Systemic Risk and Optimal Regulatory Architecture^{*}

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Abstract

Until the recent financial crisis, the safety and soundness of financial institutions was assessed from the perspective of the individual institution. The financial crisis highlighted the need to take systemic externalities seriously when rethinking prudential oversight and the regulatory architecture. Current financial reform legislation worldwide reflects this intent. However, these reforms have overlooked the need to also consider regulatory agencies' forbearance and information sharing incentives. In a political economy model that explicitly accounts for systemic connectedness, and regulators' incentives, we show that under an expanded mandate to explicitly oversee systemic risk, regulators would be more forbearing towards systemically important institutions. We also show that when some regulators have access to information regarding an institution's degree of systemic importance, these regulators may have little incentive to share it with other regulators. These findings suggest that (and we show conditions under which) a unified regulatory arrangement can reduce the degree of systemic risk *vis-a-vis* a multiple regulatory arrangement.

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Abstract

Until the recent financial crisis, the safety and soundness of financial institutions was assessed from the perspective of the individual institution. The financial crisis highlighted the need to take systemic externalities seriously when rethinking prudential oversight and the regulatory architecture. Current financial reform legislation worldwide reflects this intent. However, these reforms have overlooked the need to also consider regulatory agencies' forbearance and information sharing incentives. In a political economy model that explicitly accounts for systemic connectedness, and regulators' incentives, we show that under an expanded mandate to explicitly oversee systemic risk, regulators would be more forbearing towards systemically important institutions. We also show that when some regulators have access to information regarding an institution's degree of systemic importance, these regulators may have little incentive to share it with other regulators. These findings suggest that (and we show conditions under which) a unified regulatory arrangement can reduce the degree of systemic risk *vis-a-vis* a multiple regulatory arrangement.

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1 Introduction

Recent financial regulatory reforms include changes to the allocation of regulatory functions aimed at lessening systemic risk. For example, referring to the financial reform bill recently passed by the U.S. Congress, Senator Dodd stated: "with this bill, we have protected taxpayers from being forced to bail out companies that threaten to bring down the economy. . . regulators will no longer be able to ignore emerging threats to the economy."

This bill and other relevant legislation around the globe have been advanced under the assumption that the incentives of regulators, charged with different oversight functions, are perfectly aligned with each other. As pointed out by Repullo (2000), if the agencies charged with the different regulatory functions were social welfare maximizers, their joint action would also lead to the optimal implementation of regulatory oversight. However, this is not the case when agencies' objective functions differ across regulatory agencies. In this case, the optimal regulatory functions. This comparison is potentially more difficult when regulators are asked to explicitly take into account the systemic importance of the institutions they oversee (as in recent financial regulatory reforms).

In our view, an analytical framework that is useful for evaluating the alternative allocation of regulatory functions should explicitly model financial institutions facing two potential types of shock: a shock to their liquidity (represented by unexpected withdrawals by depositors) and a shock to their solvency (represented by a decrease in the value of their assets). And key regulatory functions to be analyzed should include lender of last resort, early intervention powers and the provision of short-term bank-liabilities insurance. Furthermore, because the regulatory reforms are intent on dealing with intermediaries' systemic connectedness risks, the model should explicitly account for systemic externalities. The model should also explicitly include political economy considerations as in, for example, Repullo (2000) and Kahn and Santos (2005), to explicitly account for regulators' strategic actions regarding data sharing and forbearance incentives.

Regulatory forbearance arises primarily because the failure of a financial intermediary is "politically costly" for a regulator. Regulators often have the incentive to keep an institution afloat, even when insolvent, because regulators strongly dislike closing institutions under their watch, especially because in some cases, given enough time, an institution may be back on its feet. Similarly, how or whether regulators share information will reflect the strategic considerations of the different agencies' objectives. For example, agencies possessing information regarding firms' degree of systemic importance may have an incentive to misrepresent this information to other agencies.

The main findings of the paper are that: i) under an expanded mandate to explicitly oversee systemic interconnectedness, regulators would be more forbearing towards systemically important institutions, because the systemically important institutions will have a more damaging effect on other institutions under the regulators' purview and stricter with non-systematic institutions; and ii) in the presence of efficient resolution mechanisms and high political costs for shutting down a financial institution, a unified regulatory arrangement could reduce systemic risk *vis-a-vis* a multiple regulatory arrangement because: a) forbearance would be reduced and b) when only a subset of regulators have access to private information regarding an institution's degree of systemic importance, these regulators may have no incentive to share it with other regulators.

Our paper contributes to the literature on the political economy of financial intermediation and regulation. Most of the literature on bank regulation has disconsidered strategic interaction among regulators by either analyzing regulatory functions separately or assuming that regulators' interests are perfectly aligned. The former includes studies on lender of last resort and the role of policy precommitment (e.g., Goodfriend and Lacker (1999) and Freixas (1999)), deposit insurance and the issue of moral hazard (e.g., Kareken and Wallace (1978) and Merton (1977)), and supervision with different closure rules (e.g., Davis and McManus (1991)). The latter is represented by studies on deposit insurance pricing and closure policies (e.g., Pennacchi (1987), Acharya and Dreyfus (1989), Allen and Saunders (1993), and Fries et al. (1997)), lender of last resort and deposit insurance policies (e.g., Kanatas (1986), Sleet and Smith (2000)), supervision and bank monitoring (e.g., Campbell et al (1992)), and deposit insurance and bank closure (e.g., Mailath and Mester (1994)). None of these papers look at the problem of what happens when you have multiple regulators, each with their own political or social agenda available to them. The first paper that we are aware of that actually does this is that of Repullo (2000), which looks at the interaction between different institutions who might be taking on the role of lender of last resort. The paper examines which one is the right one to take this role, the incentives for each, and how to compare them.

Kahn and Santos (2005) and Kahn and Santos (2006) look at this question of competition among

regulators or the games being played by regulators when they are in the presence of the dilemma of insolvency versus illiquidity – the standard dilemma of a regulator in financial intermediation. They look at the case of a regulator and a lender of last resort and the relative merits of joint versus separate regulatory powers, focusing on the problems of excessive forbearance and information. We extend this analysis to consider the consequences of systemic risk and to deal with the possibility of multiple regulated banks strategically interacting with each other. In Kahn and Santos (2005), there is just one bank. Importantly, in their paper it does not matter if there is more than one bank because they do not interact with one another. In this paper we are talking explicitly about the possibilities of failures in one bank affecting the likelihood of failures in another bank.

The remainder of the paper is organized as follows. Section 2 describes the model without private information. In Section 3, we solve the model with private information and examine its main results. Section 4 concludes the paper. All proofs are in the Appendix.

2 The Model without Private Information

In order to analyze the optimal regulatory implications of explicitly accounting for systemically important institutions, our political economy model features two "banks" indexed by $i \in \{A, B\}$. Their only source of financing for their long term projects is demand deposits. Each bank faces the possibility of two types of shocks: liquidity shocks (represented by a sudden drop in deposits) and solvency shocks (represented by low probability of success of the bank's investment project). Lastly, banks A and B are identical except in two respects: (i) bank A is the systemic institution in the sense that its failure lowers the probability that bank B 's projects succeed, and (ii) the timing of each bank's actions differ slightly in order for bank A 's performance to affect bank B.

Figure 1 summarizes the timing. There are four periods and no time discounting. At time t = 0, banks invest in long-term projects Y_i , that pay a return \tilde{R}_i at a later date. These projects are financed by capital, K_i , and short-term deposits, D_i . Banks are subject to liquidity shocks, which are publicly observable at date t = 1: if the new level of deposits \tilde{D}_i at t = 1 is such that $D_i > \tilde{D}_i$, banks are forced to seek emergency liquidity from a lender-of-last-resort to bridge the liquidity gap, defined as $\omega_i \equiv D_i - \tilde{D}_i$.¹. At t = 1, the probability of success of bank A's project,

¹The rollover risk serves the same role as does liquidity demand for early withdrawals in previous models such as Diamond and Dybvig (1983), but allows for a simpler stochastic structure.



Bank A: \longrightarrow Bank A: \longrightarrow Bank A: $Y_A = K_A + D_A$ shock $\omega_A = D_A - \widetilde{D}_A$ R_A realized u_A realized

Bank B: — — Bank B: — — Bank B: — Bank B: — Bank B: $Y_B = K_B + D_B$ shock $\omega_B = D_B - \tilde{D}_B$ u_B realized R_B realized

Figure 1: Timing of the game.

 u_A , is publicly revealed and the return of its investment project is realized at t = 2. Similarly, bank *B* 's probability of success, u_B , is publicly known at t = 2, and the return on this investment is realized at t = 3. Furthermore, as of date 0, the signal u_i containing information on \tilde{R}_i (i.e. supervisory information) has the following properties:

Assumption 1 For each bank i = A, B, the level of deposits available at date 1, D_i , are independent random variables with distribution G(D), and G'(0) > 0.

Assumption 2 The financial condition signal of bank A's portfolio of loans is given by the random variable u_A , with support $[\underline{u}_A, \overline{u}_A] \subset [0, 1]$ and distribution $F_A(u_A)$, and it is publicly observable at date 1 but not verifiable. If bank A invests Y_A in loans at period 0, it will receive $Y_A \tilde{R}_A$ in period

t = 2 where

$$\tilde{R}_A = \begin{cases} R & \text{with probability} \quad u_A \\ 0 & \text{with probability} \quad 1 - u_A \end{cases}$$

Assumption 3 The financial condition signal of bank B's portfolio of loans is given by the random variable $\tilde{u}_B - \gamma \chi$ (χ is an indicator variable which equals 1 if bank A failed at t = 2 and 0 otherwise.), where \tilde{u}_B has distribution $F_B(u_B)$ with support $[\underline{u}_B, \overline{u}_B] \subset [0, 1]$ and $\gamma \in [0, \underline{u}_B]$. The financial condition signal is publicly observable at date t = 2 but not verifiable. If bank B invests Y_B in loans at period 0, it will receive $Y_B \tilde{R}_B$ in period t = 3, where

$$\tilde{R}_B = \begin{cases} R & \text{with probability} \quad u_B - \gamma \chi \\ 0 & \text{with probability} \quad 1 - u_B + \gamma \chi \end{cases}$$

Each bank can be liquidated before the realization of the investment outcome (that is, before t = 2 for bank A and before t = 3 for bank B). If this occurs, the liquidation value of the bank's projects is L_iY_i , where $L_i \in (0, 1)$ determines the relative loss that banks suffer for canceling their investments early. In addition, there is a bankruptcy cost of $c_i > 0$, such that, as of time 0 the expected return from bank lending (net of second period bankruptcy costs) exceeds the zero return from holding liquid assets, for i = A, B,

$$E(\widetilde{u}_i)[R+c_i] > 1+c_i.$$

$$\tag{1}$$

2.1 Regulatory Architectures: Unified vs Multiple Regulators

We are interested in considering different regulatory architectures. In particular, we want to compare a unified regulator (UR) against an alternative set-up where regulatory functions are divided between two agencies: a lender of last resort (LoLR) charged with the provision of emergency liquidity to banks, and a deposit insurer (DI) responsible for guaranteeing bank deposits and that has early intervention powers (the DI can intervene even if there is no liquidity shortfall).²

All regulators have private objective functions; that is, regulators do not maximize social welfare. Instead, regulators care about their income and face a trade-off between the political cost of closing a bank in distress and the expected financial cost of not closing—that is, forbearing—the bank. In other words, on the one hand regulators are interested in avoiding the reputational cost associated with a bank closure, while on the other hand, they want to minimize their financial costs in case the bank actually fails. For the LoLR, these costs are the funds lent to a distressed bank (at a penalty rate P) that eventually defaults. For the DI, the costs are the deposit insurance payouts (to depositors) that are made if a bank eventually fails. For the UR, the costs are the sum of the LoLR and DI costs.

