C_t, N_t, I_t, K_{t+1}, R_t^k, \varpi_t, NW_t, \mathrm{P}_{t-1}, Q_t\}$ such that the following conditions are satisfied:

$$-U_N(t)/U_c(t) = MPN_t \qquad (25)$$

$$R_{t+1}^k \equiv \frac{MPK_{t+1}+(1-\delta)Q_{t+1}}{Q_t}. \qquad (26)$$

$$\mathrm{NW_t} = \gamma NW_{t-1}\bar{\kappa}_{t-1}R_t^k(1+sub)f(\varpi_t) \qquad (27)$$

$$E_t\beta R_{t+1}^k(1+sub)\left[f(\varpi_{t+1}) + U_c(t+1)g(\varpi_{t+1})\left\{\frac{-E_tR_{t+1}^kf'(\varpi_{t+1})}{E_tR_{t+1}^kU_c(t+1)g'(\varpi_{t+1})}\right\}\right] =$$

$$\left\{\frac{-E_tR_{t+1}^kf'(\varpi_{t+1})}{E_tR_{t+1}^kU_c(t+1)g'(\varpi_{t+1})}\right\}U_c(t) \qquad (28)$$

$$E_t\beta U_c(t+1)R_{t+1}^k(1+sub)\left(\frac{\bar{\kappa}_t}{\bar{\kappa}_{t-1}}\right)g(\varpi_{t+1}) = U_c(t) \qquad (29)$$

$$\varpi_t = \mathrm{P}_{t-1}\frac{(R_t^k)^{\chi-1}}{E_{t-1}(R_t^k)^{\chi-1}}.. \qquad (30)$$

$$Q_t = \phi\left(\frac{I_t}{\delta K_t}\right) + \frac{I_t}{\delta K_t}\phi'\left(\frac{I_t}{\delta K_t}\right), \tag{31}$$

$$C_t + I_t\phi\left(\frac{I_t}{\delta K_t}\right) + \mu R_t^k Q_{t-1} K_t\left[\int_0^{\varpi_{t+1}} \omega\phi(\omega)d\omega\right] = K_t^\alpha (A_t N_t)^{1-\alpha} \tag{32}$$

$$K_{t+1} = (1-\delta)K_t + I_t \tag{33}$$

Time-t monitoring costs are given by $\mu \int_0^{\varpi_t} \omega\phi(\omega)d\omega$ units of capital. The calibration used below implies that these costs are quite small in the steady-state, $\mu \int_0^{\varpi_{ss}} \omega\phi(\omega)d\omega = .0004$, compared to the calibrated value of $\delta = .02$.

3. Wedges and Inefficiency.

Equilibrium behavior in the model is defined by the employment and investment decisions. The marginal condition for employment (25) is not distorted relative to the condition in a RBC model. But capital accumulation is distorted. The lender's intertemporal condition in the agency cost model is given by:

$$U_c(t) = E_t\beta U_c(t+1)R_{t+1}^k(1+sub)\left(\frac{\bar{\kappa}_t}{\bar{\kappa}_{t-1}}\right) g(\varpi_{t+1}) \tag{34}$$

We can interpret this distortion in (34) as a particular tax or wedge in the corresponding RBC model. In particular, consider a RBC model with a tax τ_{t+1}^k on total capital accumulation so that the household budget constraint is:

$$Q_t k_{t+1} + c_t \leq [r_t + (1-\delta)Q_t]k_t(1-\tau_t^k) \tag{35}$$

The household's capital accumulation choice is given by

$$U_c(t) = E_t \beta U_c(t+1) R^k_{t+1} (1 - \tau^k_{t+1}). \tag{36}$$

This margin (36) is distorted in the same way as (34). Hence, the agency cost model is isomorphic to an RBC model with a tax on total capital accumulation, but where this tax rate varies endogenously with net worth and other macroeconomic variables.

We wish to compare (34) with the optimal or efficient behavior that would be chosen by a social planner. We would like to concentrate on the distorted accumulation equation and abstract from any (small) income effects that arise from monitoring costs. Hence, consider an agency cost model in which the monitoring costs are eliminated from the societal resource constraint, but these monitoring costs still affect contracting. That is, suppose that monitoring costs $\mu R^k_t Q_{t-1} K_t \left[\int_0^{\varpi_{t+1}} \omega \phi(\omega) d\omega\right]$, are paid for by a transfer from a party external to the economy. Recall that steady-state monitoring costs are quite small, 0.04% of steady-state capital, so that this is a modest abstraction. We will call this the zero resource cost (ZRC) model of agency costs.

The social planner in the ZRC agency cost model still must respect the financial contract and the resource constraints (33)-(34), but she can shift net worth between the lender and entrepreneur in such a way to achieve first best behavior. The planner's optimal capital accumulation equation in the ZRC model is given by

$$U_c(t) = E_t \beta U_c(t+1) R^k_{t+1} \tag{37}$$

Note that (37) is also the intertemporal condition in the RBC model. We will say that the agency model achieves ZRC efficiency if it is consistent with (37). Comparing (37) with (34), there is a time-varying distortion on capital accumulation, $(1+sub)\left(\frac{\bar{\kappa}_t}{\bar{\kappa}_{t-1}}\right)g(\varpi_{t+1})$. We first demonstrate that the agency cost model cannot achieve ZRC efficiency.

Proposition 1: A constant subsidy cannot make the agency cost model ZRC efficient.

Proof: See appendix.

Remark 1: If entrepreneurs lived forever but discounted the future more heavily at rate Γ, then (27) would be replaced by

$$1 = \beta\Gamma R_{t+1}^{k}(1+sub)f(\varpi_{t+1})\bar{\kappa}_t \tag{38}$$

In this case it is straightforward to show that this variant of the agency cost model is also unable to achieve ZRC efficiency.

The inability of the agency cost model to achieve ZRC efficiency arises from the fact that entrepreneurs do not internalize the effects that their actions have on household consumption. We will demonstrate this by showing that if entrepreneurs did internalize this effect, then the agency cost model can achieve ZRC efficiency. With e_t denoting the portion of the entrepreneur's income that is not part of next period's net worth, the entrepreneur's time-t budget constraint is given by:

$$e_t + NW_t \leq NW_{t-1}\bar{\kappa}_{t-1}R_t^k(1+sub)f(\varpi_t) \tag{39}$$

The agency cost model assumes that entrepreneurs die at rate γ and pass on their estates to households so that

$$e_t = (1-\gamma)NW_{t-1}\bar{\kappa}_{t-1}R_t^k(1+sub)f(\varpi_t). \tag{40}$$

But instead of dying, suppose that entrepreneurs discount the future more heavily, and that they choose e_t to maximize the household's valuation of the entrepreneur:

$$Entrep\ value = E_t \sum_{j=0}^{\infty} (\beta\Gamma)^j \frac{U_c(t+j)}{U_c(t)} e_{t+j} \tag{41}$$

Similar to the death assumption, the additional discounting is needed to ensure that the agency costs bind in the steady-state. The entrepreneur's optimal accumulation equation is given by

$$U_c(t) = E_t \beta\Gamma U_c(t+1) R_{t+1}^k (1+sub) f(\varpi_{t+1}) \bar{\kappa}_t \tag{42}$$

This implies that in the contracting problem we now need to discount the entrepreneur's return with household marginal utility, that is, we replace $f(\varpi_{t+1})$ with $\Gamma U_c(t+1) f(\varpi_{t+1})$ in the contracting problem. We can now state:

Proposition 2: If entrepreneurs internalize, a constant subsidy and $\chi = 1$ will make the agency cost model ZRC efficient.

Proof: See appendix.

One immediate implication of Proposition 2 is that the inability of indexation to completely eliminate the distortion in the agency cost model is a result of an externality. In particular, entrepreneurs do not internalize the effect of their behavior on aggregate conditions generally, and household consumption in particular.

