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This paper examines the degree of income tax progressivity chosen through a simple majority vote in a model with savings. Households have permanent differences with respect to their labor productivity and their discount factors. The government has limited commitment to future policy, so voting is repeated every period. Because the model features mobility within the wealth distribution, the median voter is determined endogenously. In a numerical experiment, the model is initialized to the 1992 U.S. joint distribution of income and wealth as well as several statistics of the federal income tax distribution. Support for a high degree of progressivity is widespread. In the long run, households that vote for lower progressivity have high labor productivity and/or very high wealth. A movement towards greater progressivity increases aggregate capital and income, but it effects only a small decrease in long-run income and wealth inequality.

Keywords: Heterogeneity, progressive taxation, wealth distribution, electoral competition.

JEL Codes: E21, E62, H24.

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1 Introduction

Nearly all OECD countries have statutory income tax schedules with marginal rates that increase in income. Figure 1 plots the federal statutory marginal tax rate against multiples of the lowest taxable income level in each of twelve OECD countries for the year 2007.[1] While there is considerable variation in tax rates and in tax brackets across countries, two features are common: first, every tax schedule is progressive (i.e., the average tax rate increases with income), and second, with the exception of Iceland every schedule is *marginal-rate progressive*, that is marginal rates increase with income.[2] Economists have attempted to explain this second feature of the data for many years. These efforts can be divided into two groups: one attempting to derive marginal-rate progressive taxation as a characteristic of an optimal tax code, and the second uncovering it as the outcome of a political process.

The optimal taxation approach generally does not support progressive marginal income tax schedules. In the seminal Mirrlees (1971), for instance, tje optimal income tax is very close to linear. There are some notable exceptions however. Grochulski (2007) considers an environment where households have access to a concealment technology which permits them to shelter income from the government. When marginal costs of sheltering income are increasing, it may be optimal for the government to use a marginal-rate progressive income tax. Conesa and Krueger (2006) and Conesa, Kitao, and Krueger (2009) model households who face uninsurable idiosyncratic wage risk. Capital income is taxed seperately from labor income. The optimal labor income tax approaches a two-bracket schedule: a significant exemption for low income and a flat tax for all greater income levels. The capital tax is proportional. These results are quite sensitive to the weights assigned to household types in the social welfare function. Saez (2002) finds that a similar flat tax plus exemption schedule is optimal when the wealth distribution is fixed exogenously.

The political economy approach rejects the social planner framework and instead focuses on the process through fiscal policy is decided)typically through some form of majority vote). Within this framework, tax rates should depend, at least in part, on the distributions of income and wealth. [3] In fact, a general finding within political economy models of income tax pro-

[1]Data from OECD Tax Database Table I.5.

[2]While at first glance one might think that Iceland's tax has no progressivity, in fact it has an exemption for low-income households so there is some progressivity. The other OECD country with this sort of income tax structure is the Slovak Republic. Like Iceland, it offers a basic fixed allowance so some progressivity is still present.

[3]For early examinations of this hypothesis with linear taxation, see Romer (1975), Roberts (1977), and Meltzer and Richard (1981).

gressivity is that absent rent-seeking politicians, a democracy with an electorate in which the income-poor outnumber the income-rich will demand a progressive income tax since a majority benefit from income redistribution.[4] ,[5] In these models, the income distribution is exogenously given, and a voter's preference for progressive taxes depends on the voter's resulting net tax bill (i.e., income tax less any tax revenue transfers). In a few cases, the income distribution is endogenized by allowing labor to be supplied elastically. This introduces a cost to redistribution: the distortion to the voter's labor supply decision.[6]

This literature has left the dynamic effects of income tax progressivity largely unaddresed. Intertemporal tradeoffs are not considered, and any feedback between current policy and future tax revenues is ignored. Essentially, the literature has concentrated on how politics divides the economic pie today, but has remained silent about its affects on the size and distribution of future pies. Central in the argument over how income taxes should be designed is how marginal tax rates alter incentives to save. Including a savings decision is important for three reasons. First, the distribution of wealth affects future production, and thus also future income distributions. A progressive tax not only distorts the optimal savings decisions of households by reducing future marginal returns from capital, but it places the largest marginal tax rates, and thus the strongest disincentives to saving, on households which otherwise tend to save the most.[7]

Second, in the US, wealth is much more concentrated than income and is held primarily by high-income earners.[8] From the perspective of low- and middle-income households (from which the pivotal voter likely arises), this concentration of capital income may be a tempting target for redistribution. Thus, not only could future aggregate income be reduced, but the tax base may erode as high-income households consume their wealth in response. Thus high transfers in the short run may come at the cost of lower future transfers. On the other hand, increased progressivity reduces future marginal tax rates on low-income households, making saving more attractive for them. Generally, the median income level is less than the mean income level, so it is not clear which direction aggregate wealth will move with progressivity without studying a quantitative model.

Finally, when both labor and capital are inputs to production, the capital stock influences the prices paid to each factor. Households may not only disagree on tax policy because of differences in income levels, but also because of differences in the composition of their income.

[4]Marhuenda and Ortuño-Ortín (1995).

[5]Although the literature cannot prove that a right-skewed income distribution is sufficient for a marginal-rate progressive taxation equilibrium when the space of admissible income tax functions is defined as *any* nonlinear function, many papers have shown existence within broad classes of nonlinear functions.

[6]see Klor (2003)

[7]For an overview of the suboptimality of capital income taxation see Atkeson, Chari, and Kehoe (1999).

[8]Budria Rodriguez, et. al. (2002) report a wealth gini of 0.803 and a earnings gini of just 0.553 in the 1998 wave of the SCF.

Households with a high concentration labor income (relative to capital income) have an incentive to vote for policy which increases aggregate wealth while those with a greater fraction of income from capital have an analogous incentive to see aggregate wealth reduced. In the US data, capital income and labor income are positively, but not perfectly correlated, so there may be considerable disagreement over policy even among households with similar income levels.[9] Again, inferring the direction of factor price movements in response to progressivity changes requires a quantitative model.

This paper reexamines the political demand for income tax progressivity within the neoclassical growth model. Households earn income through labor and capital some of which may be invested toward producing new capital. To capture the costs and benefits from income tax progressivity associated with wealth accumulation, this paper employs a dynamic growth model with an explicit voting mechanism. Households have permanent differences in labor productivity and in discount factors which imply heterogeneity in income and wealth. This heterogeneity, in turn, leads to disagreement over preferred tax schedules. In each period, households decide upon the progressivity of next period's income tax schedule through a simple majority vote. Because the conflict space is restricted to one dimension, the median voter theorem guarentees a political equilibrium so long as preferences are single-peaked.

For an economy calibrated to the 1992 US joint distribution of income and wealth, the highest degree of progressivity within the policy space wins election in every period. Preferences for progressivity are strongly decreasing in income and in wealth. Examination of individual household value functions shows that most households have nearly "bang-bang" preferences for progressivity. At low wealth levels, the household prefers the high progressivity, and at high levels of wealth it prefers low progressivity. Somewhere between low and high wealth, there exists a narrow interval over which intermediate degrees of progressivity are favored. Finally, as long as its labor productivity isn't too high, a household prefers more progressivity as the ratio of its labor income to its total income increases. This is because in this model higher progressivity makes effective labor more scarce to capital, inducing an rise in the wage rate.

Comparing the long-run effects from increased progressivity with an alternative case under which the parameter governing progressivity is exogenously fixed at its initial level, increased progressivity leads to higher long-run aggregate wealth and income. The elasticity of the capital stock, and therefore the elasticity of factor prices, to progressivity is very small with 12.0% increase in income tax progressivity leading to only a 1.2% increase in long-run aggregate wealth.[10] Also the equilibrium path with high tax progressivity leads to slightly more equal

[9]Carroll and Young (2009) calculates the correlation between labor income and capital income for the 1992, 1995, 1998, 2002, and 2004 SCF waves. These correlations range from 0.14 to 0.43.