²Note that this is akin to the prompt corrective actions framework of the FDIC in the United States.

The regulatory decisions of the DI and the LoLR regarding bank i are respectively given by $d_i, l_i \in \{0, 1\}$, where $d_i = 1$ if the DI decides not to close the bank and $d_i = 0$ if otherwise, and $l_i = 1$ if the LoLR provides emergency funds and $l_i = 0$ if otherwise. In terms of the model's timing this can be expressed as follows: at t = 1, signal u_A is obtained and the DI may choose to close bank A or the LoLR may refuse to provide liquidity if $\omega_A > 0$. In period t = 2, the signal u_B is observed and the regulators make similar decisions regarding bank B.

Let $s_i \equiv (d_i, l_i)$. The utility function of regulators are given by,

$$E\left[I\left(s_{A},\tilde{R}_{A}\right)-\alpha c_{A}\chi\left(s_{A},\tilde{R}_{A}\right)\theta_{A}+E\left[I\left(s_{B},\tilde{R}_{B}\right)-\alpha c_{B}\chi\left(s_{B},\tilde{R}_{B}\right)\theta_{B}\mid\chi\left(s_{A},\tilde{R}_{A}\right),\tilde{u}_{B}\right]\mid\tilde{u}_{A}\right]$$

$$(2)$$

where $I(\cdot)$ is the net income accruing to the regulator, αc_i measures the regulator's political cost of bankruptcy, and θ is an indicator variable which is 0 if and only if the regulator's action is to not close the bank while the other regulator chooses to close the bank.³

2.2 Equilibrium in a Multiple-Regulator Architecture

To find the equilibria we solve by backwards induction. The first step consists of solving the regulators' problem for bank B once it is known whether bank A is closed. Proposition 1, states "the closing rules" for bank B.

Proposition 1 Consider a multi-regulator architecture with separate LoLR and DI agencies. At t=2, once it is known whether bank A has been closed ($\chi = 0$) or not ($\chi = 1$), bank B is closed if the realized value of u_B falls below the threshold $u_B^m(\gamma)$ given by:

$$u_{B}^{m}\left(\chi\right) \equiv \max\left\{u_{B}^{DI}\left(\chi\right), u_{B}^{LoLR}\left(\chi\right)\right\},\,$$

where

$$u_B^{LoLR}(\chi) = \frac{\omega_B}{(P+1)\,\omega_B + \alpha c_B} + \gamma \chi,$$
$$u_B^{DI}(\chi) = \frac{L_B Y_B - \max\{0, \omega_B\}}{D_B - \max\{0, \omega_B\} + \alpha c_B} + \gamma \chi.$$

The intuition for the proposition is as follows. From the LoLR perspective, taking DI's actions as given, the size of the liquidity shock determines the degree of forbearance: larger injections of

³For the unified regulator $\theta = 1$ whenever the regulator chooses to close a bank.

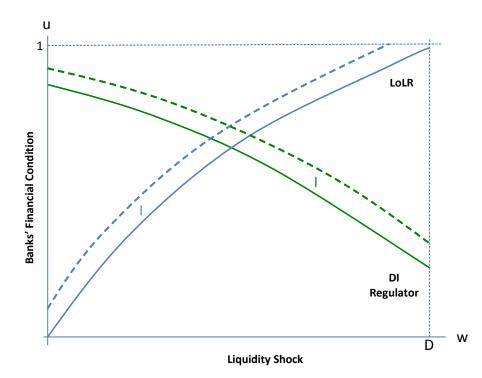


Figure 2: Higher probability of closing non-systemic institutions in crisis under multiregulator arrangements. The solid lines represent the liquidation cutoffs of the LoLR and the DI for the non-systemic bank B when bank A is not closed. The dashed lines represent the liquidation cutoffs of the LoLR and the DI for the non-systemic bank B when bank A is closed.

liquidity, ω_B , require greater likelihood of success (i.e., higher values of u_B^{LoLR}). If the LoLR does not provide the loan, the bank is forced to close and the LoLR would bear the bankruptcy cost αc . If, on the other hand, the LoLR makes the loan but the bank fails, then in addition to the bankruptcy cost, the LoLR forgoes the liquidity injection, ω_B , which the failed bank is unable to repay. Therefore, the higher the need for liquidity injections, the greater is the probability of success u required by the LoLR to extend the liquidity support.

From the DI's perspective, taking the LoLR's actions as given, the DI is more forbearing the greater the liquidity assistance supplied by the LoLR. This is because (unpaid) debts to the LoLR are outside the responsibility of the DI regulator if the bank fails *and* reduce the potential need for deposit insurance outlays.⁴ This increases the temptation for an independent DI agency to engage in forbearance as liquidity shortfalls increase.

⁴Notice that the liquidity injection transforms the bank's debt to depositors (which are insured by the DI) into debt to the LoLR (which is not insured by the DI).

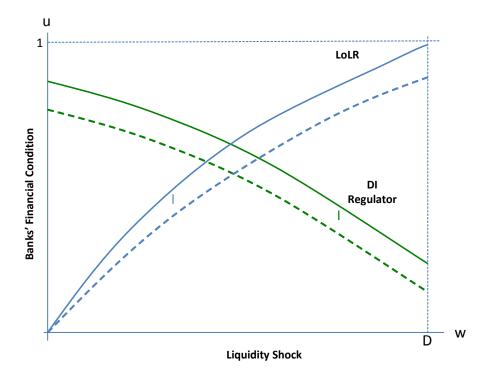


Figure 3: More forbearance toward systemic institutions under single regulator and multiregulator arrangements. The solid lines represent the liquidation cutoffs of the LoLR and the DI for the systemic bank A, in the absence of systemic risk. The dashed lines represent the liquidation cutoffs of the LoLR and the DI for the systemic bank A when bank A's degree of systemic risk is positive.

Taking as given the regulatory response towards bank B, the second step consists of solving the regulators' problem for bank A. Proposition 2, states "the closing rules" for bank A.

Proposition 2 Consider a multi-regulator architecture with separated LoLR and DI agencies. Bank A is closed for u_A values below

$$u_{A}^{m}\left(\gamma\right) \equiv \max\left\{u_{A}^{DI}\left(\gamma\right), u_{A}^{LoLR}\left(\gamma\right)\right\},\,$$

where

$$\begin{aligned} u_A^{LoLR}\left(\gamma\right) &= \frac{\omega_A}{\left(P+1\right)\omega_A + \alpha c_A + \gamma \left[\left(P+1\right)\max\left\{0,\omega_B\right\} + \alpha c_B\right]\Pr\left[u_B \ge u_B^m\left(\chi = 0\right)\right]},\\ u_A^{DI}\left(\gamma\right) &= \frac{L_A Y_A - \max\left\{0,\omega_A\right\}}{D_A - \max\left\{0,\omega_A\right\} + \alpha c_A + \gamma \left[D_B - \max\left\{0,\omega_B\right\} + \alpha c_B\right]\Pr\left[u_B \ge u_B^m\left(\chi = 0\right)\right]}.\end{aligned}$$

The insights of propositions 2 and 3 are illustrated in Figures 2 and 3, where the solid lines represent the case in which $\chi = 0$ and the dashed lines represent the case in which $\chi = 1$. The horizontal axis depicts the liquidity shock ω , whereas the vertical axis represents the bank's financial condition, u. The higher the required LoLR's liquidity injection, the less forbearing the LoLR is (i.e., higher u)—this explains why the LoLR line is upward sloping. On the other hand, higher liquidity injections reduce the size of potential outlays from the deposit insurance, thus inducing the DIagency to become more forbearing—this is represented in the downward sloping DI line in Figure 2. Figure 2 also illustrates two results arising under a mandate, to monitor the systemic importance of financial institutions. Under this mandate, both, the LoLR and the DI supervisors will be: (i) less forbearing towards non-systemic institutions (Figure 2); and (ii) more forbearing towards systemically important institutions (Figure 3). This follows from interpreting the liquidation cutoffs for which $\chi = 0$ as representing the decision to close bank B in the absence of systemic risk ($\gamma = 0$). In this case, regulation outcomes regarding bank B are the same regardless of what happens to bank A.

Figure 3 illustrates the results of Proposition 2. The solid lines represent liquidation cutoffs in the absence of systemic risk ($\gamma = 0$) and the dashed lines represent the environment in the presence of systemic risk ($\gamma > 0$). The presence of systemic risk makes regulators more forbearing towards the systemic institution. The decision to close bank A increases the probability that bank B is liquidated — due to more stringent solvency standards, increasing the expected costs associated with its failure. As a result, regulators become less willing to liquidate bank A in order to avoid the expected downside consequences of systemic risk.

2.3 Equilibrium in a Unified Regulator Architecture

2.3.1 Non-systemic bank

Because under a UR, the LoLR and a DI problems are solved simultaneously, intuition would suggest that a UR would internalize the excessive forbearance incentives faced by each individual regulator leading to the lowest degree of excessive regulatory forbearance. However, it is important to recall that what is at stake is the maximization of regulators' utility and not necessarily social welfare. Propositions 3 and 4 formally state the conditions under which UR leads to the lowest oversight forbearance of non systemic and the systemic banks, respectively.