In addition, Proposition 2 provides an important theoretical benchmark. Absent the externality of entrepreneurial decision-making, the agency cost model is ZRC efficient if the loan repayment is indexed to aggregate shocks in such a way as to make bankruptcy rates (and the risk premium) predetermined. As part of the quantitative analysis below, we will demonstrate that this level of indexation is also necessary to achieve efficiency. Further, we will calculate the welfare cost of suboptimal levels of indexation, and the welfare cost of the entrepreneurial externality.

4. Contract Indexation and the Financial Accelerator.

The degree of contract indexation matters because it alters the behavior of entrepreneurial net worth which in turn alters the behavior of the capital accumulation distortion. These effects can be most easily seen if we look at the contracting problem in log deviations (lower case variables). (The appendix contains the linear approximation of the entire model.) Equations (11) and (21)-(22) become

$$r_t^l \equiv -\frac{1}{(\kappa-1)}\kappa_{t-1} + \Theta\varpi_t + r_t^k \tag{43}$$

$$\varpi_t \equiv p_{t-1} + (\chi-1)(r_t^k - E_{t-1}r_t^k) \tag{44}$$

$$z_t \equiv p_{t-1} - \frac{1}{(\kappa-1)}\kappa_{t-1} + E_{t-1}r_t^k + \chi(r_t^k - E_{t-1}r_t^k) \tag{45}$$

where $\Theta \equiv \frac{\varpi_{ss}g\prime(\varpi_{ss})}{g(\varpi_{ss})} < 1$ and $\kappa_{t-1} \equiv (q_{t-1} + k_t - nw_{t-1})$ is the log of $\bar{\kappa}_{t-1}$. In a convenient abuse of notation we use κ to denote the steady state leverage ratio, $\kappa = \bar{\kappa}_{ss}$. To solve for p_{t-1}, we need to make use of the linearized optimal contract (see equation A1 in the appendix):

$$E_{t-1}(r_t^k - r_t^l) = E_{t-1}r_t^k - r_{t-1}^d = \nu\kappa_{t-1} \tag{46}$$

Using (46) and (43)-(44) to solve for p_{t-1}, we can express (43)-(45) as:

$$r_t^l = r_{t-1}^d + [1 + \Theta(\chi-1)](r_t^k - E_{t-1}r_t^k) \tag{47}$$

$$\varpi_t = \frac{[1-\nu(\kappa-1)]}{\Theta(\kappa-1)}\kappa_{t-1} + (\chi-1)(r_t^k - E_{t-1}r_t^k) \quad (48)$$

$$z_t \equiv \left\{\frac{1-\nu(\kappa-1)-\Theta}{\Theta(\kappa-1)}\right\}\kappa_{t-1} + E_{t-1}r_t^k + \chi\left(r_t^k - E_{t-1}r_t^k\right) \quad (49)$$

In summary, the contract is given by the promised repayment (49), the bankruptcy cut-off (48), and the lender's return (47). All of these are affected by the degree of indexation.

From (34), the agency cost distortion is given by the spread between the return on capital and the deposit rate. In log deviations this distortion is given by (46). Evidently an indexation parameter that minimizes the variance of κ_{t-1} will be preferred. That is, an optimal indexation parameter will induce net worth to move with the efficient level of capital accumulation so that leverage remains constant. From (24), net worth is given by

$$nw_t = \kappa\frac{\gamma}{\beta}\left(r_t^k - r_t^l\right) + \frac{\gamma}{\beta}(r_t^l + nw_{t-1}) + \gamma\kappa\frac{rp}{\beta}(k_t + q_t + r_t^k) \quad (50)$$

The level of net worth in time-t is a function of lagged net worth and the behavior of the spread $\left(r_t^k - r_t^l\right)$. The behavior of the spread in turn depends on the nature of contract indexation.

To gain intuition we focus on two cases. The first is the one chosen by BGG who assume that the real return r_t^l is pre-determined, $r_t^l = r_{t-1}^d$. This implies that $\chi = \frac{(\Theta-1)}{\Theta} < 0$, so that ϖ_t and z_t are both decreasing in r_t^k. Alternatively suppose that $\chi = 1$, so that the promised repayment is fully indexed to aggregate shocks, while bankruptcy rates are predetermined. As shown in Proposition 2, full indexation is ZRC efficient in the special case where entrepreneurs internalize the effect of their behavior on household consumption. With $\chi = 1$, the lender's return moves one-for-one with innovations in r_t^k so that the first term in (50) is zeroed out and innovations in r_t^k increase net worth with a coefficient of $\frac{\gamma}{\beta} \approx 1$. But under BGG indexation, there is an additional effect:

$$nw_t^{BGG} = nw_t^{\chi=1} + (\kappa - 1)\frac{\gamma}{\beta}\left(r_t^k - E_{t-1}r_t^k\right) \tag{51}$$

The difference in net worth between full indexation and BGG is quantitatively important since the calibration implies $(\kappa - 1)\frac{\gamma}{\beta} \approx 1$. Hence, the response of net worth to innovations is roughly double in BGG compared to the case of $\chi = 1$. More generally, net worth for various indexation schemes all differ from each other by a one-time innovation in net worth. These net worth innovations are then propagated forward via (50).

Innovations in the return on capital are largely driven by innovations in the price of capital. The interaction between net worth movements and the price of capital is a manifestation of the financial accelerator. To gain some insight on this feedback loop, let us set the capital stock to its steady state, and assume that shocks are iid. Appendix equations (A1) and (A10)-(A11) then become

$$nw_t = \left(\frac{\delta+\psi}{\psi}\right) q_t - \frac{1}{\nu} E_t(r_{t+1}^k - r_{t+1}^l) \tag{52}$$

This link between net worth and the price of capital comes from the market for capital and includes the capital supply curve and capital accumulation equation. The entrepreneur's demand for capital and thus the price of capital, varies positively with the future return to capital $E_t(r_{t+1}^k - r_{t+1})$, and the entrepreneur's net worth nw_t. If there were no agency costs ($\nu = 0$), then (52) becomes $E_t\left(r_{t+1}^k - r_{t+1}^l\right) = 0$, and the market for capital is unaffected by the level of net worth. Hence, one manifestation of agency costs is that the price of capital varies positively with net worth. For the case of iid shocks, the response of $E_t(r^k_{t+1} - r^l_{t+1})$ to the exogenous shocks is roughly proportional to the response of the capital price to innovations.[4] Hence we can write $E_t\left(r^k_{t+1} - r^l_{t+1}\right) \approx a_q q_t$. Equation (52) then becomes:

$$nw_t = \left[\frac{\nu(\delta+\psi)-\psi a_q}{\psi\upsilon}\right] q_t \tag{53}$$

To provide a sense of magnitudes, for our baseline calibration we have $a_q \approx -0.18$, and $\left[\frac{\nu(\delta+\psi)-\psi a_q}{\psi\upsilon}\right] \approx 5.5$.

The other link between capital prices and net worth comes from the evolution equation for net worth (50):

$$nw_t = \frac{\gamma}{\beta}[1+\Theta(\kappa-1)(1-\chi)][\epsilon q_t + (1-\epsilon)mpk_t - q_{t-1}] + \left[\frac{\gamma}{\beta} nw_{t-1} + \varepsilon_t^{nw}\right] \tag{54}$$

where we have used (47) and $rp \approx 0$. The term ε_t^{nw} is an exogenous innovation to net worth (a one-time transfer of resources from the household to the entrepreneur). Note that the evolution of net worth depends upon the value of χ. The financial accelerator is evident in the feedback effect in (53)-(54): higher levels of net worth increase the demand for capital and thus the price of capital (53); the higher price of capital then increases net worth (54), etc.