[10]Li and Sarte (2004) find that the long run effects of progressivity on GDP growth are small (-0.12% to -0.34%). Unlike their results however this paper finds that progressivity leads to an *increase* in the economy-wide level of capital. The difference in the direction of the response of the capital stock to progressivity is likely

income and wealth distributions. Interestingly, transfers are higher on the more progressive path for only the very early periods of transition. Wealthy households quickly adjust their savings in response to higher progressivity, which leads to a sharp decline in their income and thus in tax revenue as well.

Finally, predicting households' demand for progressivity based solely upon the next period distribution of net tax bills does not correctly predict either the votes of individual households nor the general direction of the equilibrium fiscal policy in terms of progressivity. While higher income levels, and by consequence net tax bills, do affect households' preferred policies, households that would pay more taxes if progressivity increases may still vote for it if their income is highly concentrated in labor income. In the long run, households with negative net taxes compose only a minority of the group voting for highly progressive policy.. This finding suggests that the simple static story of income redistribution fails to entirely capture the motivation for progressive income taxation.

2 Literature

Previous work on the popularity of progressive taxation has been developed within static models and focused its attention on the distribution of net income taxes across the population. Snyder and Kramer (1988) create a model with exogenous heterogenous earnings abilities and a two-sector labor market. When tax schedules are restricted to be non-decreasing, they find that a marginal-rate progressive tax schedule wins in a popular vote due to majority support from the middle class despite opposition from both high-income and low-income types. Marhuenda and Ortuño-Ortín (1995) prove a *popular support for progressivity theorem* which states that when the income distribution is right skewed and tax policy is revenue-neutral, continuous and non-decreasing, a progressive tax defeats a regressive tax in a majority vote under the condition that the net tax bill of the poorest agent under the progressive tax does not exceed that under the regressive tax. Under weaker assumptions, the results of Marhuenda and Ortuño-Ortín (1995) do not generally hold. Hindricks (2001) shows that within the class of quadratic tax schedules allowing a regressive tax to return a lower net tax on the poorest agents than the progressive tax does leads to a *popular support for regressivity theorem*. Thus, for any progressive tax schedule there is a regressive schedule that defeats it in election by securing support from low-income and high-income households. Combining this result with Marhuenda and Ortuño-Ortín (1995) implies that tax schedules should cycle between progressive and regressive tax schedules,

due to a difference in the way their tax schedule affects the poor. In Li and Sarte, an increase in progressivity increases the marginal tax rate on poor households, though high-income household face even larger increases. In this work, increased progressivity reduces the marginal tax rate on low- and middle-income levels. This induces some of these households to increase their savings.

however, as noted in Snyder and Kramer (1988) tax data on OECD countries has shown tax schedules to be fairly stable. Klor (2003) shows that the *popular support for progressivity theorem* does not generally apply when the tax space includes all average-rate progressive taxes nor when the income distribution is determined endogenously through household labor supply decisions.[11]

To overcome the lack of generality of previous models, some research has expanded the tax policy space to multiple dimensions. The difficulty with this is that median voter theorems do not generally apply and so voting equilibria may fail to exist. In order to guarantee an equilibrium, further restrictions must be imposed. Roemer (1999) defines a different political equilibrium concept in which two competing parties propose quadratic tax schedules, but only after each secures unanimous support from factions within their own party. The income distribution is fixed, and each voter's preference ordering over policy depends only on the amount of disposable income the policy delivers. These preferences imply linear indifference curves so voters pool at corners of the policy space. With right-skewed income distributions both parties propose progressive schedules. Following the same assumptions about preferences and tax policy as Roemer (1999), De Donder and Hindriks (2004) returns to a Downsian voting framework and gives conditions under which a Condorcet winner exists and show that when one does exist it features the maximum degree of progressivity permitted. Another line of work, turns to representative democracy to restrict the structure of the vote. In Carbonell-Nicolau and Klor (2003) two parties with exogenous preferences over income inequality pay a cost to propose a tax policy from the set of increasing piecewise linear tax functions. They find that among coalition-proof equilibria marginal-rate progressive taxation is always chosen, thus recovering the *popular support for progressivity theorem* from Marhuenda and Ortuño-Ortín (1995). Carbonell-Nicolau and Ok (2007) allow political candidates to play mixed strategies in their tax proposals. Once again the result of Marhuenda and Ortuño-Ortín (1995) obtains but only if the set of proposals is restricted to be weakly monotonic.

One shortcoming of the previous literature is the limit to which income taxation plays a distortionary role in agents decisions. Of the papers mentioned above, only Hindricks (2001) and Snyder and Kramer (1988) are the exceptions, and the only distortion considered is to labor supply. Even accounting for labor supply distortions, these static models may greatly underestimate the costs from increased progressivity since progressivity increases the marginal tax rate on high-income households and thus places the greatest distortions on the portion of the income distribution where capital is highly concentrated. Modeling voting within a dynamic model introduces more complications. First, both the wealth and income distributions are endogenous.

[11]Under an average-rate progressive tax schedule the tax burden as a fraction of income increases in income. While marginal rate progressive taxes are also average-rate progressive, the converse is not always true. As an example, consider a convex tax function with a positive value at 0.

This means that tax policy has more effects than simple income redistribution. Movements in the wealth distribution change factor prices, while changes in the income distribution alter both the concentration across households of the tax burden as well as the size redistribution. Second, when government commitment is not assumed, voting must be repeated. When taxes are flat and there is no uncertainty, the long-run distribution of wealth is indeterminate (Chatterjee 1994) and features no mobility along the transition path. In other words, the ordering of agents in terms of wealth stays the same. In such an environment, it is possible to guarantee that the pivotal voter remains within the same subset of households of identical households over time. Equilibrium policy can be uncovered simply by examining these pivotal agents' preferences. A small but growing literature of such models has arisen. In Krusell and Ríos-Rull (1999), there are three types of agents. Two groups represent 49% of the population each, and the pivotal voter resides in the remaining 2%. Within a reasonably calibrated model, a vote over a flat income tax can account for the size of redistribution in the US data. Azzimonti, de Francisco, and Krusell (2006) study an environment where agents only differ in initial wealth. Using an aggregation theorem to uncover the first-order condition of the pivotal voter, they decompose the effects of tax policy into a collection of "gaps" which distort this condition. Azzimonti, de Francisco, and Krusell (2007) explores the extent to which heterogeneity both in economic variables and in political desires aggregate up in a bond economy with elastic labor supply. In Bassetto and Benhabib (2006) agents who are heterogeneous only with respect to their initial wealth vote over sequences of flat taxes. The median voter theorem still applies despite the infinite-dimensionality of the conflict space because preferences are assumed to be Gorman aggregable. The median voter prefers the highest capital income taxation permitted for a number of periods and zero capital income taxes for all subsequent periods. In each of these models, the location of agents within the wealth and income distributions does not change so the identity of the pivotal voter can be taken as given. It is not necessary to explicitly calculate the vote nor even to solve for the policy preferences of non-pivotal agents. While changes in the wealth and income distribution may change the indirect preferences over tax policy of the pivotal agent, they never shift political power.[12]

This paper attempts to bridge these two literatures to shed more light on the demand for progressivity. The findings of Carroll and Young (2009) suggest that results derived from dynamic voting over flat taxation are likely to be quite different from what would arise under progressive taxation. In a complete markets setup with heterogeneous labor productivity, when discount factors are homogeneous and income taxation is marginal-rate progressive, there is a determinant joint distribution of income and wealth, and that distribution is grossly inconsistent with the US data. Absent exogenous borrowing constraints, income across households is equal,

[12]Bachmann and Bai (2010) use a dynamic model of the business cycle with wealth-bias in the political process to explain the procyclicality of government purchases.

making capital income and labor income perfectly negatively correlated.[13] This is true even if the deviation from linear taxation is very small. In order to build an accurate approximation to the US data, Carroll and Young (2009) suggest allowing for heterogeneity in discount factors in the spirit of Sarte (1997). This paper adopts that technique, calibrating discount factors and productivities jointly from US household level data on income and wealth. In so doing, this paper is one of the first to employ this new strategy. Giving up indeterminacy does come at a cost however. The pivotal voter must be found endogenously which makes solving the model significantly more challenging than most previous work in the dynamic voting literature.