Proposition 3 Consider the case of a unified regulator with DI and LOLR functions under its purview. At t=2, once it is known whether bank A remains open ($\chi = 0$) or not ($\chi = 1$), bank B

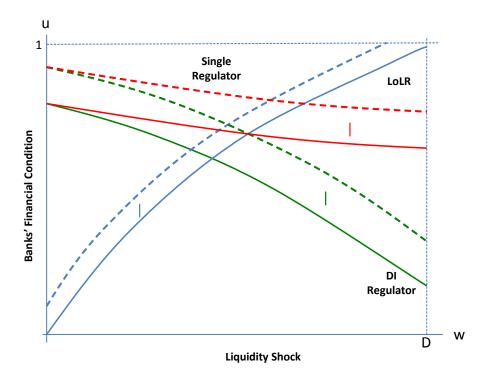


Figure 4: Higher probability of closing non-systemic institutions in crisis under single and multiregulator arrangements. The solid lines represent the liquidation cutoffs of the LoLR, the DI, and the single regulator for the non-systemic bank B when bank A is not closed. The dashed lines represent the liquidation cutoffs of the LoLR, the DI, and the single regulator for the non-systemic bank B when bank A is closed.

is closed if the realized value of u_B falls below

$$u_B^u\left(\chi\right) \equiv \frac{L_B Y_B}{P \max\left\{0, \omega_B\right\} + \alpha c_B + D_B} + \gamma \chi.$$

The unified regulator has the same payoff as that of the DI when the decision is to close bank B. However, their payoffs are different when bank B is allowed to continue operating. If bank B fails, the unified regulator has to back all the deposits while the DI only responds for those not yet covered by the LoLR. This makes the unified regulator less forbearing. If bank B succeeds, the unified regulator receives the liquidity injection times the penalty rate P while the DI has a payoff of zero. For moderate liquidity shocks, the unified regulator's payoff is more weighted towards that of when bank B fails, implying lower forbearance. Since the DI is less forbearing than the LoLR for moderate shocks, this implies that the unified architecture results in less forbearance than the multiregulator setting.

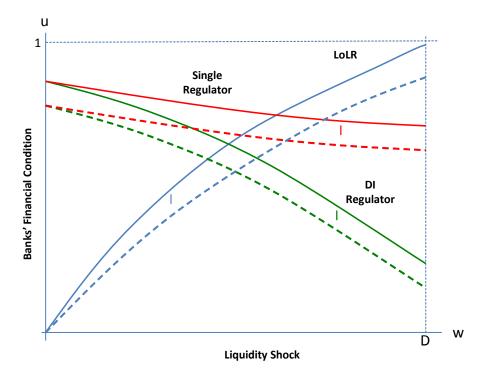


Figure 5: More forbearance toward systemic institutions under single regulator and multiregulator arrangements. The solid lines represent the LoLR, the DI, and the single regulator liquidation cutoffs for the systemic bank A in the absence of systemic risk. The dashed lines represent the LoLR, the DI, and the single regulator liquidation cutoffs for the systemic bank A when bank A's degree of systemic risk is positive.

The unified regulator has a higher payoff than that of the LoLR when the decision is to close bank B due to the salvage value. This makes the unified regulator less forbearing for a given size of liquidity shock. However, their payoff differ when bank B is not closed. If bank B succeeds, both get the liquidity injection times the penalty rate P. However, the unified regulator's payoff when bank B fails is independent of the liquidity injection — all deposits need to be backed — while the LoLR payoff is decreasing in the liquidity shock. It follows that the unified regulator's payoff when he does not close bank B is increasing in the liquidity shock, implying greater forbearance. The opposite is true in the multiregulator arrangement. Therefore, when the liquidity shock is sufficiently high, the unified regulator setting is more forbearing than the multiregulator one. This is illustrated in Figure 4.

2.3.2 Systemic bank

Proposition 4 Consider the case where a unified regulator has DI and LOLR functions. Then bank A is closed for u_A values below

$$u_{A}^{u}\left(\gamma\right) \equiv \frac{L_{A}Y_{A}}{P\max\left\{0,\omega_{A}\right\} + D_{A} + \alpha c_{A} + \gamma\left(P\max\left\{0,\omega_{B}\right\} + \alpha c_{B} + D_{B}\right)\Pr\left[u_{B} \ge u_{B}^{u}\left(\chi = 0\right)\right]}$$

The regulator will be softer on potentially systemic institutions. Consolidating standard regulatory functions without additional tools to preclude institutions from becoming systemic in the first place will not eliminate regulators' incentives to be lenient with systemically important institutions. In fact, these incentives could be exacerbated.

The failure of a systemically important institution increases the likelihood of failures among non-systemic institutions. These increased costs mean that any regulator will be more lenient with a systemically important institution, as illustrated by the downward shift in lines in Figure 5. On the other hand, since distress in the systemic institution negatively affects the chances of survival of the non-systemic institution, the regulator is less likely to save the non-systemic institution as illustrated by the upward shift of lines in Figure 4. The presence of systemic risk does not relax a regulator's standards for rescue of non-systemic institutions.

2.4 Comparing Multiple- vs Unified-Regulator Architectures

In this subsection we compare the multiregulator and unified regulator arrangements in terms of forbearance. We derive several comparative statics associating the levels of forbearance in each case with the model's primitive parameters.

2.4.1 Non-systemic bank

Given the liquidity shock ω_B^* , the liquidation cutoffs of the unified regulator and the multiregulator cross each other, i.e., both regulators are equally forbearing. Liquidity shocks below (above) this point imply that the multiregulator arrangement is more (less) forbearing than the unified one. Therefore, increases in ω_B^* reduces the "dominance region", i.e., the set of liquidity shocks for which the unified regulator arrangement is less forbearing than multiregulator arrangement. The following proposition states that the dominance region is increasing in the liquidation value, the bankruptcy cost, the penalty rate, and decreasing in the level of deposits. **Proposition 5** Consider a multiregulator arrangement, which has agencies LoLR and DI, and a unified regulator, which has DI and LOLR functions. Let $u_B^m(\cdot)$ and $u_B^u(\cdot)$ be their respective liquidation thresholds for bank B. Then

1. If $\omega_B = 0$, then $u_B^u(\cdot) = u_B^m(\cdot)$, i.e., the unified regulator and the multiregulator arrangements are equally forbearing.

2. Suppose $\omega_B > 0$.

(i) If $u_B^u(\cdot) > u_B^{LoLR}(\cdot)$ for all $\omega_B \in (0, D_B]$, then the multiregulator arrangement is always more forbearing than the unified regulator arrangement.

(ii) If $\exists \omega_B^* \in (0, D_B]$ such that $u_B^u(\cdot) = u_B^{LoLR}(\cdot)$, the the multiregulator arrangement is more forbearing than the unified regulator arrangement for $\omega_B \in (0, \omega_B^*)$ and less forbearing for $\omega_B \in [\omega_B^*, D_B]$. In addition, if α is sufficiently large, then ω_B^* increasing in L_B , c_B , P, and decreasing in D_B .

The intuition behind the results is as follows. Increases in the liquidation value L_B make the unified regulator less forbearing — it increases the payoff when bank B is closed — and does not change the behavior of the LoLR. Therefore, the LoLR and the unified regulator are equally forbearing only at higher levels of liquidity shock, which make the LoLR less forbearing and the unified regulator more forbearing. It follows that increases in the liquidation value improves the unified regulator arrangement over the multiregulator setting. Higher values of political cost c_B make both the unified regulator and the multiregulator more forbearing. The reason is that closing bank B now results in higher costs with certainty while the realization of these costs when bank B is not closed is uncertain — only if bank B fails. However, because the unified regulator has to back all the deposits (while the LoLR only loses the liquidity injection) when bank B fails, the decrease in the liquidation cutoff of the unified regulator is relatively smaller. Therefore, the impact of c_B on forbearance is lower for the unified regulator. It follows that higher political costs reduce the region for which the multiregulator setting is less forbearing than the unified regulator arrangement. A similar logic applies to changes in P. Increases in D_B make the unified regulator more forbearing — increases the costs of closing the bank — and do not change the incentives of the LoLR. As a result, the higher the amount of deposits the larger the relative performance of the multiregulator arrangement.

2.4.2 Systemic bank

Proposition 6 Consider a multiregulator arrangement, which has agencies LoLR and DI, and a unified regulator, which has DI and LOLR functions. Suppose $\omega_B \leq 0$ and let $u_A^m(\gamma)$ and $u_A^u(\gamma)$ be their respective liquidation thresholds for bank A. Then

1. If $\omega_A = 0$, then $u_A^u(\gamma) = u_A^m(\gamma)$, i.e., the unified regulator and the multiregulator arrangements are equally forbearing.

2. Suppose $\omega_A > 0$.

(i) If $u_A^u(\gamma) > u_A^{LoLR}(\gamma)$ for all $\omega_A \in (0, D_A]$, then the multiregulator arrangement is more forbearing than the unified regulator arrangement.

(ii) If $\exists \omega_A^* \in (0, D_A]$ such that $u_A^u(\gamma) = u_A^{LoLR}(\gamma)$, the the multiregulator arrangement is more forbearing than the unified regulator arrangement for $\omega_A \in (0, \omega_A^*)$ and less forbearing for $\omega_A \in [\omega_A^*, D_A]$. In addition, if α is sufficiently large, then ω_A^* increasing in L_A , c_A , c_B , D_B , P, γ , and decreasing in D_A .

From the last proposition, we can see that the region where the unified regulator is less forbearing than the multiple regulators is increasing in the liquidation value, bankruptcy costs, penalty rate, systemic risk, and level of deposits of bank B, and decreasing in the level of deposits of bank A. The results regarding L_A , c_A , P, and D_A parallel those for non-systemic bank. The other results concerning the attributes of bank B follow from the link between the two banks due to systemic risk. The intuition for the other results is as follows. Higher values of c_B increase the expected costs associated with the closure of bank B — either due to regulatory action or bank failure. This also increases the losses resulting from either closure or failure of bank A as both outcomes increase the probability that bank B is closed (due to systemic risk). Closure of bank A increases the insolvency risk of bank B with certainty while allowing bank A to continue results in the same outcome only if bank A fails. Therefore, increases in c_B make regulators more forbearing towards the systemic bank. Because the unified regulator has to fully guarantee the deposits (while the *LoLR* only loses the liquidity injection) the reduction in the liquidation threshold associated with the bank A for the unified regulator is lower. As a result, higher c_B increases the dominance region, i.e., the region for which the unified regulator setting is less forbearing than the unified regulator arrangement.

3 The Model with Private Information

In the previous section we assumed that the degree of systemic risk imposed by bank A (that is, the parameter γ) was publicly known. In this section we extend our analysis to an environment where there is incomplete information regarding the value of γ . Thus, assume that γ has publicly known distribution $Z(\gamma)$ with support $[\underline{\delta}, \overline{\delta}]$ and expected value $\underline{\gamma} \equiv E[\widetilde{\gamma}]$. The DI can always choose to observe, costlessly, the realized γ . If the DI chooses to observe γ , then the LoLR automatically observes a signal from a coarser information set. The DI can then supplement the LoLR's information set takes the following form.

$$\{\{\gamma\}|\gamma<\gamma_*\}\cup\{[\gamma_*,1]\}$$

In other words, there exists a value γ_* such that, if the DI receives information $\gamma < \gamma_*$ then the LoLR receives identical information. If $\gamma \ge \gamma_*$ then the LoLR does not observe γ ; and instead only knows that it lies in the interval $[\gamma_*, 1]$. Let $\overline{\gamma} \equiv E[\widetilde{\gamma}|\gamma \ge \gamma_*]$.