Solving (53)–(54) we have

$$nw_t = \frac{M[(1-\epsilon)mpk_t - q_{t-1}]}{1-M}\left[\frac{\nu(\delta+\psi)-\psi a_q}{\epsilon\psi\upsilon}\right] + \frac{\left[\frac{\gamma}{\beta}nw_{t-1}+\varepsilon_t^{nw}\right]}{1-M} \tag{55}$$

$$q_t = \left[\frac{\psi\upsilon}{\nu(\delta+\psi)-\psi a_q}\right] nw_t \tag{56}$$

[4] This approximation is exact for iid net worth shocks, but is only approximate for iid technology shocks. However, even in this case the approximation is good for χ not too large.

where

$$M \equiv \frac{\frac{\gamma}{\beta}[1+\Theta(\kappa-1)(1-\chi)]\epsilon\psi\upsilon}{\nu(\delta+\psi)-\psi a_q}, \tag{57}$$

denotes the multiplier effect at work here. Innovations in net worth in (55) are multiplied into much larger changes in net worth ($\frac{1}{1-M}$) by the financial accelerator feedback loop. Note the interaction between leverage and indexation. In expressions (55)-(57), only the product $(\kappa - 1)(1-\chi)$ appears. That is, up to a first-order approximation, what matters is the product $(\kappa-1)(1-\chi)$, and not the individual value of κ or χ. Suppose that the optimal level of indexation satisfies $(\kappa-1)(1-\chi) = \text{DD}$, or $\chi = 1 - \frac{\text{DD}}{(\kappa-1)}$. This suggests that the optimal χ will converge to unity as leverage (κ) increases.

The size of the financial accelerator depends critically upon the value of χ. An important cut-off point is where M goes to unity or the net worth multiplier ($\frac{1}{1-M}$) goes to plus (or minus) infinity. This χ value is given by:

$$\underline{\chi} \equiv \frac{\gamma\epsilon\psi\upsilon[1+\Theta(\kappa-1)]-\beta[\nu(\delta+\psi)-\psi a_q]}{\gamma\epsilon\psi\upsilon\Theta(\kappa-1)} \tag{58}$$

Again, to provide a sense of magnitudes, the benchmark calibration implies $\underline{\chi} \approx -4$. As χ approaches $\underline{\chi}$ from above, M approaches unity from below, and the slopes of (53) and (54) come together. In this case the multiplier effect goes to positive infinity.[5] Such large responses of net worth and prices are welfare-reducing so that optimal indexation will always satisfy $\chi \gg \underline{\chi}$.

[5] As χ moves below $\underline{\chi}$, M exceeds unity and the multiplier becomes negative.

Note that $\underline{\chi}$ is increasing in κ, so that the optimal degree of indexation will be increasing in the size of leverage.

Figure 1 demonstrates the effect of χ on the financial accelerator by graphing (53) and (54) in nw-q space. The figure demonstrates the effect of a one unit shock in net worth, ε_t^{nw} = 1. The figure ignores any modest effect of net worth on employment and thus the marginal product of capital by setting mpk_t= 0 (this would be exactly correct, for example, if labor supply were inelastic). Equation (53) cuts through the origin and is not shifted by shocks. If there were no agency costs, then the price of capital would not depend upon net worth so that (53) would be a vertical line in Figures 1-2. The remaining lines are equation (54) drawn under different values of χ. The flat line is χ = 2.06, and corresponds to the case where the multiplier is one (M = 0, or the slope of (54) is zero). Full indexation (χ = 1) and the BGG model ($\chi = \frac{\Theta-1}{\Theta} \approx -.01$) are also presented. To illustrate the effect of a larger multiplier, Figure 1 also considers the case of χ = -1. All four of these cut the vertical axis at unity because the shock is ε_t^{nw} = 1. In each case the increase in net worth increases the price of capital. This is the agency cost effect. But the size of the feedback of this price change back on to net worth varies inversely with the degree of indexation. In each case (except for the M = 0 case), the ultimate effect on net worth is magnified relative to the exogenous innovation, with the degree of this magnification decreasing in χ.

Figure 2 presents the complementary experiment for the case of an iid productivity shock that increases the marginal product of capital by 10%, mpk_t= 10 and ε_t^{nw} = 0. As before, equation (53) cuts through the axis and equation (54) is drawn for the four different values of χ. All four of these cut the horizontal axis at $q_t = -\left(\frac{1-\epsilon}{\epsilon}\right)10$, because this is the movement in asset prices that holds net worth fixed (see (54)). The productivity shock increases the price of capital, but has differing effects on net worth depending upon the nature of the contract (this corresponds to their location on the vertical axis). These net worth movements are then multiplied up into larger movements in net worth and the price of capital. As before, this amplification is inversely related to the degree of indexation χ.

5. Quantitative Analysis.

Calibration

Our calibration will largely follow BGG. The discount factor β is set 0.99. Utility is assumed to be logarithmic in consumption (σ=1), and the elasticity of labor is assumed to be 1/3 (θ = 3). The production function parameters include α = 0.35, investment adjustment costs ψ = 0.25, and quarterly deprecation is δ = .025. As for the credit-related parameters, we calibrate the model to be consistent with: (i) a spread between $R^k(1 + sub)$ and R^s of 200 bp (annualized), (ii) monitoring costs μ = 0.12, and (iii) a leverage ratio of κ = K/NW = 1.954. These values imply a death rate of γ = 0.98, a standard deviation of the idiosyncratic productivity shock of 0.28, and a quarterly bankruptcy rate of .75% (ϖ_{ss} = 0.486). This then implies ν = 0.041.

We assume that total factor productivity follows an AR(1) process. We follow BGG and assume that technology shocks are nearly permanent with an AR coefficient very close to one, ρ^A = 0.99. The net worth shocks are also serially correlated with ρ^{nw} = 0.8.

Optimal Indexation.

We calculate the optimal indexation using unconditional household welfare as the metric. For a given level of indexation, we first compute a second-order perturbation solution using the methods of Schmitt-Grohe and Uribe (2004) as implemented in the software package DYNARE. Using these decision rules, the software computes the unconditional mean of the value function of households. We then search on a fine grid for the value of χ that maximizes this welfare measure. As a robustness check, we also include a conditional welfare measure in our analysis. Unconditional welfare neglects the transitional dynamics of the endogenous variables from a particular initial condition to their new long run distribution. For simplicity and in line with many other studies, we assume that the initial state is the deterministic steady state.

We conjecture that optimal indexation will come close to minimize the resulting movements in the spread, as the spread is a manifestation of the agency cost distortion. Recall that the spread is given by

$$E_t\left(r_{t+1}^k - r_{t+1}^l\right) = \nu\kappa_t = \nu[q_t + k_{t+1} - nw_t]$$

Hence, to minimize movements in the spread, the movement in net worth must be comparable to the movement in Pareto efficient capital spending. The indexation parameter determines the response of net worth to shock innovations. But the subsequent behavior of net worth is independent of indexation as it is given by the savings behavior of entrepreneurs. If entrepreneurs internalize the effects of their behavior on households, then the subsequent path of net worth will be optimal. But if entrepreneurs do not internalize their behavior, then net worth decays at the exogenous death rate. In this case optimal contract indexation is a second-best problem as it is typically not possible to achieve the necessary level of net worth both in the period of the shock and along an entire dynamic path.

Table 1 presents the optimal indexation parameters for the baseline calibration. Recall that if entrepreneurs internalize the effects of their savings decision, then Proposition 2 demonstrates that in a ZRC economy the optimal indexation is $\chi = 1$. More realistically, if we move away from the assumption of internalization, the optimal level of indexation exceeds unity. Positive productivity shocks increase R_t^k and shift wealth towards entrepreneurs, an effect that varies inversely with χ. Since the movement in efficient capital levels is small, the needed movement in net worth is also small. To prevent net worth from moving too much, optimal indexation is given by $\chi > 1$. For net worth shocks, the ZRC efficient capital accumulation response is zero, so that optimal indexation calls for an even larger χ to recoup this net worth movement.