3 Model

The model economy consists of three sectors: households, firms, and a government. In this outline of the model, capital letters denote aggregate variables and lower case letters denote individual-specific variables.

3.1 Households

This sector is comprised of a unit continuum of infinitely-lived households which differ with respect to their subjective discount factor, β, and permanent labor productivity, ε. Each household belongs to one of a finite number, I, of types. A type i is a pair (β_i, ε_i), and ψ_i is the fraction of the total population comprised by type i. Each household has the same period utility function $u(c)$ which is assumed to be strictly increasing and concave and to obey the Inada conditions. Lifetime utility for a household of type i is given by the time-separable function

$$\max_{\{c_{it},k_{i,+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta_i^t u(c_{it}) \tag{1}$$

subject to

$$\begin{aligned} s.t.\ c_{it} + k_{i,t+1} &\leq y_{it} - \tau(y_{it}) + T_t + k_{it} && (2)\\ y_{it} &= w_t \varepsilon_i \bar{h} + r_t k_{it} && (3)\\ k_t &\geq 0 && (4) \end{aligned}$$

where c_{it} and k_{it} are household i's consumption and wealth, respectively, in period t. k_b is a non-positive borrowing limit and may be as low as the natural debt limit. Households supply a

[13] The results of Carroll and Young (2009) do not extend to environments where risk is not completely insurable. For a dynamic model of flat taxation under uncertainty, see Corbae, D'Erasmo, and Kuruscu (2009).

fixed number of hours, $\bar{h}$, and therefore a fixed number of effective hours as well.[14] w_t and r_t are the payments to effective labor and to capital. Total income, y_{it} is the sum of income from labor and from capital. $\tau(y_{it})$ is the total tax bill paid on income of y_i at time t before any transfers.

3.1.1 The tax bill

Both $\tau(y)$, the tax bill function, and $\tau_y(y)$, the marginal tax rate, are assumed to be nonnegative for all income and strictly monotonic increasing $\forall t$ with $\tau(y) = 0$ at $y = 0$. In words, the tax bill is progressive according to the definition from Musgrave and Thin (1948) since the average tax bill $\frac{\tau(y)}{y}$ is increasing in income. It is also marginal rate progressive; that is, at the margin, an additional unit of income increases the tax bill by more than the previous unit did.[15]

The tax bill takes the follwing function form:

$$\tau(y) = y\xi\left(\frac{y}{z}\right)^{\phi} \tag{5}$$

for some z. This progressive tax function is used in Li and Sarte (2004).[16] It has the convenient property that ϕ captures the degree of progressivity in the tax schedule, measured as the marginal tax rate, $\tau_y(y)$, divided by the average tax rate, $\frac{\tau(y)}{y}$[17], higher ϕ implies more progressivity.

As ϕ changes, marginal tax rates may not remain well-ordered across all income levels. As a result, there is a built-in potential for the single-peakedness over ϕ' to fail. To illustrate, figure 2 plots the marginal tax function for three values of ϕ. For income level below y_{low} (above y_{high}), marginal tax rates fall (rise) with progressivity; however, for incomes between y_{low} and y_{high}, the highest marginal tax rate occurs when ϕ takes an intermediate value.[18] Because savings responses are sensitive to the marginal tax rate, it is possible for a household to prefer either high progressivity or low progressivity over a value between which violates single-peakedness.

To eliminate this potential problem, the marginal tax function above is altered slightly to allow z to depend upon ϕ. Ideally, the function should pivot about a central tax rate as ϕ changes. If ϕ increases (i.e., more progressivity), then marginal tax rates above the pivot rate

[14] Allowing for elastic labor supply would introduce significantly greater computational challenge while not changing the resulting path of votes. The households that would adjust their hours the most have high labor productivity and zero wealth, and they reduce their hours. This would drive up the wage which, as the analysis below indicates, increases the support for progressivity. In this way, the model is biased against progressivity.

[15] Under the restriction that a household's tax burden cannot exceed its total income, a tax function being marginal-rate progressivity is equivalent to it being strictly convex.

[16] In Li and Sarte (2004), z equals mean income.

[17] $\tau_y(y) = \frac{(1+\phi)}{y}\tau(y)$ so $\frac{\tau_y(y)}{\frac{\tau(y)}{y}} = 1 + \phi$.

[18] While the size of this income interval may seem small, it contains approximately 16% of households in the final income distribution, and a even larger fraction of househould pass through this interval at some point during the evolution of the distribution to its terminal steady state.

will rise while those below will decline. Given any two marginal tax functions, one described by ϕ_0 and the other by ϕ_1, $z(\phi)$ must satisfy the condition

$$\tau_y(\tilde{y};\phi_0) = \tau_y(\tilde{y};\phi_1),$$

where $\tilde{y}$ is the income level associated with the pivot rate, implying

$$z(\phi_1) = \left\{ \left[\frac{(1+\phi_1)}{(1+\phi_0)} \right] \tilde{y}^{\phi_1-\phi_0} z(\phi_0)^{\phi_0} \right\}^{\frac{1}{\phi_1}}.$$ [19], [20]

Figures 3 plots the marginal tax rate for several values of ϕ given $\tilde{y} = 1$ and $z_0 = 1$. Notice that as ϕ increases the marginal tax remains the same for $\tilde{y}$, rises for all $y > \tilde{y}$, and falls for all $y < \tilde{y}$. It should be pointed out that this normalization procedure is not sufficient to guarentee single-peakedness since there other general equilibrium factors which influence a household's preference over ϕ'. Nevertheless, it does address one potential pitfall.

3.2 Firms and Government

Each period, households rent their effective labor, N, and capital, K, to a stand-in firm in return for wages and rent. With labor and capital as inputs, the firm produces a good which may be consumed or invested for future production. Let the production technology be Cobb-Douglas with capital's share denoted by α. Under the assumption that markets are competitive, factors of production are paid their marginal product so that

$$w_t = (1-\alpha) K_t^{\alpha} N_t^{-\alpha} \tag{6}$$

$$r_t = \alpha K_t^{\alpha-1} N_t^{1-\alpha} - \delta \tag{7}$$

The government raises tax revenue to finance wasteful government spending, $\bar{G}$. Any surplus revenue is returned to the households as a lump-sum transfer,

$$T_t = \sum_i^I \psi_i \tau(y_{it}) - \bar{G}. \tag{8}$$

T_t is restricted to be non-negative so lump-sum taxation is not a policy instrument available to the government. Furthermore, the government does not have access to a commitment technology.

[19] Notice that since z does not depend upon income y, this augmented tax function preserves the identity from 5, $1+\phi = \frac{\tau_y(y)}{\frac{\tau(y)}{y}}$.

ϕ_0 will be uncovered from calibration, and $z(\phi_0)$ will be defined to equal 1.

[20] Because τ_y is not bounded above by 1, an upper bound of 0.999 is imposed in the quantitative experiment.