3.1 Information sharing

We begin by assuming that the DI elects to receive the information. For $\gamma < \gamma_*$, there are no strategic information sharing considerations and our former results go through. The same is true for values $\gamma \ge \gamma_*$ such that $u_A^{DI}(\gamma) > u_A$. In this case, bank A is liquidated, which is exactly the action desired by the DI.

3.1.1 The DI has incentive to act strategically

Consider the case in which $\gamma \geq \gamma_*$ and $u_A^{DI}(\gamma) \leq u_A$, i.e., the *DI* does not want to liquidate institution *A*. When $u_A^{LoLR}(\overline{\gamma}) \leq u_A$, the *LoLR* would have incentive to provide liquidity support to this institution *A*. A report sent by the *DI* to the *LoLR* saying γ is such that $u_A^{LoLR}(\gamma) > u_A$, if believed, would cause the *LoLR* to refuse to provide liquidity support to bank *A*. The resulting outcome is the opposite of that desired by the *DI*.

When $u_A < u_A^{LoLR}(\overline{\gamma})$, a talking strategy from the DI stating γ is such that $u_A^{LoLR}(\gamma) \leq u_A$, if believed, would cause the LoLR to provide liquidity assistance to bank A, which is the outcome wished by the DI. As a result, information sharing is strategically important and suggests the

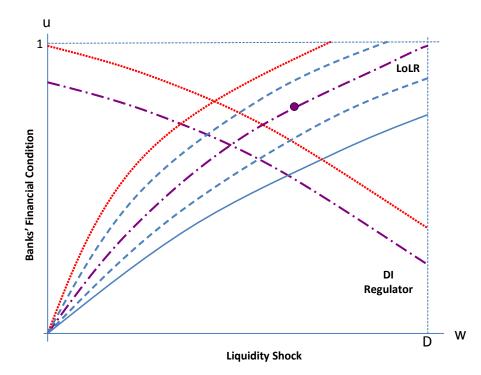


Figure 6: DI's incentive to reveal information. The purple point represents a realization of $(\omega_A, \omega_B, u_A)$. The dashed-dotted purple lines represent the LoLR and DI liquidation cutoffs given γ^* , which is the value of gamma such that the LoLR's cutoff goes through the realization $(\omega_A, \omega_B, u_A)$. The solid blue line represents the LoLR's liquidation cutoff given $\overline{\gamma}$. The dotted red lines represent the LoLR and DI liquidation cutoffs given γ_* . The dashed blue lines represent the LoLR and DI liquidation cutoffs given γ_* . The dashed blue lines represent the LoLR liquidation cutoffs for realized γ above and below γ^* . The DI has (does not have) incentive to report truthfully for realized gamma above (below) γ^* .

DI has an incentive to draft a report in such a way as to induce LoLR to act according to DI's preferences. In what follows, we assume $\gamma \geq \gamma_*$ and $u_A^{DI}(\gamma) \leq u_A$ unless stated otherwise.

Let the DI send a message $m : [0,1] \to M$ to the LoLR, where without loss of generality it can be assumed that M is a finite set. The LoLR observes the message, forms belief $\overline{\gamma}(m) = E[\widetilde{\gamma}|m]$, and takes action $l_A \in \{0,1\}$. The decision of the LoLR will be based on the u threshold associated with $\overline{\gamma}(m)$. We denote this threshold by $u_A^{LoLR}(\overline{\gamma}(m))$. Let γ^* be such that $u_A^{LoLR}(\gamma^*) = u_A$.

3.1.2 Communication is ineffective

If $\gamma \geq \gamma_* \geq \gamma^*$, then for any message $m \in M$ we have $\overline{\gamma}(m) \geq \gamma^*$, which implies $u_A = u_A^{LoLR}(\gamma^*) \geq u_A^{LoLR}(\overline{\gamma}(m))$. Therefore, bank A is not liquidated. Consider now the cases in which $\gamma_* \leq \gamma < \gamma^*$ and $\gamma_* < \gamma^* \leq \gamma$. If $\gamma_* < \gamma^* \leq \gamma$, which is a situation described by the lower dashed blue line

in Figure 5, the DI has incentives to fully reveal information on the degree of A's systemic importance. This follows from the fact that, if the LoLR believes the report, he provides liquidity support to bank A. However, the DI does not have incentive to reveal his information to the LoLR if $\gamma_* \leq \gamma < \gamma^*$, which is the case represented by the upper dashed blue line in Figure 5. The reason is that, if the LoLR believes the report, he does not provide liquidity assistance to bank A.

One must note that if $\gamma_* < \gamma^* \leq \gamma$, then any message m such that $\overline{\gamma}(m) \geq \gamma^*$ implies $u_A^{LoLR}(\overline{\gamma}(m)) \leq u_A^{LoLR}(\gamma^*) = u_A$. In this case, the LoLR provides liquidity to bank A and the bank is not liquidated. This would also be the action of the LoLR if he observed γ , which implies the he would be better off after communication. However, if $\gamma_* \leq \gamma < \gamma^*$, then any message m such that $\overline{\gamma}(m) \geq \gamma^*$ implies $u_A^{LoLR}(\overline{\gamma}(m)) \leq u_A^{LoLR}(\gamma^*) = u_A < u_A^{LoLR}(\gamma)$. As a result, the LoLR would not liquidate the bank. The LoLR is worse off with communication since his decision would be to liquidate the bank if he knew γ . This analysis suggests that fully separating equilibria do not exist. The proposition below confirms this intuition.

Proposition 7 If $\gamma \geq \gamma_*$ and $u_A^{DI}(\gamma) \leq u_A$, then babbling equilibria exist, i.e., equilibria in which $m(\gamma) = m \forall \gamma \geq \gamma_*$. In addition, every equilibrium has the same outcome as that of a babbling equilibrium, i.e., communication is ineffective.

The result of the last proposition parallel those of Kahn-Santos in that regulators with private information fail to share information. In our model, the DI fails to share information that induces the LoLR to take an action different than he would have taken in the absence of communication. Therefore, if $\gamma \geq \gamma_*$, then bank A will not be liquidated if γ_* is large enough and will be liquidated if otherwise.

Corollary 1 If $\gamma \geq \gamma_*$, then bank A is not liquidated if $\exists \ \hat{\gamma} \in [0,1]$ such that $E[\widetilde{\gamma}|\gamma_* \leq \widetilde{\gamma} \leq \widehat{\gamma}] = \gamma^*$ and is liquidated if otherwise.

The above corollary states the following. Given that the posterior belief regarding the degree of systemic risk is higher than the initial conjecture, the better the bank's financial condition lower liquidity shocks (ω) and higher solvency signals (u) — the more likely the *LoLR* revises the decision to not provide liquidity injections.

3.2 Information gathering

We now investigate the incentive of the DI to gather information given γ_* . Although a higher revealed degree of systemic risk results in better outcomes form the DI's point of view, it is not always the case that the DI wants to learn about γ . The reason is that, if the revealed γ is low, the LoLR might liquidate the bank while the DI wants its continuation. Our next result states that the DI might not gather information even if it is free to do so.

Proposition 8 Let $\underline{\gamma}$ be such that $u_A^{DI}\left(\underline{\gamma}\right) = u_A$. If 1. $\max\left\{u_A^{LoLR}\left(\underline{\gamma}\right), u_A^{DI}\left(\underline{\gamma}\right)\right\} \leq u_A$, 2. $\underline{\gamma} \leq \underline{\delta}$, i.e., $\underline{\gamma}$ is below the lower bound of the support of $Z\left(\gamma\right)$. 3. $\nexists \widehat{\gamma} \in [0, 1]$ such that $E\left[\widetilde{\gamma}|\gamma_* \leq \widetilde{\gamma} \leq \widehat{\gamma}\right] = \gamma^*$, and 4. If $\gamma_* \leq \gamma^*$, then in equilibrium the DI chooses not to observe γ .

This result shows that a multiregulator setting decreases the amount of information gathered even if information can be collected at no cost. The reason is that, if the expected outcome of revealing information makes a regulator worse off, then that regulator has less incentive to gather the information in the first place. Therefore, as a basic result, there is better gathering and using of information by a unified regulator than by a divided one.

4 Concluding Remarks

An important ingredient missing from most recent reform proposals mandating the oversight of systemic risk is the analysis of regulators' incentives. This includes "regulatory forbearance" for the incentive to keep institutions afloat when they should be unwound—which will likely vary across the alternative ways the regulatory functions could be allocated.

We show how adding a systemic risk monitoring mandate to the regulatory mix without a set of associated policy tools does not alter the basic regulator's incentives at the heart of some of the regulatory shortcomings leading to this crisis. Regulators often have the incentive to keep an institution afloat, even when insolvent, because regulators strongly dislike closing institutions under their watch, especially because in some cases, given enough time, an institution may be back on its feet. Therefore, in the absence of concrete methods formally to limit financial institutions' ability to become systemically important in the first place—regardless of how regulatory functions are allocated—regulators may well be more forgiving with systemically important institutions compared to those that are not. This is because the systemically important institutions will have a more damaging effect on other institutions under the regulators' purview. Moreover, competing regulators are less likely to gather information and may have an incentive not to share information once gathered.

While we acknowledge the limitations of our model, we see its main value in the illustration of the importance of analyzing strategic considerations in the optimal allocation of regulatory functions —particularly as a counterweight to analyses which ignore regulatory incentives.