Table 2 provides sensitivity analysis. Note that for TFP shocks, the optimal level of indexation approaches unity as leverage increases. This same behavior is illustrated in Figure 3 which charts the optimal level of indexation (for TFP shocks only) as we vary steady-state leverage from $\kappa = 1.5$ to $\kappa = 5$. The Figure presents the optimal level of indexation for three different metrics: (i) unconditional welfare, (ii) conditional welfare, and (iii) the unconditional variance of the capital accumulation distortion. Note that for all three metrics, the optimal level of indexation approaches unity as leverage increases.[6]

Figure 3 illustrates an interesting phenomenon. For low levels of steady-state leverage the difference between the conditionally-optimal χ and the unconditionally-optimal χ is significant. The conditionally-optimal χ is the value that maximizes welfare assuming that the economy begins with all state variables at the steady-state, while the unconditionally-optimal χ

[6] At the optimal level of indexation, we have numerically confirmed that there is no welfare gain to also indexing the contract to innovations in aggregate consumption.

integrates this conditional criterion over all possible combinations of the state variables, weighted by their likelihood. In this model with several state variables it is difficult to isolate exactly the reason for the difference in conditional vs. unconditional analysis. But for the case of iid net worth shocks, equation (55) is informative:

$$nw_t = \frac{M[(1-\epsilon)mpk_t - q_{t-1}]}{1-M}\left[\frac{\nu(\delta+\psi)-\psi a_q}{\epsilon\psi\upsilon}\right] + \frac{\left[\frac{\gamma}{\beta}nw_{t-1}+\varepsilon_t^{nw}\right]}{1-M} \tag{55}$$

The ZRC-efficient response of real activity to a net worth shock is "no response." Hence, if we assume that all state variables are at their steady state, then we can eliminate ε_t^{nw} from (55) by setting χ as large as possible (so that M goes to negative infinity). This is the conditionally-optimal χ for net worth shocks. But if the initial states are not all at steady-state, then this is suboptimal. From (55), suppose that the lagged share price is away from steady-state. Then a large χ will eliminate the ε_t^{nw}term, but will accentuate the q_{t-1} term. To eliminate the q_{t-1} term, we would need M = 0 or χ of about 2. The unconditionally optimal criterion will choose the indexation parameter that maximizes welfare given these alternative initial states, weighted by their likelihood of occurrence. As this net worth example suggests, the conditionally-optimal and unconditionally-optimal χ can vary substantially. Our focus will be on the unconditional criterion.[7]

Figure 4 presents the welfare cost of alternative indexation schemes using unconditional welfare as the metric (again, for TFP shocks). The level of welfare is reported as the welfare improvement over the BGG indexation (χ = -0.01). Note that in the case of high leverage, κ = 4,

[7] A similar effect arises for iid productivity shocks. In this case the efficient response of asset prices and capital accumulation is essentially zero (for the benchmark parameter values). This implies that net worth should not respond to these shocks. From a conditional perspective this suggests that the optimal χ ≈ 2 so that M = 0. But from an unconditional perspective, such an M value accentuates the initial level of net worth, nw_{t-1}, and is thus suboptimal.

the welfare costs can be substantial: welfare under optimal indexation exceeds BGG by about 2% of consumption. These welfare costs are clearly asymmetric: being below the optimal χ is significantly worse than being above the optimal χ. Evidently this is because low levels of indexation approach the $\underline{\chi}$ cut-off (see (58)) where the financial accelerator becomes infinite and induces large and welfare-reducing responses to TFP shocks.

Figure 5 presents the impulse response function to a TFP shock under three different levels of indexation for the benchmark calibration which includes monitoring costs in the resource constraint. The three different indexation parameters are: (1) the unconditionally optimal $\chi = 1.28$, (2) the conditionally optimal $\chi = 0.51$, and (3) BGG's $\chi = -0.01$. The behavior of output is amplified for the BGG indexation scheme. The TFP shocks increases the return to capital which raises the firm's net worth and thus reduces borrowing costs, albeit by less than 10 annualized basis points. For the unconditionally optimal indexation, net worth is more insulated from the increase in the return to capital, because loan rates adjust nearly one-to-one with the return to capital. Thus, the behavior of output is significantly muted. The counterpart to this muted response of output under optimal indexation is that credit spreads and bankruptcy rates move by less.

Figure 6 shows the corresponding response to a net worth shock of size 1 percent which evolves an AR(1) with persistence of 0.8. Here, the indexation values have been re-optimized for this particular shock. The value for χ which maximizes unconditional welfare is 2.45, while it is 60 for conditional welfare (see our earlier discussion of why this value is large for net worth shocks). Qualitatively, the results are similar as for TFP shocks: the BGG indexation scheme results in the largest aggregate fluctuations. There is a significant financial accelerator effect, as the 1 percent exogenous increase in net worth is scaled up threefold due to the feedback between net worth and asset prices. The conditionally and unconditionally optimal indexation schemes all result in much smaller fluctuations of net worth and output. As with TFP shocks, no indexation scheme is able to fully undo the aggregate effects of the net worth shock. This arises because loan rates are indexed to the unforecastable components in the return to capital, which occur only in the first period. Thus, the optimal indexation must strike a balance between mitigating distortions in that particular period and in subsequent periods which it can only indirectly influence via the net worth accumulation equation of entrepreneurs.

6. Conclusion.

This paper examines the BGG model from the vantage point of contract indexation. To reiterate our principle results: First, the agency cost model is isomorphic to a real business cycle (RBC) model but with an endogenous and time-varying distortion on total capital accumulation. Second, this agency cost distortion arises because entrepreneurs do not internalize the effect of their behavior on aggregate conditions generally and household consumption in particular. Third, for TFP shocks, the optimal level of indexation is typically close to the loan repayment being fully indexed to movements in aggregate conditions. Finally, under optimal indexation, the financial accelerator is significantly muted. Of course, other models can be constructed in which financial frictions have large business cycle effects despite full indexation, see the highly stylized example in Suarez and Sussmann (1997).

There are several interesting implications and extensions of the analysis. First, our analysis could be extended to study whether there is a difference between the socially optimal degree of indexation and the indexation that is chosen by an atomistic private agent. Private agents take market prices as given and therefore do not internalize the impact that their indexation has on aggregate net worth via this channel. The emerging literature on Pigouvian taxation in economies with financial frictions suggest that in general such pecuniary externalities imply that private allocations are not constrained efficient, see Jeanne and Korinek (2010) or Lorenzoni (2008).

Second, future work could examine whether a simple scheme that makes debt contingent on a single statistic like the return to aggregate capital is performing reasonably well when the economy is hit by several different shocks.

Third, our framework has empirical implications that could be studied. Since indexation is more important in economies with high leverage, one would anticipate that we would see more debt indexation in highly-levered sectors. Further the analysis suggests that economies with suboptimal indexation would be more volatile compared to indexed economies. An interesting question for future work is to include the estimation of the indexation parameter as part of a broader DSGE model estimation.

APPENDIX

1. Linearized Model.

$$E_t(r^k_{t+1} - r^l_{t+1}) = \nu\kappa_t \quad (A1)$$

$$nw_t = \kappa\frac{\gamma}{\beta}\left(r^k_t - r^l_t\right) + \frac{\gamma}{\beta}(r^l_t + nw_{t-1}) + \gamma\kappa\frac{rp}{\beta}(k_t + q_t + r^k_t) \quad (A2)$$

$$r^l_t = r^d_{t-1} + [1 + \Theta(\chi - 1)](r^k_t - E_{t-1}r^k_t) \quad (A3)$$

$$\varpi_t = \frac{[1-\nu(\kappa-1)]}{\Theta(\kappa-1)}\kappa_{t-1} + (\chi - 1)(r^k_t - E_{t-1}r^k_t) \quad (A4)$$

$$z_t \equiv \left\{\frac{1-\nu(\kappa-1)-\Theta}{\Theta(\kappa-1)}\right\}\kappa_{t-1} + E_{t-1}r^k_t + \chi\left(r^k_t - E_{t-1}r^k_t\right) \quad (A5)$$

$$r^k_t = \epsilon q_t + (1-\epsilon)mpk_t - q_{t-1} \quad (A6)$$

$$\kappa_{t-1} = (q_{t-1} + k_t - nw_{t-1}) \quad (A7)$$

$$\sigma c_t + \eta n_t = \alpha k_t + (1-\alpha)a_t - \alpha n_t \quad (A8)$$

$$r^d_t = \sigma(E_t c_{t+1} - c_t) \quad (A9)$$

$$q_t = \psi(i_t - k_t) \quad (A10)$$

$$k_{t+1} = \delta i_t + (1-\delta)k_t \quad (A11)$$

$$\left(1 - \frac{\alpha\beta\delta}{1-\epsilon}\right)c_t + \left(\frac{\alpha\beta\delta}{1-\epsilon}\right)i_t = \alpha k_t + (1-\alpha)(a_t + n_t) \quad (A12)$$

where $\epsilon \equiv \frac{1-\delta}{mpk_{ss}+(1-\delta)} = \beta(1-\delta)$, the second equality holding because the ss is efficient. Also we have $\kappa \equiv K_{SS}/NW_{ss}$, Q_{ss}= 1, $R^s_{ss} = R^k_{ss} = 1/\beta$, $\Theta \equiv \frac{\varpi_{ss}g\prime(\varpi_{ss})}{g(\varpi_{ss})}$ < 1. Finally, we set $\mu\int_0^{\varpi_{ss}}\omega\phi(\omega)d\omega \approx 0$ so that monitoring costs do not appear in (A12).