3.3 Voting

In each period t, the degree of progressivity in $t+1$ is determined through simple majority rule in pairwise competition. The median voter theorem generally does not hold for multidimensional policy spaces, and voting equilibria are not guaranteed to exist. The existence of equilibria in political economy models rarely comes without conceding to some restrictive assumptions. If a richer policy space were permitted, very strong assumptions on voter preferences would have to be made in order to guarantee the existence of a Condorcet winner.[21]

4 Recursive Problem

Let Γ be the distribution of wealth and assume that it follows the law of motion $\Gamma^{'} = H(\Gamma, \phi)$. For simplicity, denote by Γ_i the wealth holdings of an i-th type household. I assume that the progressivity of taxation evolves over time according to $\Psi(\Gamma, \phi)$. It should be stressed here that the assumptions about H and Ψ imply that this analysis is restricted to Markov equilibria. That these functions depend only upon Γ and ϕ is the concept of a "minimum state variable" as discussed in Krusell and Ríos-Rull (1999). Together, the distribution of wealth and the degree of progressivity provide sufficient information to calculate current prices and transfers. It is assumed that the markets for investment, consumption, and labor clear every period. Mathematically,

$$K = \sum_i \psi_i k_i \tag{9}$$

$$N = \bar{h} \sum_i \psi_i \varepsilon_i \tag{10}$$

$$\sum_i \psi_i c_i + K' - K + G = F(K, N) - \delta K \tag{11}$$

The household problem may be expressed recursively as the following dynamic programming problem:

$$v_i(k, \Gamma, \phi) = \max_{c,k'} u(c) + \beta_i v_i(k', \Gamma', \phi')$$

subject to

[21] Common solutions to the difficulties arising from multidimensional conflict are not feasible in this model primarily because of the multidimensionality of heterogeneity. Generally, these solutions amount to projecting the multidimensional conflict down into a unidimensional characteristic space over which policy preferences are easily ordered. In this model, the relationship between β and ε and preferred policy is not easily reduced to a single dimension.

$$
\begin{aligned}
c + k' &\leq y - \tau(y;\phi) + k + T && (12)\\
y &= w\bar{h}\varepsilon_i + rk && (13)\\
k' &\geq 0 && (14)\\
\Gamma' &= H(\Gamma,\phi) && (15)\\
\phi' &= \Psi(\Gamma,\phi). && (16)
\end{aligned}
$$

Solving this problem yields decision rules $c = g_i(k,\Gamma,\phi)$ and $k' = h_i(k,\Gamma,\phi)$ for consumption and savings, respectively. Following Krusell and Ríos-Rull (1999), this next section distinguishes a *competitive economic equilibrium* and a *politico-economic equilibrium*.

4.1 Competitive Economic Equilibrium

A competitive economic equilibrium (CEE) takes the evolution of tax policy, $\Psi(\Gamma,\phi)$, as given. As will be seen in the next section, to find its preferred value of ϕ', each household must evaluate the outcome associated with any candidate ϕ'. When the economy evolves according to Ψ and H, any ϕ will lead to a sequence of future tax progressivities and wealth distributions. From this sequence, a household can determine its welfare associated with a given ϕ' and rank all ϕ' in the policy space accordingly. The definition of a recursive competitive economic equilibrium is now formally stated.

Definition 1 *Given Ψ, a CEE is a set of functions $\left\{\{v_i,g_i,h_i\}_{i\in I}, H, r, w, T\right\}$ such that:*

1. *Given $\{H,r,w,T\}$, v_i, g_i, and h_i solve the recursive problem for type i households for all $i \in I$.*

2. *Factor markets clear.*

3. *T clears the government budget constraint.*

4. *The economy-wide resource constraint is satisfied.*

5. *$\Gamma_i = H_i(\Gamma,\phi) = h_i(\Gamma_i,\Gamma,\phi)$. In words, the i-th element of the wealth distribution implied by the law of motion H is consistent with the optimal saving decision of the i-th type for all i.*

4.2 Politico-Economic Equilibrium

In a politico-economic equilibrium (PEE), $\Psi(\Gamma, \phi)$ is determined endogenously. To uncover the equilibrium Ψ, households solve the *one-period deviation* problem. Every household knows that ϕ' will follow Ψ from tomorrow onward, but today ϕ' is permitted to deviate from the rule $\Psi(\Gamma, \phi)$. Because of this, the distribution of wealth may no longer evolves according to H. Instead, in the period in which the vote occurs, the Γ' will follow a new rule $\tilde{H}(\Gamma, \phi, \phi')$, after which it resumes following H in all future periods. Formally, the problem is stated as follows:

$$\tilde{v}_i(k, \Gamma, \phi; \phi') = \max_{c,k'} \{u(c) + \beta_i v_i(k', \Gamma', \phi')\} \tag{17}$$

subject to

$$\begin{aligned} c + k' &\leq y - \tau(y; \phi) + k + T && (18)\\ y &= w\bar{h}\varepsilon_i + rk && (19)\\ k' &\geq 0 && (20)\\ \Gamma' &= \tilde{H}(\Gamma, \phi, \phi'). && (21) \end{aligned}$$

$\tilde{v}_i$ here differs from v_i in that it depends directly upon the ϕ' chosen in the current period. Clearly, different values of ϕ' induce different savings decisions and thus differents paths of aggregate wealth and income, as well as transfers and factor prices.

To arrive at its preferred policy, a household must also account not only for how its choice of ϕ' will affect future wealth distributions, but also how it will affect future elections. The pivotal voter today will be the pivotal voter in any future vote with zero probability. There are two reasons for this. First, a household's position in the wealth and income distribution is not fixed over time so the ordering of households' preferences for policy will not remain constant over time either. Second, voting is competitive in the sense that even if the same household type contained the median voter every period, the median-voter household would be selected randomly from among the infinite households of that type. Each household, then, considers how its vote today, should it be decisive, would influence the entire sequence of future wealth distributions and policy decisions, though the laws of motion

$$\begin{aligned} \phi'' &= \Psi\left(\tilde{H}(\Gamma, \phi, \phi'), \phi'\right)\\ \Gamma'' &= H\left(\tilde{H}(\Gamma, \phi, \phi'), \phi'\right)\\ \phi''' &= \Psi\left(H\left(\tilde{H}(\Gamma, \phi, \phi'), \phi'\right), \Psi\left(\tilde{H}(\Gamma, \phi, \phi'), \phi'\right)\right)\\ \Gamma''' &= H\left(H\left(\tilde{H}(\Gamma, \phi, \phi'), \phi'\right), \Psi\left(\tilde{H}(\Gamma, \phi, \phi'), \phi'\right)\right)\\ &\ldots \end{aligned}$$

Solving this problem returns decision rules $\tilde{g}_i\left(k;\Gamma,\phi,\phi'\right)$ and $\tilde{h}_i\left(k;\Gamma,\phi,\phi'\right)$ for consumption and savings, respectively.

Given knowledge of the future effects of each ϕ', the household selects it most preferred progressivity value

$$\phi^i = \arg\max_{\phi'} \tilde{v}_i. \tag{22}$$

Definition 2 *A PEE then can be formally stated as a set of functions* $\left\{\{v_i, g_i, h_i\}_{i\in I}, H, r, w, T\right\}$ *and* $\left\{\tilde{v}_i, \tilde{g}_i, \tilde{h}_i, \tilde{H}\right\}$ *such that:*

1. $\left\{\{H, r, w, T\}, \{v_i, g_i, h_i\}_{i\in I}\right\}$ is a CEE.
2. $\left\{\tilde{v}_i, \tilde{g}_i, \tilde{h}_i\right\}_{i\in I}$ solve (17)-(21) and $\tilde{H}$ implies $\Gamma'_i = \tilde{H}_i = \tilde{h}_i$ for all i.
3. For all i, ϕ^i satisfies (22).
4. ϕ^i_{med} is such that $\sum_{i:\phi^i\leq\phi^i_{med}}\psi_i = \sum_{i:\phi^i\geq\phi^i_{med}}\psi_i = 0.5$.
5. $\Psi(\Gamma,\phi) = \phi^i_{med}(\Gamma_{med},\Gamma,\phi)$

4.3 Steady State

Although a full solution to this model can only be achieved using numerial solution methods, some insights can be gained from a brief analytical study of its steady state. In the long-run, the marginal rate of substitution of consumption goes to unity so the optimal savings decision for a type-i household is described by the following equation:

$$1 \geq \beta_i\left[(1-\tau_y(y_i;\phi))\,r + 1\right] \tag{23}$$

where

$$y_i = w\varepsilon_i\bar{h} + rk_i \tag{24}$$

and (23) holds with equality if and only if $k_i > 0$.

Given that τ_y is monotonic increasing, two facts are immediately apparent:

1. There is only one value of income which can satisfy (23) with equality for β_i.
2. Among households with wealth above the borrowing limit in the long-run, a higher discount factor is associated with higher income.