Appendix

Proof of Proposition 1. Let $s_j \in \{l_B, d_B\}$ be the strategy of regulator j in the game starting at t = 2 and define $s_B \equiv (s_j, s_{-j})$. The utility function of regulator j is given by

$$U_{j}(s_{j}, s_{-j}) \equiv E\left[I_{j}\left(s_{B}, \tilde{R}_{B}\right) - \alpha c_{B}\chi\left(s_{B}, \tilde{R}_{B}\right)\theta_{j}\left(s_{B}\right) \mid \chi\left(s_{A}, \tilde{R}_{A}\right), \tilde{u}_{B}\right] = u_{B}\left[I_{j}\left(s_{B}, R_{B}\right) - \alpha c_{B}\chi\left(s_{B}, R_{B}\right)\theta_{j}\left(s_{B}\right)\right] + (1 - u_{B})\left[I_{j}\left(s_{B}, 0\right) - \alpha c_{B}\theta_{j}\left(s_{B}\right)\right] - \chi\left(s_{A}, \tilde{R}_{A}\right)\gamma\left[I_{j}\left(s_{B}, R_{B}\right) - I_{j}\left(s_{B}, 0\right) - \alpha c_{B}\chi\left(s_{B}, R_{B}\right)\theta_{j}\left(s_{B}\right) + \alpha c_{B}\theta_{j}\left(s_{B}\right)\right].$$

In order to find the best response of regulator j given the strategy of regulator -j we calculate

$$\begin{split} U_{j}\left(1, s_{-j}\right) &- U_{j}\left(0, s_{-j}\right) = \\ & u_{B}\left[\begin{array}{c} I_{j}\left((1, s_{-j}), R_{B}\right) - I_{j}\left((0, s_{-j}), R_{B}\right) - \alpha c_{B}\chi\left((1, s_{-j}), R_{B}\right)\theta_{j}\left(1, s_{-j}\right) \\ &+ \alpha c_{B}\chi\left((0, s_{-j}), R_{B}\right)\theta_{j}\left(0, s_{-j}\right) \\ &+ \left(1 - u_{B}\right)\left[I_{j}\left((1, s_{-j}), 0\right) - I_{j}\left((0, s_{-j}), 0\right) - \alpha c_{B}\theta_{j}\left(1, s_{-j}\right) + \alpha c_{B}\theta_{j}\left(0, s_{-j}\right)\right)\right] \\ &- \chi\left(s_{A}, \tilde{R}_{A}\right)\gamma\left[\begin{array}{c} I_{j}\left((1, s_{-j}), R_{B}\right)\theta_{j}\left(1, s_{-j}\right) + \alpha c_{B}\chi\left((0, s_{-j}), R_{B}\right)\theta_{j}\left(0, s_{-j}\right) \\ &- \alpha c_{B}\chi\left((1, s_{-j}), R_{B}\right)\theta_{j}\left(1, s_{-j}\right) + \alpha c_{B}\chi\left((0, s_{-j}), R_{B}\right)\theta_{j}\left(0, s_{-j}\right) \\ &- I_{j}\left((1, s_{-j}), 0\right) + I_{j}\left((0, s_{-j}), 0\right) + \alpha c_{B}\theta_{j}\left(1, s_{-j}\right) - \alpha c_{B}\theta_{j}\left(0, s_{-j}\right) \\ \end{array}\right]$$

If a bank is closed by any regulator, the income accruing to each regulator is the same regardless of the regulator responsible for the closure. This translates into $I_j((0,0), \cdot) = I_j((1,0), \cdot) =$ $I_j((0,1), \cdot)$, which we use to find that $U_j(1,0) - U_j(0,0) = \alpha c_B > 0$. As a result, regulator jalways choose $s_j = 1$ given $s_{-j} = 0$. We also find that

$$U_{j}(1,1) - U_{j}(0,1) = u_{B} - \chi\left(s_{A},\tilde{R}_{A}\right)\gamma - \frac{I_{j}\left((0,1),0\right) - I_{j}\left((1,1),0\right)}{I_{j}\left((1,1),R_{B}\right) - I_{j}\left((1,1),0\right) + \alpha c_{B}}$$

It follows that, given $s_{-j} = 1$, regulator j chooses $s_j = 1$ for

$$u_{B} \ge u_{B}^{j}(\chi) \equiv \frac{I_{j}((0,1),0) - I_{j}((1,1),0)}{I_{j}((1,1),R_{B}) - I_{j}((1,1),0) + \alpha c_{B}} + \chi\left(s_{A},\tilde{R}_{A}\right)\gamma$$

and chooses $s_j = 0$ if otherwise. Therefore, we have the following Nash equilibria: $\left(s_j^*, s_{-j}^*\right) = (0, 1)$ for $u_B < u_B^j(\chi)$, $\left(s_j^*, s_{-j}^*\right) = (1, 0)$ for $u_B < u_B^{-j}(\chi)$, and $\left(s_j^*, s_{-j}^*\right) = (1, 1)$ for $u_B \ge u_B^m(\chi) \equiv \max\left\{u_B^j(\chi), u_B^{-j}(\chi)\right\}$. The result from the proposition now follows from the observation that $I_{LoLR}((0, 1), 0) = 0$, $I_{LoLR}((1, 1), 0) = -\omega_B$, $I_{LoLR}((1, 1), R_B) = P\omega_B$, $I_{DI}((0, 1), 0) = L_B Y_B - D_B$, $I_{DI}((1, 1), 0) = -(D_B - \max\{0, \omega_B\})$, $I_{DI}((1, 1), R_B) = 0$.

Proof of Proposition 2. Let $s_j \in \{l_A, d_A\}$ be the strategy of regulator j in the game starting at t = 1 and define $s_A \equiv (s_j, s_{-j})$. The utility function of regulator j is given by

$$\begin{split} U_{j}\left(s_{j}, s_{-j}\right) &\equiv \\ E\left[I\left(s_{A}, \tilde{R}_{A}\right) - \alpha c_{A}\chi\left(s_{A}, \tilde{R}_{A}\right)\theta\left(s_{A}\right) + E\left[I\left(s_{B}^{*}, \tilde{R}_{B}\right) - \alpha c_{B}\chi\left(s_{B}^{*}, \tilde{R}_{B}\right)\theta\left(s_{B}^{*}\right)|\chi\left(s_{A}, \tilde{R}_{A}\right), \tilde{u}_{B}\right]|\tilde{u}_{A}\right] \\ &= \int \left\{ \begin{array}{l} u_{A} \left[\begin{array}{c} I_{j}\left(s_{A}, R_{A}\right) - \alpha c_{A}\chi\left(s_{A}, R_{A}\right)\theta\left(s_{A}\right) + u_{B}\left(I_{j}\left(s_{B}^{*}, R_{B}\right) - \alpha c_{B}\chi\left(s_{B}^{*}, R_{B}\right)\theta_{B}\left(s_{B}^{*}\right)\right) \\ &+ \left(1 - u_{B}\right)\left(I_{j}\left(s_{B}^{*}, 0\right) - \alpha c_{B}\theta_{B}\left(s_{B}^{*}\right)\right) \\ &- \chi\left(s_{A}, R_{A}\right)\gamma\left(I_{j}\left(s_{B}^{*}, R_{B}\right) - I_{j}\left(s_{B}^{*}, 0\right) - \alpha c_{B}\chi\left(s_{B}^{*}, R_{B}\right)\theta_{B}\left(s_{B}^{*}\right) + \alpha c_{B}\theta_{B}\left(s_{B}^{*}\right)\right) \\ &+ \left(1 - u_{A}\right)\left[\begin{array}{c} I_{j}\left(s_{A}, 0\right) - \alpha c_{A}\theta\left(s_{A}\right) + \left(u_{B} - \gamma\right)\left(I_{j}\left(s_{B}^{*}, R_{B}\right) - \alpha c_{B}\chi\left(s_{B}^{*}, R_{B}\right)\theta_{B}\left(s_{B}^{*}\right)\right) \\ &+ \left(1 - u_{B} + \gamma\right)\left(I_{j}\left(s_{B}^{*}, 0\right) - \alpha c_{B}\theta_{B}\left(s_{B}^{*}\right)\right) \end{array} \right] \right\} dF_{B} \end{split}$$

In order to find the best response of regulator j given the strategy of regulator -j we calculate

$$\begin{split} U_{j}\left(1, s_{-j}\right) &- U_{j}\left(0, s_{-j}\right) = \\ & \int \left\{ \begin{array}{l} & I_{j}\left((1, s_{-j}), R_{A}\right) - \alpha c_{A}\chi\left((1, s_{-j}), R_{A}\right)\theta\left(1, s_{-j}\right) \\ & -\chi\left((1, s_{-j}), R_{A}\right)\gamma\left(\begin{array}{c} & I_{j}\left(s_{B}^{*}, R_{B}\right) - I_{j}\left(s_{B}^{*}, 0\right) \\ & -\alpha c_{B}\chi\left(s_{B}^{*}, R_{B}\right)\theta_{B}\left(s_{B}^{*}\right) + \alpha c_{B}\theta_{B}\left(s_{B}^{*}\right) \end{array} \right) \end{array} \right\} dF_{B}. \\ & + \left(1 - u_{A}\right)\left[I_{j}\left((1, s_{-j}), 0\right) - \alpha c_{A}\theta\left(1, s_{-j}\right)\right] \\ & - \int \left\{ \begin{array}{c} u_{A} \left[\begin{array}{c} & I_{j}\left((0, s_{-j}), R_{A}\right) - \alpha c_{A}\chi\left((0, s_{-j}), R_{A}\right)\theta\left(0, s_{-j}\right) \\ & -\alpha c_{B}\chi\left(s_{B}^{*}, R_{B}\right) - I_{j}\left(s_{B}^{*}, 0\right) \\ & -\alpha c_{B}\chi\left(s_{B}^{*}, R_{B}\right)\theta_{B}\left(s_{B}^{*}\right) + \alpha c_{B}\theta_{B}\left(s_{B}^{*}\right) \end{array} \right) \end{array} \right\} dF_{B} \end{split}$$

If a bank is closed by any regulator, the income accruing to each regulator is the same regardless of the regulator responsible for the closure. This translates into $I_j((0,0), \cdot) = I_j((1,0), \cdot) =$ $I_j((0,1), \cdot)$, which we use to find that $U_j(1,0) - U_j(0,0) = \alpha c_A > 0$. As a result, regulator always choose $s_j = 1$ given $s_{-j} = 0$. We also find that

$$\begin{split} U_{j}\left(1,1\right) - U_{j}\left(0,1\right) &= \\ u_{A}\left[I_{j}\left((1,1\right), R_{A}\right) - I_{j}\left((0,1\right), R_{A}\right) + \alpha c_{A}\right] + \left(1 - u_{A}\right)\left[I_{j}\left((1,1\right), 0\right) - I_{j}\left((0,1\right), 0\right)\right] \\ &+ \int u_{A}\gamma\left[I_{j}\left(s_{B}^{*}, R_{B}\right) - I_{j}\left(s_{B}^{*}, 0\right) - \alpha c_{B}\chi\left(s_{B}^{*}, R_{B}\right)\theta_{B}\left(s_{B}^{*}\right) + \alpha c_{B}\theta_{B}\left(s_{B}^{*}\right)\right]dF_{B} \\ &= u_{A}\left[I_{j}\left((1,1\right), R_{A}\right) - I_{j}\left((0,1\right), R_{A}\right) + \alpha c_{A}\right] + \left(1 - u_{A}\right)\left[I_{j}\left((1,1\right), 0\right) - I_{j}\left((0,1\right), 0\right)\right] \\ &+ \int_{u_{B}^{m}(\chi=0)} u_{A}\gamma\left[I_{j}\left((1,1\right), R_{B}\right) - I_{j}\left((1,1), 0\right) + \alpha c_{B}\right]dF_{B} \end{split}$$