2. The Derivation of A1.

Ignoring aggregate uncertainty in (20) we have

$$\frac{R_{t+1}^{k}(1+sub)}{R_{t+1}^{L}} = \frac{-f'(\varpi_{t+1})}{[g'(\varpi_{t+1})f(\varpi_{t+1}) - g(\varpi_{t+1})f'(\varpi_{t+1})]} \quad \text{(A13)}$$

This implicitly defines a mapping

$$\varpi_{t+1} \equiv \Omega(\frac{R_{t+1}^{k}(1+sub)}{R_{t+1}^{L}}). \quad \text{(A14)}$$

Rearranging (19) we have:

$$1 - \frac{R_{t+1}^{k}}{R_{t+1}^{L}} g\left[\Omega(\frac{R_{t+1}^{k}(1+sub)}{R_{t+1}^{L}})\right] = \frac{NW_t}{Q_t k_{t+1}} \quad \text{(A15)}$$

For convenience let us define the spread as $s_t \equiv \frac{R_{t+1}^{k}}{R_{t+1}^{L}}$. Linearizing (A15) we have:

$$-\frac{s_{ss}[g_{ss} + s_{ss} g_{ss}' \Omega_{ss}']}{1 - s_{ss} g_{ss}} (r_{t+1}^{k} - r_{t+1}^{l}) = n_{t+1} - q_t - k_{t+1} \quad \text{(A16)}$$

Let us define

$$\upsilon \equiv \frac{1 - s_{ss} g_{ss}}{s_{ss}[g_{ss} + s_{ss} g_{ss}' \Omega_{ss}']} \quad \text{(A17)}$$

Hence we have (A1).

3. Steady-states and subsidies.

The steady state is defined by the following three equations:

$$\beta \frac{\kappa}{\kappa - 1} R_{ss}^{k}(1 + sub) g(\varpi_{ss}) = 1 \quad \text{(A18)}$$

$$1 = \gamma\kappa R^k_{ss}(1+sub)f(\varpi_{ss}) \tag{A19}$$

$$f'(\varpi_{ss}) + \frac{f(\varpi_{ss})}{g(\varpi_{ss})}g'(\varpi_{ss})(\kappa-1) = 0 \tag{A20}$$

A18 comes from the lender's participation constraint. A19 comes from the entrepreneur's accumulation. A20 is an implication of the optimal contract. The subsidy is chosen to make the steady-state efficient, $\beta R^k_{ss} = 1$, and γ is chosen to satisfy A19. We can then express (18)-(A20) as

$$(1+sub) = \frac{\kappa-1}{\kappa g(\varpi_{ss})} \tag{A21}$$

$$1 = \frac{\gamma}{\beta}(\kappa-1)\,\frac{f(\varpi_{ss})}{g(\varpi_{ss})} \tag{A22}$$

$$f'(\varpi_{ss}) + \frac{f(\varpi_{ss})}{g(\varpi_{ss})}g'(\varpi_{ss})(\kappa-1) = 0 \tag{A23}$$

These are three equations in five unknowns: κ, ϖ_{ss}, $(1+sub)$, γ, and σ (the standard deviation of the idiosyncratic shock which defines the functions f and g). Hence, we need two more restrictions to determine the equilibrium. These other two restrictions come from calibrating the model with "risk" premium = $\frac{R^k_{ss}(1+sub)}{R} = \beta R^k_{ss}(1+sub) = (1+sub)$, and κ = 1.954. These imply a quarterly bankruptcy rate of .03/4. Hence, we can vary κ and hold the risk premium fixed, but allow γ and σ to vary. Note that this implies a change in the bankruptcy rate.

4. Proofs.

Proposition 1: A constant subsidy cannot make the agency cost model ZRC efficient.

Proof: We prove this by contradiction. The AC model will be consistent with (37) only if

$$E_t \beta U_c(t+1) R_{t+1}^k (1+sub) \left(\frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1}\right) g(\varpi_{t+1}) = E_t \beta U_c(t+1) R_{t+1}^k \quad \text{(A24)}$$

Recall that $\left(\frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1}\right)$ is chosen in time-t. Hence, for (A24) to hold, $\left(\frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1}\right)$ must be given by:

$$\left(\frac{\bar{\kappa}_t}{\bar{\kappa}_t - 1}\right) \equiv \frac{E_t U_c(t+1) R_{t+1}^k}{(1+sub) E_t U_c(t+1) R_{t+1}^k g(\varpi_{t+1})}$$

The level of net worth needed to support efficient capital accumulation is then given by

$$NW_t = \frac{Q_t K_{t+1}}{\bar{\kappa}_t}$$

where $Q_t K_{t+1}$ is the level of capital accumulation from the ZRC agency cost model. Notice that NW_t is necessarily forward-looking. But net worth accumulation in the agency cost model is given by the backward-looking (27) which is a contradiction. QED.

Proposition 2: If entrepreneurs internalize, a constant subsidy and $\chi = 1$will make the agency cost model ZRC efficient.

Proof: Combining (34) and (41) we have:

$$E_t \beta U_c(t+1) R_{t+1}^k (1+sub)[\Gamma f(\varpi_{t+1}) + g(\varpi_{t+1})] = U_c(t) \quad \text{(A25)}$$

With internalization, the counterpart to (18) is given by:

$$E_t U_c(t+1) R_{t+1}^k (1+sub)[\Gamma f(\varpi_{t+1}) + \lambda_t g(\varpi_{t+1})] = \lambda_t \frac{U_c(t)}{\beta} \quad \text{(A26)}$$

Consistency between these two equations imply that $\lambda_t = 1$. It is convenient to define

$$m_{t+1} \equiv R_{t+1}^k \frac{\beta U_c(t+1)}{U_c(t)}$$

$$\mathrm{F}(\varpi_{t+1}) \equiv \Gamma f(\varpi_{t+1}) + g(\varpi_{t+1})$$

Note that if the ZRC economy is efficient, then (39) is satisfied and $E_t m_{t+1} = 1$. Equations (17)-(19) are then given by

$$E_t m_{t+1} \mathrm{F}'(\varpi_{t+1}) = 0 \tag{A27}$$

$$E_t m_{t+1} \mathrm{F}(\varpi_{t+1}) = (1 + sub)^{-1} \tag{A28}$$

$$E_t m_{t+1} \left(\frac{\bar{\kappa}_t}{\bar{\kappa}_{t-1}}\right) g(\varpi_{t+1}) = (1 + sub)^{-1} \tag{A29}$$

Consider ω-indexation schemes of the following form:

$$\varpi_{t+1} = \mathrm{P}_t \frac{(R_{t+1}^k)^{\chi-1}}{E_t (R_{t+1}^k)^{\chi-1}} \tag{A30}$$