I now formally define a β-*group*.

Definition 3 *A β-group is a collection of all households in I with the same value of β.*

All households in a β-group have the same discount factor, but they may differ in their labor productivity. The long-run savings behavior of households within a β-group follow the general results in Carroll and Young (2009). Within a β-group, all households with positive assets have the same long run level of income. Among these households, those with greater labor productivity earn a greater fraction of their income from labor than do less productive households in the same β-group. Households that are especially productive in labor may hit the lower bound on savings. These households will have higher income than the positive wealth households in the group.

4.3.1 The effect of changes in tax policy on the long-run income and savings of households

When ϕ changes, there are several effects on a household's long-run wealth and income. Because the tax structure is nonlinear, the net result of these effects will differ from household to household depending upon each one's characteristics. [22] To understand the long-run effects of changes in progressivity on each household, it is helpful to express (23) as

$$\frac{\beta_i^{-1} - 1}{r} \geq 1 - \tau_y\left(y_i; \phi\right). \tag{25}$$

The tax rate effect An increase in ϕ, raises τ_y for types with income above $\tilde{y}$ and discouraging saving. High-income households that are not already at the borrowing limit gradually reduce their wealth, decreasing their income and their marginal tax rates over time. Eventually, either τ_y declines enough that saving once again becomes optimal or the borrowing limit will be reached. The tax rate effect works in the reverse for low income types. They will increase their savings and their income. In aggregate, the tax rate effect reduces long run income and wealth inequality.

The factor price effect. A higher long-run interest rate implies a higher marginal benefit from saving. For all households with wealth above the borrowing limit this effect decreases long-run wealth and income. A higher value of r decreases the LHS of (25). For equality to be restored, τ_y, and therefore also y_i, must rise as well. Because wages and interest rates move in opposite directions, an increase in r implies less labor income for all households. Since in the long-run income rises, capital income must increase to offset the change in labor income. The magnitude of the change in labor income will be largest for households with high ε so these households will also have large changes in wealth. Households at the lower limit after tax policy

[22] Here an increase in ϕ will be examined. The results from a decrease in progressivity are the opposite.

changes will experience an unambiguous decline in long-run income, due entirely to the decrease in wages.

Figures 4 and 5 compare the two effects when ϕ increases and r increases.[23] Both types hold positive wealth before and after the policy change. To find the tax rate effect, calculate long-run income when r is held fixed, that is keeping the LHS of (25) constant. Since (25) holds with equality, while $\tau_y(y)$ decreases, y must rise so that the marginal tax rate is the same before and after the increase in ϕ. Turning to the figures, this can be seen as the distance from point A to point B. This change in income keeps the marginal tax rate the same before and after reform. In equilibrium, however, the household ends up at point C. This additional increase in income is due to the factor price effect through r. Comparing the low-income and high-income cases, the factor price effect moves income in the same direction for both types, while the tax rate effect is positive for a low-income household and negative for a high-income household. The change in the marginal tax function is particularly severe for high income households, and clearly the tax rate effect strongly dominates causing income to decline in the long run. For low income households however the marginal tax function does not decline much as progressivity rises. As a consequence most of the increase in income can be attributed to the change in r.

4.3.2 The Steady-State Distributions of Income and Wealth

As stated above, all else equal, increasing the progressivity of the tax schedule reduces income inequality. The relationship between the long-run wealth distribution and household heterogeneity is best seen in the definition of income. Rearranging (24) yields

$$k_i = \frac{y_i\left(\beta_i, \tau_y, r\right)}{r} - \frac{w\bar{h}}{r}\varepsilon_i.$$

For households with the same labor productivity, those with larger discount factors will have more wealth. Within a β-group long-run wealth declines with ε at a rate of $\frac{w\bar{h}}{r}$. Notice that this implies wealth inequality within every β-group is negatively related to r. Looking across β-groups an increase in r causes the LHS of (25) to decrease. The factor price effect increases the long-run income of all households with positive wealth, however because the marginal tax rate function is concave high-income types must increase income more than low-income types for equality in (25) to be restored. This leads to an increase wealth inequality between β-groups. Whether overall wealth inequality increases or decreases in response to a change in factor prices is ambiguous The question is further complicated by the existence of a lower bound on assets. Once assets have been depleted to zero, a household income is composed entirely of labor income

[23] The values of income in these plots are taken from the the initial steady state $(\phi, r) = (0.71, 0.138)$ and the final steady state. $(\phi, r) = (0.8, 0.139)$.

which it cannot reduce to lower its tax burden. This suggests that, to the extent that some households in the model have zero wealth, removing the non-negativity constraint would lead to lower income inequality and greater wealth inequality.

Appendix A details the computational algorithm for solving the model.

5 A Quantitative Experiment

5.0.3 Initialization

Period utility is assumed to be a CRRA function of consumption

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma},$$

with $\gamma = 2$.

A representative sample of the US economy is constructed using observations of the US income and wealth distribution from the 1992 wave of the Survey of Consumer Finances data set. Each observation i in the SCF contains of an income value, $\tilde{y}_i$, a wealth value, $\tilde{k}_i$, and a population weight, $\tilde{\psi}_i$, which is assigned by the survey. The 1992 wave contains 3906 households. Unfortunately, it is not computationally feasible to use so many types. Instead, the sample joint distribution of income and wealth must be approximated by a coarser distribution. The coarse distribution has 51 types of households. This means that 153 parameters describing the preferences, productivities, and population weights of the household types are inferred from the data. In addition, ξ and ϕ, the tax function parameters, are set to match the average tax rate and the average marginal tax rate from the NBER TAXSIM data for 1992. Finally the transfer, $\bar{T}$ clears the government budget constraint.

As shown in Carroll and Young (2009), in a deterministic models with heterogeneous productivities a non-degenerate long run distribution of wealth implies that the distribution of household discount factors have a one-to-one relation with the marginal tax function. To initialize, the model assumes a steady state in 1992 and that no household were restricted from borrowing. Given a market clearing interest rate, β_i can be backed out as a function of household i's marginal tax rate,

$$\beta_i = \left[(1 - \tau_y(y_i))\, r + 1\right]^{-1}.$$

If $\tau_y(y)$ is strictly increasing then every y is assigned to a unique $\tau_y(y)$ and therefore also to a unique β.[24]

[24] If $\tau_y(y)$ is flat however any y is associated with the same marginal tax rate and therefore mapped to the same β. In order to get a wealth distribution with a significant upper tail, I place a limit on the marginal tax rate of 0.396. This is the highest tax bracket in the US statutory income tax code for 1992. By capping the marginal tax rate at 0.396, the calibrated distribution can capture more of the right-skewness in income and asset holdings

α is set to 0.36 to match labor's share of income in the data. G_t is assumed to be $\bar{G} = 0.08$. δ is 0.05 so that in the initial steady state investment is 15% of aggregate income. $\bar{Y}$, the aggregate level of income is set equal to 1. $\bar{K}$ the initial aggregate capital stock is set to 3.0. $\bar{h}$ is assumed to be 0.33. z_0 and $\tilde{y}$, the initial base value of z and the income for which marginal taxes remain constant are set equal to initial mean income.[25]

One advantage of this initialization strategy is its ability to well approximate the US distributions of income and wealth. Table 1 compares moments from the data with those from the coarse distribution. In general, the coarse distribution does a good job of characterizing the inequality in income and wealth: considerable variance in income and extreme variance in wealth, a strong positive covariance, and significant right-skewedness.[26] Although the inequality in the data is still larger than in the coarse distribution, this is primarily due to observations in the data of very wealthy individuals. With sufficient grid points, the coarse approximation could do a better job, but for these gains one must increase the number of types and thus tradeoff large amounts of computational time. Preliminary work with 267 types showed insignificant changes to the results.