It follows that, given $s_{-j} = 1$, regulator j chooses $s_j = 1$ for

$$u_{A} \ge u_{A}^{j}(\gamma) \equiv I_{j}((0,1),0) - I_{j}((1,1),0)$$
$$I_{j}((1,1),R_{A}) - I_{j}((1,1),0) + \alpha c_{A} + \gamma \left[I_{j}((1,1),R_{B}) - I_{j}((1,1),0) + \alpha c_{B}\right] \Pr \left[u_{B} \ge u_{B}^{m}(\chi = 0)\right]$$

and chooses $s_j = 0$ if otherwise. Therefore, we have the following Nash equilibria: $\left(s_j^*, s_{-j}^*\right) = (0, 1)$ for $u_A < u_A^j(\gamma)$, $\left(s_j^*, s_{-j}^*\right) = (1, 0)$ for $u_A < u_B^{-j}(\gamma)$, and $\left(s_j^*, s_{-j}^*\right) = (1, 1)$ for $u_A \ge u_A^m(\gamma) \equiv \max\left\{u_A^j(\gamma), u_A^{-j}(\gamma)\right\}$. The result from the proposition now follows from the observation that $I_{LoLR}((0, 1), 0) = 0$, $I_{LoLR}((1, 1), 0) = -\omega_A$, $I_{LoLR}((1, 1), R_A) = P\omega_A$, $I_{DI}((0, 1), 0) = L_A Y_A - D_A$, $I_{DI}((1, 1), 0) = -(D_A - \max\{0, \omega_A\})$, $I_{DI}((1, 1), R_A) = 0$.

Proof of Proposition 3. The problem of the unified regulator is analogous to that of a regulator in the multiregulator setting assuming that the other regulator always choose not to close the bank. If we let $s \in \{0, 1\}$ be the action of the unified regulator starting at t = 2 and U(s) be the utility function, we find that the unified regulator does not close bank B if:

$$U(1) - U(0) = u_B - \chi\left(s_A, \tilde{R}_A\right)\gamma - \frac{I(0,0) - I(1,0)}{I(1,R_B) - I(1,0) + \alpha c_B} \ge 0.$$

It follows that the unified regulator does not close bank B if

$$u_{B} \ge u_{B}^{u}\left(\chi\right) \equiv \frac{I\left(0,0\right) - I\left(1,0\right)}{I\left(1,R_{B}\right) - I\left(1,0\right) + \alpha c_{B}} + \chi\left(s_{A},\tilde{R}_{A}\right)\gamma$$

The result from the proposition now follows from the observation that $I(0,0) = LY_B - D_B$, $I(1,0) = -D_B$, $I(1,R_B) = P \max\{0,\omega_B\}$.

Proof of Proposition 4. The problem of the unified regulator is analogous to that of a regulator in the multiregulator setting assuming that the other regulator always choose not to close the bank. If we let $s \in \{0, 1\}$ be the action of the unified regulator starting at t = 1 and U(s) be the utility function, we find that the unified regulator does not close bank A if:

$$U(1) - U(0) = u_A \left[I(1, R_A) - I(0, R_A) \right] + (1 - u_A) \left[I(1, 0) - I(0, 0) - \alpha c_A \right] \\ + \int_{u_B^u} u_A \gamma \left[I((1, R_B) - I((1, 0) + \alpha c_B) dF_B \ge 0 \right] dF_B \le 0$$

It follows that the unified regulator does not close bank B if

$$u_{A} \ge u_{A}^{u}(\gamma) \equiv \frac{I(0,0) - I(1,0)}{I(1,R_{A}) - I(1,0) + \alpha c_{A} + \gamma \left[I(1,R_{B}) - I(1,0) + \alpha c_{B}\right] \Pr\left[u_{B} \ge u_{B}^{u}(\chi = 0)\right]}$$

The result from the proposition now follows from the observation that $I(0,0) = LY_A - D_A$, $I(1,0) = -D_A$, and $I(1,R_A) = P \max\{0,\omega_A\}$.

Proof of Proposition 5. (i) If $\omega_B = 0$ then we have $u_B^u(\chi = 1) = \frac{L_B Y_B}{\alpha c_B + D_B} + \gamma = \max \left\{ u_B^{DI}(\chi = 1), u_B^{LoLR}(\chi = 1) \right\} = u_B^m(\chi = 1)$

(ii) If $\omega_B > 0$ we need to compare

$$u_{B}^{u}(\chi = 1) = \frac{L_{B}Y_{B}}{P\omega_{B} + \alpha c_{B} + D_{B}} + \gamma \text{ and } u_{B}^{m}(\chi = 1) = \max\left\{u_{B}^{LoLR}(\chi = 1), u_{B}^{DI}(\chi = 1)\right\},$$

where

$$u_B^{LoLR} (\chi = 1) = \frac{\omega_B}{(P+1)\omega_B + \alpha c_B} + \gamma,$$
$$u_B^{DI} (\chi = 1) = \frac{L_B Y_B - \omega_B}{\alpha c_B + D_B - \omega_B} + \gamma.$$

One must note that

$$\frac{\partial u_B^u}{\partial \omega_B} < 0, \frac{\partial u_B^{LoLR}}{\partial \omega_B} > 0, \text{ and } \frac{\partial u_B^{DI}}{\partial \omega_B} < 0 \text{ for } \alpha \text{ big enough.}$$

If $u_B^u > u_B^{LoLR} \forall \omega_B \in (0, D_B]$, then the unified regulator arrangement is always less forbearing than the multiregulator arrangement. To see this, note that for $\omega_B = 0$, $u_B^{DI} = u_B^u$, and if α is big enough, $u_B^{DI} < u_B^u$ for $\omega_B > 0$. Therefore $u_B^u > u_B^m = \max \{u_B^{LoLR}, u_B^{DI}\}$ for α sufficiently large.

Suppose $\exists \omega_B^* \in (0, D_B]$ such that $u_B^u = u_B^{LoLR}$. Since $u_B^{DI} < u_B^u$ for $\omega_B > 0$ and α big enough, $u_B^{LoLR}(\omega^*) = u_B^u(\omega^*) > u_B^{DI}(\omega^*)$. In addition, $u_B^{DI} > u_B^{LoLR}$ for $\omega_B = 0$. Therefore, $\exists \omega_B^{**} \in (0, \omega_B^*)$ such that $u_B^{DI}(\omega_B^{**}) = u_B^{LoLR}(\omega_B^{**})$. Because u_B^{DI} is decreasing and u_B^{LoLR} is increasing we have $u_B^u > u_B^{DI} = \max \{u_B^{LoLR}, u_B^{DI}\} = u_B^m$ for $\omega_B \in (0, \omega_B^{**})$. Since u_B^u is decreasing and u_B^{LoLR} is increasing is increasing, we have $u_B^u > u_B^{LoLR} = \max \{u_B^{LoLR}, u_B^{DI}\} = u_B^m$ for $\omega_B \in [\omega_B^{**}, \omega_B^*)$, and $u_B^m = \max \{u_B^{LoLR}, u_B^{DI}\} = u_B^m$ for $\omega_B \in [\omega_B^{**}, \omega_B^*)$, and $u_B^m = \max \{u_B^{LoLR}, u_B^{DI}\} = u_B^u$ for $\omega_B \in [\omega_B^{**}, \omega_B^*)$, and $u_B^m = \max \{u_B^{LoLR}, u_B^{DI}\} = u_B^u$ for $\omega_B \in [\omega_B^{**}, \omega_B^*)$.

The critical point ω_B^* solves $u_B^{LoLR} = u_B^u$. It is straightforward to see that γ is cancelled out, which makes ω_B^* independent of γ . Differentiating both sides with respect to L_B we get

$$\frac{\partial \omega_B^*}{\partial L_B} = \frac{Y_B \left[\left(P + 1 \right) \omega_B^* + \alpha c_B \right]}{P \left(\omega_B^* - L_B Y_B \right) + P \omega_B^* - L_B Y_B + \alpha c_B + D_B},$$

which is clearly positive for α sufficiently large.

Differentiating both sides of $u_B^{LoLR} = u_B^u$ with respect to D_B we get

$$\frac{\partial \omega_B^*}{\partial D_B} = \frac{-\omega_B^*}{P\left(\omega_B^* - L_B Y_B\right) + P\omega_B^* - L_B Y_B + \alpha c_B + D_B},$$

which is clearly negative if α is big enough.

Differentiating both sides of $u_B^{LoLR} = u_B^u$ with respect to c_B and P we get:

$$\frac{\partial \omega_B^*}{\partial c} = \frac{\alpha \left(L_B Y_B - \omega_B^* \right)}{P \left(\omega_B^* - L_B Y_B \right) + P \omega_B^* - L_B Y_B + \alpha c_B + D_B},$$

$$\frac{\partial \omega_B^*}{\partial P} = \frac{\omega_B^* \left(L_B Y_B - \omega_B^* \right)}{P \left(\omega_B^* - L_B Y_B \right) + P \omega_B^* - L_B Y_B + \alpha c_B + D_B}.$$

If $\varphi(L_B) \equiv L_B Y - \omega_B^* < 0$, then we have $\frac{\partial \omega_B^*}{\partial c_B}, \frac{\partial \omega_B^*}{\partial P} < 0$ for α big enough. If $\varphi(L_B) > 0$, then we have that $\frac{\partial \omega_B^*}{\partial c_B}, \frac{\partial \omega_B^*}{\partial P} > 0$ for α sufficiently large. Therefore, we need to investigate $\varphi(L_B)$. One

can easily check that $\varphi(0) = 0$. In addition, $\varphi(L_B) = 0$ and $u_B^{LoLR} = u^u$ can be simultaneously satisfied for $\overline{L}_B = \frac{D_B}{Y_B}$. If we differentiate $\frac{\partial \omega_B^*}{\partial L_B}$ with respect to L_B we get

$$\frac{\partial^{2}\omega_{B}^{*}}{\partial L_{B}^{2}} = \frac{-\frac{\partial\omega_{B}^{*}}{\partial L_{B}}2\left(P\frac{\partial\omega_{B}^{*}}{\partial L_{B}} - Y_{B}\left(P+1\right)\right)}{P\left(\omega_{B}^{*} - L_{B}Y_{B}\right) + P\omega_{B}^{*} - L_{B}Y_{B} + \alpha c_{B} + D_{B}},$$

which is positive for α big enough. In this case $\varphi''(L_B) < 0$. By the Mean Value Theorem $\exists L_B^* \in (0, \overline{L}_B)$ such that $\varphi'(L_B^*) = 0$, which implies that $\varphi(L_B)$ achieves its maximum at L_B^* . Since $\varphi'(L_B)$ is decreasing, $\varphi'(L_B) > 0$ for $L_B < L_B^*$ and $\varphi'(L_B) < 0$ for $L_B > L_B^*$.