An indexation scheme (A30) is consistent with (37) if and only if it satisfies (A27)-(A29). Suppose that $\chi = 1$. Equation (A27) then becomes $\mathrm{F}'(\mathrm{P}_t) = 0$ so that the contract is defined by a unique and constant value of ϖ such that $\mathrm{F}'(\varpi) = 0$. The needed subsidy then comes from (A28), and the constant level of κ then comes from (A29). Net worth then evolves as

$$NW_t = \frac{Q_t K_{t+1}}{\kappa} \tag{A31}$$

where $Q_t K_{t+1}$ is the behavior consistent with (37). Hence, $\chi = 1$ and P_t constant are sufficient for achieving optimal capital accumulation. QED

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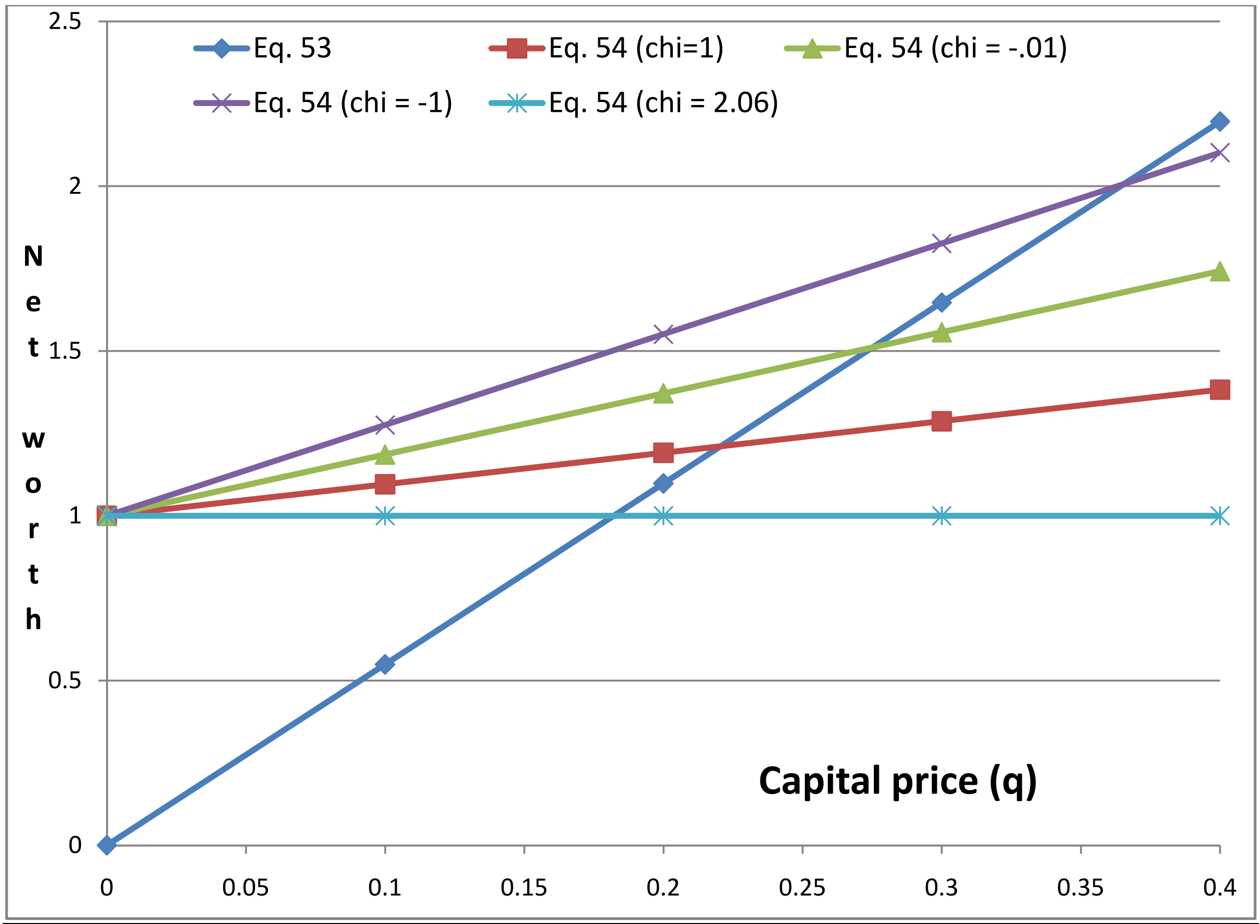
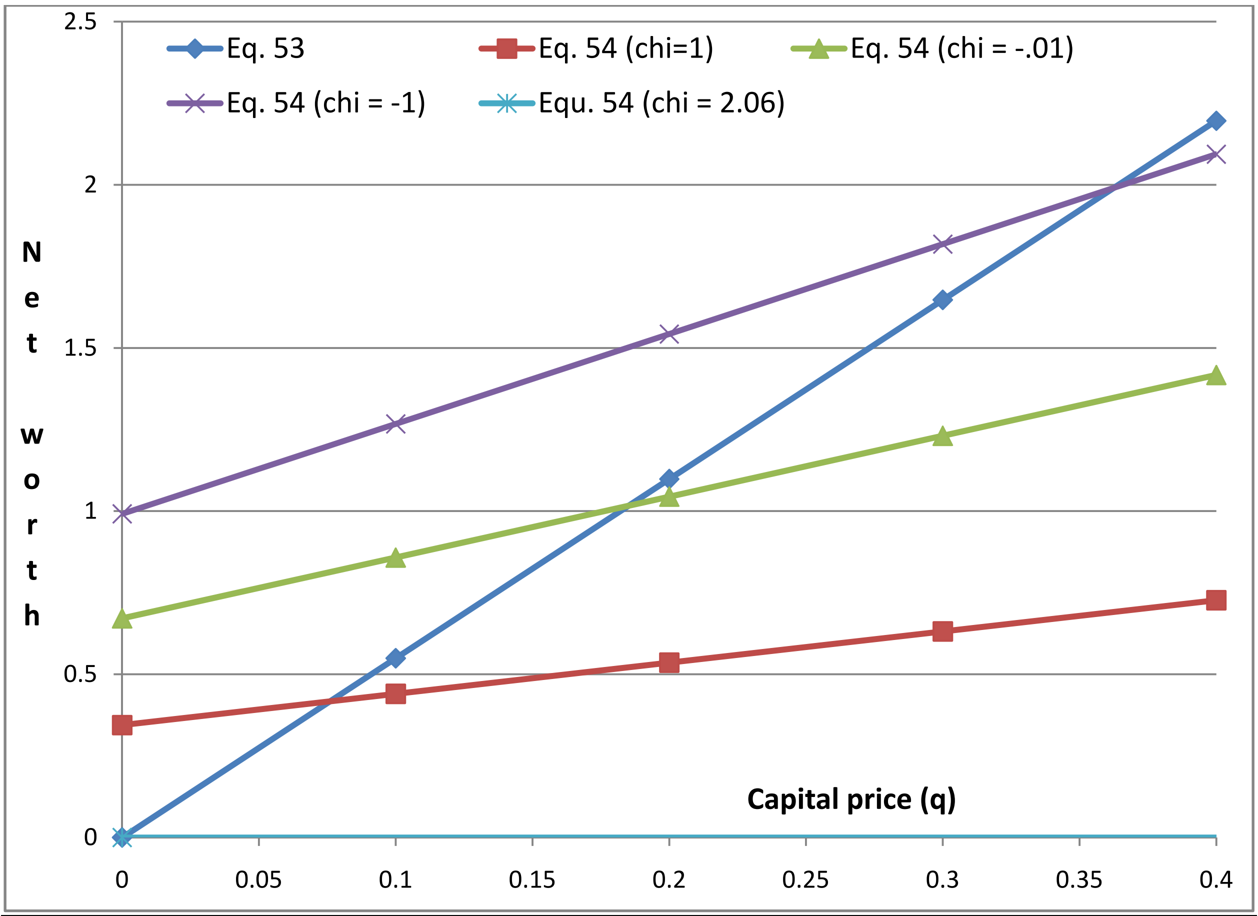
FIGURE 1.