Looking at each marginal distribution in closer detail, while the coarse approximation underestimates wealth in the middle to upper part of the distribution, it does a very good job of characterizing the income distribution along income percentiles. Figures 6 and 7 compare the cumulative distribution functions of income and wealth, respectively, from the model approximation to those from the SCF. Given the greatly reduced number of household types, the coarse distributions follows the SCF distributions relatively well. This is especially true for the wealth distribution which is encouraging given this study's particular emphasis on savings.

Each period, households vote on a value for ϕ' between 0.2 and 0.8. Single-peaked preferences cannot be guaranteed for every household in every possible state. When households consider off-equilibrium paths, they do so presuming that other households vote sincerely. Single-peakedness is tested numerically for every possible state. Given that each individual household has zero population mass, the assumption of sincere voting may be thought of as arising from very large coordination costs[27].

found in the US data without resorting to extremely high discount factors.

This paper uses value function iteration to solve the model. High discount factors are known to make convergence of the value function very slow. Moreover, with high discount factors small approximation errors can disrupt convergence making the process potentially unstable. Since the effects of progressivity on high-income households are clear in the results, assigning extremely high discount factors to rich households would not provide any additional insight.

In all voting periods however, the marginal tax function associated with a given ϕ will be strictly increasing.

[25] For more detail on the calibration method see Appendix B.

[26] Because variance is not a unit-independent measure, Table 1 reports the coefficient of variation.

[27] In the baseline case described, only 9040 of a possible 2409750 (or 0.38%) state variable and household combinations display non-single-peaked preferences. In these cases, the value function is essentially flat, only

Because the initial distribution is calibrated assuming a marginal tax function with a flat region, while the policy space over which households vote contains only marginal tax functions that are strictly increasing, the economy would not remain in the initial steady state even if ϕ remained unchanged from its initial calibrated level in every period. Figure 8 compares the marginal tax function with the ceiling to the one with the same value of ϕ and no ceiling. High-income households face considerably greater marginal tax rates once the ceiling on this function is removed. All else equal, the adjustment to strictly increasing functions decreases these households' desire to save which will have a sizeable effect on the capital stock in the economy. For this reason, two experiments are run: one where tax progressivity is determined by a vote and another where tax policy stays fixed at its initial value. All results arising from the voting case will be referred to as *on the political equilibrium path.* The economy with voting does approach a steady state in the limit. This steady state is called the *political equilibrium steady state.* In the alternative scenario, where just the ceiling is removed and no voting takes place, the initial value of ϕ is called the *status quo* value, the transition path is called the *fixed policy path*, and the steady state is called the *fixed policy steady state.*

6 Results

6.0.4 Steady State Comparison

6.0.5 Inequality

Table 2 shows the long-run effects on the distribution of wealth in both the political equilibrium and fixed policy steady states. Comparing the political equilibrium and initial steady states, wealth is more spread out with more progressive taxes. Wealth among the highest income quintile declines by 36.5% while wealth among the remaining 80% of households increases substantially. The starkest example of this behavior comes from the model's bottom quinitile which increases savings by an astounding $5,410\%$! There is a denominator effect in operation here. This quintile holds only a 0.2% share of the capital stock initially. Table 3 shows the share of steady state wealth held by each quintile. While wealth holdings are still significantly skewed over the income distribution, capital in the economy is much more evenly distributed in the political equilibrium steady state.

The reduction of wealth inequality comes primarily from removing the flat portion of the marginal tax function as evidenced by the fixed policy steady state.

Long-run income equality is reduced only slightly with more progressivity. The variances, skewnesses, and Gini coefficients of the long-run income and wealth distributions are reported in Table 4. Under both the voting and fixed policy cases, there are large reductions in the

minute "wiggles" cause single-peakedness to be violated.

variance and skewness of income and wealth from the initial steady state. Within each β-group households trade off labor income and capital income one-for-one, and wealth holdings are very sensitive to changes in marginal tax function and in factor prices. For many types after-tax return is too low to induce them to save. When the equilibrium and alternative steady states are compared, however, the effect of income tax progressivity on inequality is modest. A 7% increase in ϕ produces a reduction 2.9% in the income gini and of 2.3% in the wealth gini. In both cases, a majority of the population holds assets at or very near to the borrowing limit. The median capital holding in the political equilibrium steady state is 2.1 while in the fixed policy steady state, it is 1.93.

6.0.6 Aggregates

Because roughly two-thirds of the initial capital stock is owned by these households, this decline in wealth at the upper end has a significant effect on aggregate wealth. Table 5 compares the percentage change in economy-wide variables between the initial and political equilibrium steady states to those between the initial and fixed policy steady states. Under the fixed policy case the aggregate capital stock remains very near its initial level, but the political equilibrium steady state, with a more progressive tax, has a capital stock that is a little larger. This may seem suprising because in both cases marginal tax rates on high-income households increase considerably. The after-tax return to saving declines dramatically for these households, leading them to dissave rapidly. The aggregate capital stock rises in the long-run however because the dissaving of the high-income households is eventually offset by increased saving from low and middle-income households. Aggregate income does increase with more progressive tax rates however the change is very small. There is also a small decline in average consumption driven by large reductions in the top quintile. Less afluent quintiles increase consumption.

When voting is not permitted, the marginal tax function increases only on the top 13%. Nevertheless, wealth holdings of poorer households still change in response to the interest rate. When the effects of removing the upper ceiling on the initial marginal tax function are controlled for, progressive taxation leads to a slight increase in the long-run capital stock.

6.1 Transitional Dynamics

While steady state analysis is helpful for understanding households' decisions, given the forward-looking behavior of voters in this model, the transition path induced by policy change plays a critical role. This is especially true for economies which converge slowly to their steady states since long run consumption levels may not be close approximations to consumption levels in early transitional periods, and because due to discounting, early consumption levels have many times more weight in a voter's decision than consumption in distant periods. For some initial distributions, early transitional dynamics may make some policies politically unpopular even if those policies lead to steady states in which a majority of households enjoy more consumption than under the status quo.

Figure 9 plots the paths of aggregate wealth, mean income, the government transfer, and mean consumption. The transition path can be divided into two stages. The first stage is characterized by declines in aggregate wealth, income, and consumption, and for most periods, transfers as well. The second stage is marked by a somewhat more gradual rise in aggregate behavior. The path with fixed policy follows a similar pattern implying that this "dip" behavior arises from the removal of the ceiling on the marginal tax rate function. The dip can be understood by considering that households' responses to the fiscal policy differ both in direction (some households increase wealth, others decrease wealth) as well as in rate. Under either the political equilibrium path or the fixed policy path, high income households immediately face a dramatic rise in their marginal tax rates. In the political equilibrium, this is primarily due to the removal of the marginal tax rate ceiling while in the fixed policy path, it is due *exclusively* to the ceiling's removal. High marginal tax rates discourage savings. This effect, along with significantly greater tax bills after the policy change, causes wealth to decrease rapidly.

Meanwhile, low-income households increase their wealth for two possible reasons. First, along the political equilibrium, these households' marginal tax rates are decline, making consumption more expensive at the margin. Second, in both the voting and non-voting cases, the rapid depletion of aggregate capital increases the interest rate. The factor price effect discussed earlier showed that in isolation r rising causes households to increase their income. Essentially, the tax rate effect dominates the factor price effect for high-income households while the opposite is true for low-income households. Figure 10 plots the wealth paths of two households, one low income and the other high income. In the first stage of transition, high-income households reduce their wealth very quickly while low-income households increase wealth but more slowly. During this time, the interest rate rises, incentivizing low-income households to increase wealth accumulation and high-income households to slow their dissaving. As transition enters stage two, the dissaving by high-income households has slowed sufficiently that saving by the other households causes aggregate wealth to begin rising. Notice that in the political equilibrium (increased progressivity combined with the removal of the tax ceiling), the tax effect is even more powerful than along the status quo equilibrium. Aggregate wealth (and income) declines more quickly and the second stage arrives sooner on the political equilbrium transition.