We now claim that $\frac{\partial \omega_B^*}{\partial c_B}$, $\frac{\partial \omega_B^*}{\partial P} > 0$ for all $L_B \in (0, \overline{L}_B)$. We first show that the claim holds for all $L_B \in [L_B^*, \overline{L}_B)$. Because $\varphi'(L_B) < 0$ for $L_B \in (L_B^*, 1)$ and $\varphi(\overline{L}_B) = 0$, it follows that $\varphi(L_B) > 0$ for $L_B \in [L_B^*, \overline{L}_B)$. Therefore, $\frac{\partial \omega_B^*}{\partial c_B}$, $\frac{\partial \omega_B^*}{\partial P} > 0$ for $L_B \in [L_B^*, \overline{L}_B)$. We now show that the claim holds for all $L_B \in (0, L_B^*)$. This follows from the fact that $\varphi(L_B) > 0$ for $L_B \in (0, L_B^*)$. Suppose not, i.e., $\varphi(L_B) = 0$ for some $L'_B \in (0, L_B^*)$. Because $\varphi'(L_B) > 0$ for $L_B \in (0, L_B^*)$, it follows that $\varphi(L_B) < 0 \forall L_B \in [0, L'_B)$. But this contradicts the fact that $\varphi(0) = 0$.

Proof of Proposition 6. (i) If $\omega_A = 0$, then

$$u_A^u(\gamma) = \frac{L_A Y_A}{D_A + \alpha c_A + \gamma \left(\alpha c_B + D_B\right) \Pr\left[u_B \ge u_B^u\left(\chi = 0\right)\right]} = \max\left\{0, \frac{L_A Y_A}{D_A + \alpha c_A + \gamma \left(D_B + \alpha c_B\right) \Pr\left[u_B \ge u_B^m\left(\chi = 0\right)\right]}\right\} = u_A^m(\gamma)$$

(ii) If $\omega_A > 0$ we compare

$$u_{A}^{u}(\gamma) = \frac{L_{A}Y_{A}}{P\omega_{A} + D_{A} + \alpha c_{A} + \gamma \left[\alpha c_{B} + D_{B}\right] \Pr\left[u_{B} \ge u_{B}^{u}\left(\chi = 0\right)\right]} \text{ and } u_{A}^{m}\left(\gamma\right) = \max\left\{u_{A}^{DI}\left(\gamma\right), u_{A}^{LoLR}\left(\gamma\right)\right\}$$

where

$$u_A^{LoLR}(\gamma) = \frac{\omega_A}{(P+1)\,\omega_A + \alpha c_A + \gamma \alpha c_B \Pr\left[u_B \ge u_B^m\left(\chi = 0\right)\right]},$$
$$u_A^{DI}(\gamma) = \frac{L_A Y_A - \omega_A}{D_A - \omega_A + \alpha c_A + \gamma \left(D_B + \alpha c_B\right) \Pr\left[u_B \ge u_B^m\left(\chi = 0\right)\right]}.$$

It is straightforward to check that

$$\frac{\partial u_A^u}{\partial \omega_A} < 0, \frac{\partial u_A^{LoLR}}{\partial \omega_A} > 0, \text{ and } \frac{u_A^{DI}}{\partial \omega_A} < 0 \text{ for } \alpha \text{ big enough.}$$

If $u_A^u > u_A^{LoLR} \ \forall \omega_A \in (0, D_A]$, then the unified regulator arrangement is always less forbearing than the multiregulator arrangement. To see this, note that for $\omega_A = 0$, $u_A^{DI} = u_A^u$, and if α is big enough, $u_A^{DI} < u_A^u$ for $\omega_A > 0$. Therefore $u_A^u > u_A^m = \max \{u_A^{LoLR}, u_A^{DI}\}$ for α sufficiently large.

Suppose $\exists \omega_A^* \in (0, D_A]$ such that $u_A^u = u_A^{LoLR}$. Since $u_A^{DI} < u_A^u$ for $\omega_A > 0$ and α big enough, $u_A^{LoLR}(\omega_A^*) = u_A^u(\omega_A^*) > u_A^{DI}(\omega_A^*)$. In addition, $u_A^{DI} > u_A^{LoLR}$ for $\omega = 0$. Therefore, \exists

$$\begin{split} \omega_A^{**} &\in (0, \omega_A^*) \text{ such that } u_A^{DI} \left(\omega_A^{**} \right) = u_A^{LoLR} \left(\omega_A^{**} \right). \text{ Because } u_A^{DI} \text{ is decreasing and } u_A^{LoLR} \text{ is increasing we have } u_A^{DI} > u_A^{DI} = \max \left\{ u_A^{LoLR}, u_A^{DI} \right\} = u_A^m \text{ for } \omega_A \in (0, \omega_A^{**}). \text{ Since } u_A^u \text{ is decreasing and } u_A^{LoLR} \text{ is increasing, we have } u_A^u > u_A^{LoLR} = \max \left\{ u_A^{LoLR}, u_A^{DI} \right\} = u_A^m \text{ for } \omega_A \in [\omega_A^{**}, \omega_A^*), \text{ and } u_A^m = \max \left\{ u_A^{LoLR}, u_A^{DI} \right\} = u_A^u \text{ for } \omega_A \in [\omega_A^{**}, \omega_A^*), \text{ and } u_A^m = \max \left\{ u_A^{LoLR}, u_A^{DI} \right\} = u_A^{LoLR} > u_A^u \text{ for } \omega_A \in [\omega_A^*, D_A]. \end{split}$$

The critical point ω_A^* solves $u_A^{LoLR}(..., \omega_A^*, \gamma) = u_A^u(..., \omega_A^*, \gamma)$. Differentiating both sides with respect to L we get

$$\frac{\partial \omega_A^*}{\partial L_A} = \frac{Y_A \left[\alpha \left(c_A + \gamma c_B \operatorname{Pr} \left(\cdot \right) \right) + \left(P + 1 \right) \omega_A^* \right]}{P \left(\omega_A^* - L_A Y_A \right) + P \omega_A^* - L_A Y_A + D_A + \alpha c_A + \gamma \left(\alpha c_B + D_B \right) \operatorname{Pr} \left(\cdot \right)}$$

Because ω_A^* is bounded, this is clearly positive for α big enough.

Differentiating both sides of $u_A^{LoLR} = u_A^u$ with respect to D_A we get

$$\frac{\partial \omega_A^*}{\partial D_A} = \frac{-\omega_A^*}{P\left(\omega_A^* - LY_A\right) + P\omega_A^* - LY_A + D_A + \alpha c_A + \gamma \left(\alpha c_B + D_B\right) \Pr\left(\cdot\right)}$$

which is clearly negative for α sufficiently large.

Differentiating both sides of $u_A^{LoLR} = u_A^u$ with respect to c_B , γ , c_A , D_B , and P we get:

$$\begin{split} \frac{\partial \omega_A^*}{\partial c_B} &= \frac{\gamma \alpha \operatorname{Pr}\left(\cdot\right) \left(L_A Y_A - \omega_A^*\right) + \gamma \alpha c_B \frac{\partial \operatorname{Pr}\left(\cdot\right)}{\partial c_B} \left(L_A Y_A - \omega_A^*\right) - \gamma D_B \frac{\partial \operatorname{Pr}\left(\cdot\right)}{\partial c_B} \omega_A^*}{P\left(\omega_A^* - L_A Y_A\right) + P\omega_A^* - L_A Y_A + D_A + \alpha c_A + \gamma \left(\alpha c_B + D_B\right) \operatorname{Pr}\left(\cdot\right)}{\frac{\partial \omega_A^*}{\partial c_A}} &= \frac{\operatorname{Pr}\left(\cdot\right) \alpha c_B \left(L_A Y_A - \omega_A^*\right) - D_B \operatorname{Pr}\left(\cdot\right) \omega_A^*}{P\left(\omega_A^* - L_A Y_A\right) + P\omega_A^* - L_A Y_A + D_A + \alpha c_A + \gamma \left(\alpha c_B + D_B\right) \operatorname{Pr}\left(\cdot\right)}{\frac{\partial \omega_A^*}{\partial D_B}} &= \frac{\gamma \alpha c_B \frac{\partial \operatorname{Pr}\left(\cdot\right)}{\partial D_B} \left(L_A Y_A - \omega_A^*\right) - \gamma \omega_A^* \left(\operatorname{Pr}\left(\cdot\right) + D_B \frac{\partial \operatorname{Pr}\left(\cdot\right)}{\partial D_B}\right)}{P\left(\omega_A^* - L_A Y_A\right) + P\omega_A^* - L_A Y_A + D_A + \alpha c_A + \gamma \left(\alpha c_B + D_B\right) \operatorname{Pr}\left(\cdot\right)}}{\frac{\partial \omega_A^*}{\partial P}} &= \frac{\gamma \alpha c_B \frac{\partial \operatorname{Pr}\left(\cdot\right)}{\partial D_B} \left(L_A Y_A - \omega_A^*\right) - \gamma \omega_A^* \left(\operatorname{Pr}\left(\cdot\right) + D_B \frac{\partial \operatorname{Pr}\left(\cdot\right)}{\partial D_B}\right)}{P\left(\omega_A^* - L_A Y_A\right) + P\omega_A^* - L_A Y_A + D_A + \alpha c_A + \gamma \left(\alpha c_B + D_B\right) \operatorname{Pr}\left(\cdot\right)}}{\frac{\partial \omega_A^*}{\partial P}} &= \frac{\gamma \alpha c_B \frac{\partial \operatorname{Pr}\left(\cdot\right)}{\partial P} \left(L_A Y_A - \omega_A^*\right) + \omega_A^* \left(L_A Y_A - \gamma D_B \frac{\partial \operatorname{Pr}\left(\cdot\right)}{\partial P} - \omega_A^*\right) - \gamma \omega_A^* \left(\operatorname{Pr}\left(\cdot\right) + D_B \frac{\partial \operatorname{Pr}\left(\cdot\right)}{\partial P}\right)}{P\left(\omega_A^* - L_A Y_A\right) + P\omega_A^* - L_A Y_A + D_A + \alpha c_A + \gamma \left(\alpha c_B + D_B\right) \operatorname{Pr}\left(\cdot\right)}} \right)} \right) \\ \end{array}$$