Legend: The figure demonstrates the effect of a one unit shock in net worth, $\varepsilon_t^{nw} = 1$. The figure ignores any modest effect of net worth on mpk_t which are set to zero. Equation (53) cuts through the origin as it is not affected by shocks. The remaining lines are equation (54) drawn under different indexation assumptions. The four cases are: (i) no multiplier ($\chi = 2.06$), (ii) FI ($\chi = 1$), (iii) BGG ($\chi = \frac{\Theta-1}{\Theta} \approx -.01$) , and (iv) $\chi = -1$. All four of these cut the vertical axis at unity because the shock is $\varepsilon_t^{nw} = 1$.

FIGURE 2.



Legend: The figure demonstrates the effect of a 10% increase in the marginal product of capital, mpk_t= 10. Equation (53) cuts through the origin as it is not affected by shocks. The remaining lines are equation (54) drawn under different indexation assumptions. The four cases are: (i) no multiplier (χ = 2.06), (ii) FI (χ = 1), (iii) BGG ($\chi = \frac{\Theta-1}{\Theta} \approx -.01$), and (iv) χ = -1. All four of these cut the vertical axis at different points because the innovation in the marginal product of capital has differing effects on net worth via the level of indexation. All four of these cut horizontal axis at $q_t = -\left(\frac{1-\epsilon}{\epsilon}\right)10$, because this is the movement in asset prices that holds net worth fixed (see (54)).

Figure 3.

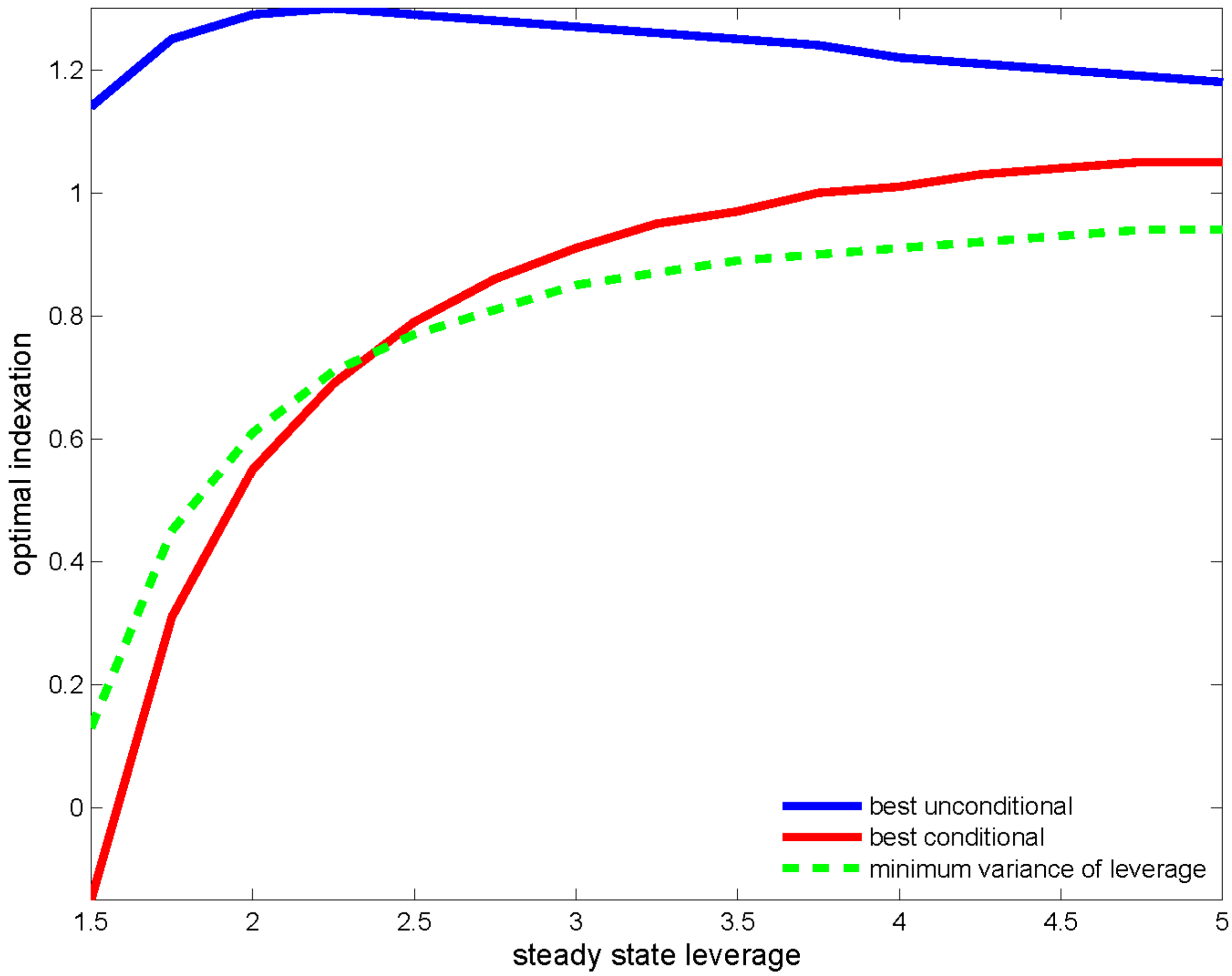

1.2
1
0.8
0.6
0.4
0.2
0
optimal indexation
1.5
2
2.5
3
3.5
4
4.5
5
steady state leverage
best unconditional
best conditional
minimum variance of leverage

Figure 4.