Under either case the transfer increases initially after the first vote because of the increased tax bill on high income households. Not surprisingly, the transfer increase is larger for the more progressive ϕ'.[28] The additional amount of transfer quickly diminishes as aggregate income

[28]The average tax bill in the economy equals the transfer plus a fixed level of government spending. Therefore, this account of the transfer holds true for the average tax bill as well.

declines through the first stage of transition. In fact, despite being initially higher, the transfer under the political equilibrium falls below the transfer level for the non-voting equilibrium after the sixth period. This highlights the importance modeling redistribution with an endogenous income distribution. In this case, a large transfer can only be supported temporarily since it distorts the savings decisions of the high-income households. Interestingly, unlike aggregate income, the transfer does not rise in the second stage of transition. This is because a progressive tax schedule attempts to collect the largest share of taxes from the portion of the income distribution that is shrinking while lowering the tax burden on the portion that is growing. For this reason, the total tax collected does not rise when average income does.

Finally, there are several other intersting facts to note from comparing the two transition paths. First, it takes a considerable amount of time for the voting equilbrium to overtake the non-voting equilibrium in terms of aggregate wealth and income. It takes 13 periods for the political equilibrium path to overtake the status quo path. Second, the status quo path leads to slightly lower aggregate capital and wealth than in the initial steady state. Increasing the marginal tax rate on high income households does not lead to greater aggregate wealth. A tax rate effect must also be active for low-income households in order to get gains in aggregate wealth. Finally, aggregate variables converge to their steady values more rapidly than individual household variables, suggesting that the effects of progressive tax policy changes on inequality take much longer to evolve than its effects on economy-wide measures.

6.2 Voting Decision

6.2.1 On the equilibrium path

The most important factors determining a household's preference over tax progressivity are its current income and its labor productivity. Not surprisingly, as a household's income increases, all else equal, it prefers less progressivity. Figure 11 plots households' preferred tax policy against their income in the long run (i.e., 5000 periods after the initial vote) and demonstrates clear negative relationship between income and progressivity. A similar pattern appears in every voting period. In fact, the bottom 81% of households by income always vote for the most progressive tax policy. The economic logic for why income and preferred progressivity are negatively related is a recurrence fo the lesson from previous static models: low-income households prefer a more progressive tax because it imposes a lower net tax to them; high-income households oppose progressivity because it raises their net tax. This logic however only captures part of the story. From the logic above, one would expect that the distribution of preferred tax policy across income would be degenerate. There would be an income level below which all households would want the maximum progressivity in the policy space and above which all households would want the least progressive choice available. Figure 11 however, shows that there is a wide range of income over which households want intermediate levels of progressivity. In some cases, even households with same level of income vote for considerably different policies.

The reason that some households vote for intermediate progressivity levels is that they can differ dramatically in the way that they income is composed. Households with low labor productivity will, in the long-run, have income derived almost entirely from capital. Meanwhile, those with high labor productivity will have very little (or zero) capital income. Because tax policy affects savings incentives, and therefore equilibrium ratio of capital to labor input, it alters the prices paid to capital and labor. Since a more progressive policy leads to a higher capital stock, it also increases the aggregate wage and decreases the rental rate for capital because labor is supplied inelastically. Households that earn most of their income from labor gain from the change in factor prices. Figure 12 plots preferred tax policy across ε for several income levels. All the lowest income households vote for very high progressivity regardless of productivity level. The direct effect of tax policy on net transfers dominates for them. Among higher income levels, preferred progressivity is increasing in ε. At these income levels, the net tax effect still dominants for the low productivity households, however at sufficiently high levels of ε, the positive effect of progressivity on wages leads them to vote for more progressive policy.

6.2.2 Off the equilibrium path

One advantage of using the approach in this paper is that the voting rule $\phi^i(k, \Gamma, \phi)$ is uncovered in the computation. This rule reveals how a household would vote in states of the world which do not appear along the equilibrium path. Figure 13 shows preferred ϕ' as a function of k for three households from the same β-group but with different productivities. Because ϕ^i has a high-dimension, the values of the other states are chosen from those corresponding to the nearest gridpoints to their long run equilibrium values. Preferences are nearly bang-bang. For each type, there is a narrow interval $[k_{low}, k_{high}]$ such that it prefers maximum progressivity for all wealth levels below k_{low} and minimum progressivity for wealth above k_{high}. Within $[k_{low}, k_{high}]$, the preferred ϕ' decreases sharply in wealth. As ε increases, k_{low} and k_{high} decrease so that higher productivity types stop supporting maximum progressivity at lower levels of wealth.

The impact of the discount factor, β, on preferences for ϕ' as a function of wealth is small. Despite nearly identical preferences for ϕ', two households with the same ε may vote for different ϕ' because they differ in wealth. Given two households with the same ε, the less patient household will generally have lower income and therefore less wealth.[29] In some cases, this wealth difference may be large enough so that the low-β household always votes for the highest level of progressivity while the high-β household always votes for the lowest value.

[29] It is possible for two households to have the same income level in the long run despite having different discount factors. This would happen if both households have zero wealth.

As for other state variables, current progressivity, ϕ, has an quantitatively negligible effect on the preferences of households. For greater values of R, if a household supports high progressivity at some levels of individual wealth, the wealth level for which that household begins to favor lower levels of progressivity increases.[30] K has a small effect on households' policy preferences, and the direction of effect depends upon how concentrated total income is in a single factor. For households who earn income almost exclusively from labor (capital) an increase in K tends to increase (decrease) the preference for progressivity. In a capital abundant environment wages are relatively high meaning that households with high labor productivity will be relatively income-rich, inducing them to vote against more progressive income taxes. By the same logic, households with low labor productivity (and thus highly concentrated in capital income) will be relatively income-poor and thus value progressivity.

6.2.3 Net taxes as a predictor of votes

Most of the literature on the demand for income tax progressivity has focused on the distribution of net tax burden (taxes less transfers) to predict policy. The idea is that a winning policy must induce a reduction in tax burden for a majority of voters. In this model, that intuition does not hold up for every household. Although it is true that all households with negative net tax bills support the highest degree of progressivity, these households do not form a decisive majority on their own. Other households join them in voting for high progressivity despite facing a positive tax bill. These other households have low, but positive, net taxes and a high concentration of total income from labor.

The best analog to the prediction from the static literature, however, is between the net tax bill under the equilibrium policy and the net tax bill from a one-time deviation in progressivity. Viewed this way, the net tax burden does a good job of predicting the winning vote and a decent job of predicting individual votes. To see this, the following counterfactual is run. For each household, compare the net tax bills resulting from two different one-period deviations The first deviation is to $\phi' = \phi^*$, the household's preferred policy, and the second is to $\phi' = 0.2$, the lowest degree of progressivity in the policy space. To do this, the tax bill under both policies is calculated assuming that income does not change in one period[31]. The counterfactual transfer under each policy is derived using the equilibrium law of motion for government revenue. Given the tax bill and transfer for each policy, the net tax bills can be calculated.

[30] Since R does not clear the government budget constraint at the grid points, contemplating alternative R amounts to uncoupling tax progressivity and tax revenue. Therefore, households can reduce their tax bills by imposing higher progressivity without affecting their redistribution.

[31] While it would be best to allow income to be endogenous for these counterfactuals, it is not possible to do so. Fixing these income values for a one-period deviation should induce only a small error since the counterfactual policy under consideration is not too drastic a change and one period is not long for income to evolve.