If $\varphi(L_A) \equiv L_A Y_A - \omega_A^* < 0$, then we have $\frac{\partial \omega_A^*}{\partial c_B}, \frac{\partial \omega_A^*}{\partial \gamma}, \frac{\partial \omega_A^*}{\partial c_A}, \frac{\partial \omega_A^*}{\partial D_B}, \frac{\partial \omega_A^*}{\partial P} < 0$ for α big enough. If $\varphi(L_A) > 0$, then we have that $\frac{\partial \omega_A^*}{\partial c_B}, \frac{\partial \omega_A^*}{\partial \gamma}, \frac{\partial \omega_A^*}{\partial c_A}, \frac{\partial \omega_A^*}{\partial D_B}, \frac{\partial \omega_A^*}{\partial P} > 0$ for α sufficiently large. Therefore, we need to investigate $\varphi(L_A)$. One can easily check that $\varphi(0) = 0$. In addition, $\varphi(L_A) = 0$ and $u_A^{LoLR} = u_A^u$ can be simultaneously satisfied for $\overline{L}_A = \frac{D_A + \gamma D_B \operatorname{Pr}(\cdot)}{Y_A}$. If we differentiate $\frac{\partial \omega_A^*}{\partial L_A}$ with respect to L_A we get

$$\frac{\partial^{2}\omega_{A}^{*}}{\partial L_{A}^{2}} = \frac{-\frac{\partial\omega_{A}^{*}}{\partial L_{A}}2\left(P\frac{\partial\omega_{A}^{*}}{\partial L_{A}} - Y_{A}\left(P+1\right)\right)}{P\left(\omega_{A}^{*} - L_{A}Y_{A}\right) + P\omega_{A}^{*} - L_{A}Y_{A} + D_{A} + \alpha c_{A} + \gamma\left(\alpha c_{B} + D_{B}\right)\Pr\left(\cdot\right)},$$

which is positive for α big enough. In this case $\varphi''(L_A) < 0$. By the Mean Value Theorem $\exists L_A^* \in (0, \overline{L}_A)$ such that $\varphi'(L_A^*) = 0$, which implies that $\varphi(L_A)$ achieves its maximum at L_A^* . Since $\varphi'(L_A)$ is decreasing, $\varphi'(L_A) > 0$ for $L_A < L_A^*$ and $\varphi'(L_A) < 0$ for $L_A > L_A^*$.

Since $\varphi'(L_A)$ is decreasing, $\varphi'(L_A) > 0$ for $L_A < L_A^*$ and $\varphi'(L_A) < 0$ for $L_A > L_A^*$. We now claim that $\frac{\partial \omega_A^*}{\partial c_B}, \frac{\partial \omega_A^*}{\partial \gamma}, \frac{\partial \omega_A^*}{\partial c_A}, \frac{\partial \omega_A^*}{\partial D_B}, \frac{\partial \omega_A^*}{\partial P} > 0$ for all $L_A \in (0, \overline{L}_A)$. We first show that the claim holds for $[L_A^*, \overline{L}_A)$. Because $\varphi'(L_A) < 0$ for $L_A \in (L_A^*, 1)$ and $\varphi(\overline{L}_A) = 0$, it follows that $\varphi(L_A) > 0$ for $L_A \in [L_A^*, \overline{L}_A)$. Therefore, $\frac{\partial \omega_A^*}{\partial c_B}, \frac{\partial \omega_A^*}{\partial c_A}, \frac{\partial \omega_A^*}{\partial D_B}, \frac{\partial \omega_A^*}{\partial P} > 0$ for $L_A \in [L_A^*, \overline{L}_A)$. We now show that the claim holds for $L_A \in (0, L_A^*)$. This follows from the fact that $\varphi(L_A) > 0$ for all $L_A \in (0, L_A^*)$. Suppose not, i.e., $\varphi(L_A) = 0$ for some $L'_A \in (0, L_A^*)$. Because $\varphi'(L_A) > 0$ for $L_A \in (0, L_A^*)$, it follows that $\varphi(L_A) < 0 \forall L_A \in [0, L'_A)$. But this contradicts the fact that $\varphi(0) = 0$.

Proof of Proposition 7. The result that babbling equilibria exist comes from the observation that if the LoLR's belief treats all messages as uninformative, i.e., $\overline{\gamma}(m) = \overline{\gamma}$, and if messages are independent of types, $m(\gamma) = m \forall \gamma \geq \gamma_*$, then the optimal action taken by the LoLR given $\overline{\gamma}(m) = \overline{\gamma}$ and the message strategy adopted by the DI are best responses to each other.

Consider the case in which $\exists \ \widehat{\gamma} \in [0,1]$ such that $E[\widetilde{\gamma}|\gamma_* \leq \widetilde{\gamma} \leq \widehat{\gamma}] = \gamma^*$. As a result, $\overline{\gamma} \geq \gamma^*$ and $u_A^{LoLR}(\overline{\gamma}) \leq u_A^{DI}(\gamma^*) = u$, which implies that the outcome of a babbling equilibrium has the LoLR not liquidating bank A for all $\gamma \geq \gamma_*$.

Suppose there is an equilibrium in which the LoLR liquidates bank A for some $\gamma'' \geq \gamma_*$. If $\forall \gamma' \geq \gamma_*$ we have $m(\gamma') = m(\gamma'') = m$, then the equilibrium is a babbling equilibrium, in which case consistency of beliefs requires $\overline{\gamma}(m) = \overline{\gamma}$ and the LoLR does not liquidate bank A for all $\gamma \geq \gamma_*$, which is a contradiction. Let $m(\gamma') \neq m(\gamma'')$ for some γ' and let M' be the set of different messages sent in equilibrium. For every $m' \in M'$ we define the set $\Gamma(m') = \{m^{-1}(m')\}$. Clearly, $\cup \Gamma'(m') = [\gamma_*, 1]$.

Suppose the LoLR liquidates bank A for all $m' \in M'$. Let μ be the measure associated with the distribution Z. Optimality requires $\overline{\gamma}(m') < \gamma^* \forall m' \in M$. However, one must note that $\sum_{m' \in M'} \frac{\mu(\Gamma(m'))}{1-Z(\gamma_*)} \int_{\Gamma(m')} \gamma \frac{z(\gamma)}{\mu(\Gamma(m'))} d\gamma = \overline{\gamma}$ must hold. But since $\overline{\gamma}(m') < \gamma^*$, we have a contradiction as $\sum_{m' \in M'} \frac{\mu(\Gamma(m'))}{1-Z(\gamma_*)} \overline{\gamma}(m') < \sum_{m' \in M'} \frac{\mu(\Gamma(m'))}{1-Z(\gamma_*)} \gamma^* = \gamma^* \leq \overline{\gamma}$. Therefore, there must $\exists m' \in M'$ such that $\overline{\gamma}(m') \geq \gamma^*$, i.e., the LoLR does not liquidate bank A upon seeing message m'. But by assumption the LoLR liquidates bank A upon receiving $m(\gamma'') \neq m'$. Therefore, the DI that observes $\gamma \in \Gamma(m(\gamma''))$ has an incentive to deviate and report m', which contradicts the assumption that there is an equilibrium in which the LoLR liquidates bank A for some $\gamma'' \geq \gamma_*$.

Now consider the case $\nexists \ \widehat{\gamma} \in [0,1]$ such that $E[\widetilde{\gamma}|\gamma_* \leq \widetilde{\gamma} \leq \widehat{\gamma}] = \gamma^*$. In this case $\overline{\gamma} < \gamma^*$ and $u_A^{LoLR}(\overline{\gamma}) > u_A^{DI}(\gamma^*) = u$, which implies that the outcome of a babbling equilibrium has the LoLR liquidating bank A for all $\gamma \geq \gamma_*$.

Suppose there is an equilibrium in which the LoLR does not liquidate bank A for some $\gamma'' \ge \gamma_*$. If $\forall \gamma' \ge \gamma_*$ we have $m(\gamma') = m(\gamma'') = m$, then the equilibrium is a babbling equilibrium, in which case consistency of beliefs requires $\overline{\gamma}(m) = \overline{\gamma}$ and the LoLR liquidates bank A for all $\gamma \ge \gamma_*$, which is a contradiction. Let $m(\gamma') \ne m(\gamma'')$ for some γ' and let M' be the set of different messages sent in equilibrium. For every $m' \in M'$ we define the set $\Gamma(m') = \{m^{-1}(m')\}$. Clearly, $\cup \Gamma'(m') = [\gamma_*, 1]$.

Suppose the LoLR does not liquidate bank A for all $m' \in M'$. Optimality requires $\overline{\gamma}(m') \geq \gamma^* \forall m' \in M$. However, one must note that $\sum_{m' \in M'} \frac{\mu(\Gamma(m'))}{1-Z(\gamma_*)} \int_{\Gamma(m')} \gamma \frac{z(\gamma)}{\mu(\Gamma(m'))} d\gamma = \overline{\gamma}$ must hold. But since $\overline{\gamma}(m') \geq \gamma^*$, we have $\sum_{m' \in M'} \frac{\mu(\Gamma(m'))}{1-Z(\gamma_*)} \overline{\gamma}(m') \geq \sum_{m' \in M'} \frac{\mu(\Gamma(m'))}{1-Z(\gamma_*)} \gamma^* = \gamma^* > \overline{\gamma}$. Therefore, we have a contradiction. As a consequence, $\exists m' \in M'$ such that $\overline{\gamma}(m') < \gamma^*$, i.e., the LoLR liquidates bank A upon seeing message m'. But by assumption, the LoLR does not liquidate bank A upon

receiving $m(\gamma'') \neq m'$. Therefore, the *DI* that observes $\gamma \in \Gamma(m')$ has an incentive to deviate and report m'', which contradicts the assumption that there is an equilibrium in which the *LoLR* does not liquidates bank *A* for some $\gamma'' \geq \gamma_*$.

Proof of Proposition 8. If the *DI* chooses not to learn about γ , then bank *A* is not liquidated since max $\{u_A^{LoLR}(\underline{\gamma}), u_A^{DI}(\underline{\gamma})\} \leq u_A$. Suppose the *DI* chooses to learn about γ . Because $\underline{\gamma}$ is below the lower bound of the support of $Z(\gamma)$, for any observed γ we have $u_A^{DI}(\gamma) \leq u_A^{DI}(\underline{\gamma}) = u_A$, which implies that the *DI* prefers bank *A* not to be liquidated. If $\gamma \geq \gamma_*$, then because $\nexists \ \widehat{\gamma} \in [0, 1]$ such that $E[\widetilde{\gamma}|\gamma_* \leq \widetilde{\gamma} \leq \widehat{\gamma}] = \gamma^*$, bank *A* is liquidated and the *DI* is worse off. Let $\gamma < \gamma_*$. Since $\gamma_* \leq \gamma^*$, we have that $u_A^{LoLR}(\gamma) > u_A^{LoLR}(\gamma_*) \geq u_A^{LoLR}(\gamma^*) = u_A$, which implies that the *LoLR* always liquidate bank *A*. Therefore, the *DI* is worse off if he chooses to learn γ .

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