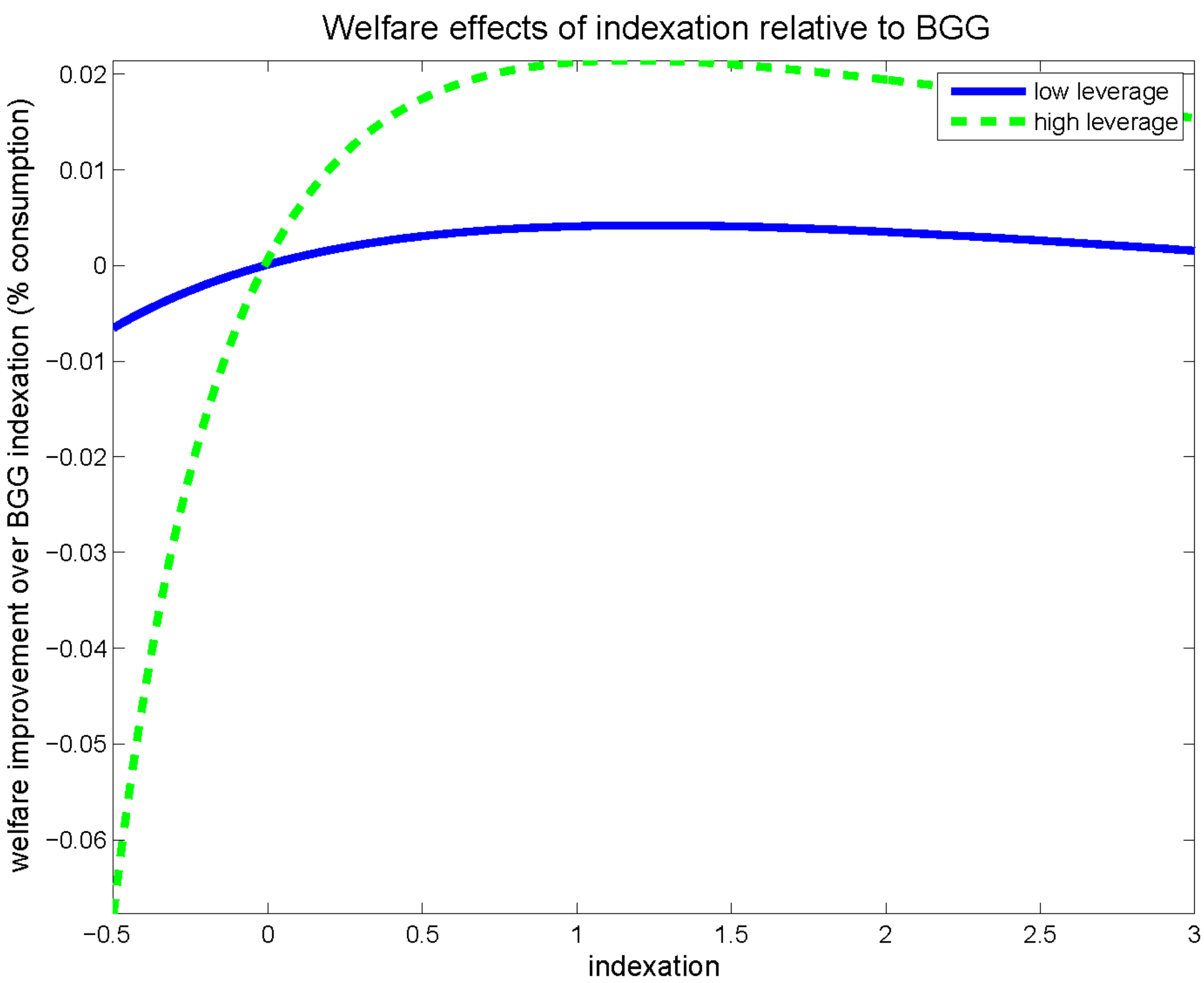

Welfare effects of indexation relative to BGG
welfare improvement over BGG indexation (% consumption)
0.02
0.01
0
−0.01
−0.02
−0.03
−0.04
−0.05
−0.06
−0.5
0
0.5
1
1.5
2
2.5
3
indexation
low leverage
high leverage

Figure 5

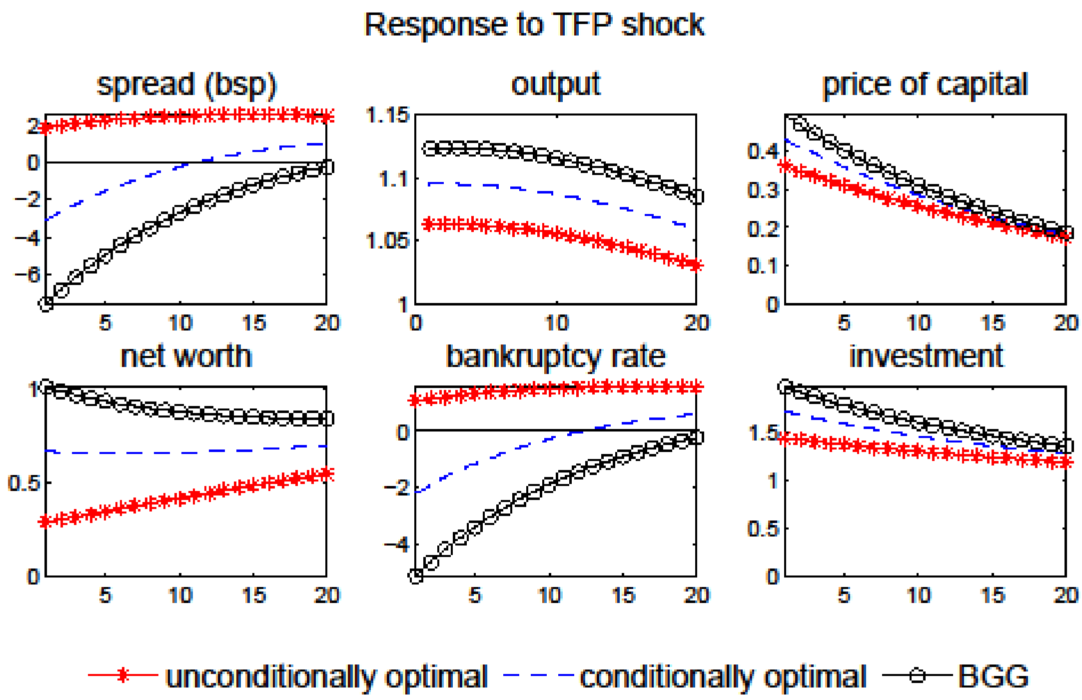


Figure 6

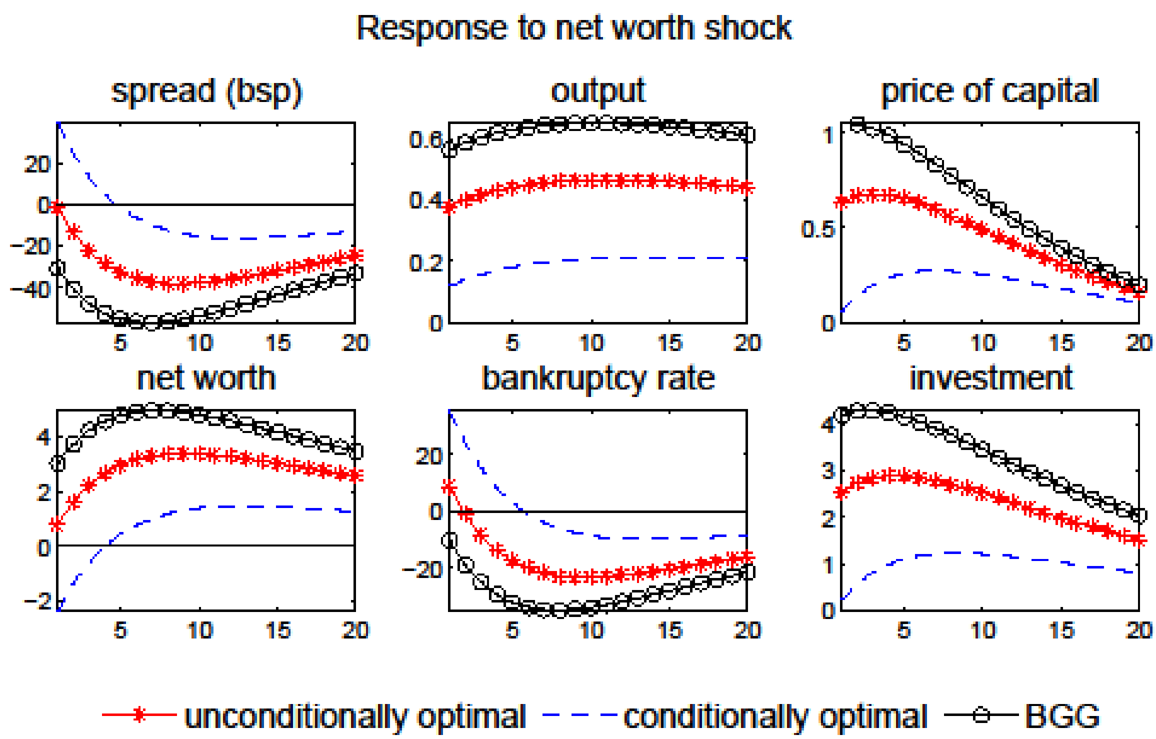


Table 1: Optimal Indexation.

	Optimal indexation
TFP shocks, monitoring costs in resource constraint	**1.28**
TFP shocks, no monitoring costs in resource constraint (ZRC)	**1.23**
Net worth shocks, monitoring costs in resource constraint	**2.44**
Net worth shocks, no monitoring costs in resource constraint (ZRC)	**1.85**

Table 1: Optimal values for the indexation parameter based on unconditional welfare. The table assumes the baseline parameter values.

Table 2: Optimal Indexation

Sensitivity Analysis

	TFP shocks	Net worth shocks
Baseline calibration (κ = 1.954, ψ = 0.25)	**1.28**	**2.44**
Higher leverage (κ = 4, ψ = 0.25)	**1.22**	**0.98**
Extreme leverage (κ = 10, ψ = 0.25)	**1.08**	**0.77**
Higher adj. costs (κ = 1.954,ψ = 1.0)	**1.18**	**1.23**
Higher adj. costs (κ = 1.954,ψ = 2.0)	**1.03**	**0.86**

Table 2: Optimal values for the indexation parameter based on unconditional welfare. This is for the case with monitoring costs in the resource constraint and no internalization by the entrepreneur.