If every household votes for the policy which yields the lowest net tax bill to them, then for any household,subtracting the net tax bill under its preferred policy from the smallest net tax bill under either $\phi' = 0.2$ or $\phi' = 0.8$, should yield a non-negative number. In other words, if the intution from the static literature holds up, then the net tax bill under the net tax bill can be no larger than that under some other policy. Figure 14 displays the absolute difference between the net tax bill with $\phi' = \phi^*$ and the net tax bill under the best alternative policy (i.e., whichever ϕ' from the set [0.2 0.8] leads to a smaller net tax bill) plotted by labor productivity. The blue circles mark households whose votes are consistent with the static literature's prediction. These household's vote for a progressivity level that yields a net tax that is no greater than that under the best alternative policy. For many of these households the difference is zero, meaning that their preferred policy and the best alternative are identical. The red diamonds are households who would have a lower net tax if $\phi' = 0.2$ but vote for a $\phi' > 0.2$. These households have labor productivity above the mean (2.03) though they are not the most productive. Nevertheless, they have a high concentration of total income from labor, and so the wage benefit of higher progressivity alters their votes. Finally, the green squares are the counterparts to the red diamonds. These households would enjoy a lower net tax if $\phi' = 0.8$, but vote for less progressivity instead. They have lower than average labor productivity and a high concentration of income from capital. Because the rental rate declines with progressivity, they vote for $\phi' < 0.8$. Despite failing to correctly predict the votes of these households, the net tax bill does a good job of predicting the winning policy. The blue circles account for 95% of the households in the model.

6.3 Extensions

6.3.1 Fixed Factor Prices

While the impact of factor prices on household decisions was discussed in the baseline model, it is illustrative to point out the quantitative differences between the baseline and an alternative case in which factor prices are fixed. Table 5 highlights some differences between the two economies. Because the economy is initialized with a greater capital stock than emerges in the long run with voting, the rental rate is a bit lower in the fixed prices case. The first difference is that with fixed factor prices, progressivity receives even more support than in the baseline. As shown in Carroll and Young (2011), in this model environment, a higher rental rate implies higher long run income for every household with positive wealth, given the same marginal tax function. The reason for this rests in the steady state Euler equation (25). A higher rental rate reduces the LHS of (25). Equality can only be restored if the long run marginal tax rate is higher, or equivalently, long run income is higher. All households with positive wealth are necessarily poorer in the long run when prices are fixed. Because policy cannot alter future factor prices, only the direct impact of progressivity on the net tax bill matters, and so these poorer households show more support for progressivity.

Aggregate activity is lower overall. Table 6 reports these values for the experiment along with those from the baseline The long run capital stock and aggregate income are only 83% and 95%, respectively, of their baseline values. The transfer is also reduced in the fixed price case as well. Both wealth and income inequality are a bit higher. Figure 15 compares the transition path under fixed factor prices to that under the baseline. The dip portion of the transition path is only noticeable after a very long period time. Without price corrections to incentivize savings and speed a recovery in the capital stock, it takes a very long time for low-to-middle income households to build up aggregate wealth.

6.3.2 Alternative γ

As a robustness check, the voting experiment is run for other values of intertemporal elasticity of substitution. Results show that it has no impact on the long run levels of income or capital. More importantly, it does not change the voting outcome in any period; the most progressive policy is preferred in every vote. Furthermore it does not change the basic path of transition. There is a "dip" period followed by a smooth rise to the new steady state capital stock level. The rate of transition to the new steady state, however, is different. Generally, as γ increases the capital stock adjusts more slowly, reaching the bottom of the dip earlier and emerging sooner as well.

7 Conclusion

This paper has examined the popular choice for progressivity of income taxation in a neoclassical growth model. Within a model that is calibrated to well-estimate the distributions of wealth and income in the United States, there is strong support for progressive income taxation even accounting for its effects on future income distributions and tax revenues. In the long run support for a high level of progressivity comes from a coalition of low-income households and middle-income households whose income is primarily derived from labor. Although the distribution of net tax bills predicts the winning policy, for some households the impact of progressivity on factor prices causes them to vote for policies which increase their net tax bills.

In contrast to much of the other dynamic models of progressive income taxation, income tax progressivity in this model leads to a slightly higher long run capital stock due to the increase in savings by low- and middle-income households. Switching from a ceiling to an increasing marginal tax for rich households greatly reduces income inequality from initial levels, but once this change is accounted for higher progressivity leads to only a small reduction in both income and wealth inequality.

The model presented here only begins to scratch the surface of the political economy of progressive income taxation within a model with savings. One potentially interesting research topic would be to include some income uncertainty to the environment. Carroll and Young (2011) shows that the long-run consequences of progressive tax reforms are qualitatively different in environments where wealth and income heterogeneity result from permanent differences in preferences and labor productivity from other results in the literature where heterogeneity arises from uninsurable income shocks. Adding such shocks to the model here would reduce the sensitivity of savings to taxation since households would have a precautionary saving motive as in Aiyagari (1994) so the impact of policy on factor prices could be quite different. Uncertainty would also create another reason for a below-average income household to vote for progressive taxation: social insurance. If such a household faced an earnings shock process with a reasonable degree of persistence, then its income is likely to be low in the future as well. Without a redistribution mechanism, this household would self-insure to prevent very low future consumption. If this household were the pivotal voter, it would have a desire to reduce self-insurance through social insurance by redistributing income. It is likely that the structure of a progressive tax would make social insurance even more attractive to this household since unlike a proportional tax, an increase in progressivity yields larger transfers (though perhaps only in the short run) without increasing the pivotal voter's tax rate.

In this work, it has been assumed that progressive taxation can be described by just one policy variable. This is sufficient for examining the degree of progressivity desired by a population, but it greatly restricts the shape of policy. One interesting phenomenon in the tax schedules of OECD countries has been a general decrease in the tax rate on the highest income brackets. Studying the implications for this sort of reform would require allowing agents to vote over the degree of progressivity within income brackets. Within the framework studied here, a flat tax on the highest income earners would allow for much greater wealth accumulation among income-rich households so that higher transfers could potentially be feasible in equilibrium. Tax policy would be at least two-dimensional so restrictions would almost certainly need to be imposed to find equilibria. There are two strategies to deal with this complication. The first would separate the vote into two unidimensional contests.[32] The order in which the vote occurs could be fixed or determined randomly. Although it is unlikely that a political equilibrium would be invariant to the ordering of the votes, it could be useful to examine how "close" the outcomes are under some distance measure. Another strategy would abandon the Condorcet winner concept all together and look instead for sets of policies which have a greater likelihood of winning pairwise elections. Careful study of the size and scope of policy within the *uncovered set* may yield

[32]For an introductory discussion of "agenda setting" as a solution to the problem of multidimensional conflict see Persson and Tabellini (2000) pp. 37-38.

insight into what sorts of policies are likely to arise in multidimensional conflicts.[33][34]

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[33] Given x, y, and z elements of the conflict space and the common preference relation $\succ$, y *covers* x if the following is true:

$$z \succ y \Rightarrow z \succ x.$$

Members of the uncovered set then are elements for which this condition is not true. For a more complete examination of the properties of covering sets see (Miller 1980)

[34] Dolmas (2008) allows households to vote simultaneously over average consumption, labor income, and capital income taxes in an AK production model. The uncovered set is very large. In particular, it contains half of the OECD schedules studied.

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Appendices

A Algorithm for Solving Recursive Politico-Economic Equilibrium

Due to the large size of the model, a hybridization of the computational algorithms of Krusell and Smith (1998), Krusell and Ríos-Rull (1999), and Corbae, D'Erasmo, and Kuruscu (2008) is used. In order to solve the household's savings decision, tomorrow's prices, r' and w', the next period transfer, T', and the tax policy two periods in the future, ϕ'' must be known. These values depend directly upon the current distribution of wealth, Γ, however the high dimensionality of Γ makes using this state variable computationally unfeasible. Instead this paper develops a deterministic hybrid of Krusell and Smith (1998).[35] As in Krusell and Smith, the key idea is to approximate Γ with a finite set of moments. The moments used in this paper are the mean level of capital K and the current period tax revenue, R. Along with the current tax policy ϕ, the laws of motion for these state variables are approximated by a set of log-linear equations.

$$\log\left(K'\right) = a_0 + a_1 \log(K) + a_2 \log(\phi) + a_3 \log(R) + a_4 \log\left(\phi'\right) \tag{26}$$

$$\log\left(\phi'\right) = b_0 + b_1 \log(K) + b_2 \log(\phi) + b_3 \log(R) + b_4 \log\left(\phi'\right) \tag{27}$$

$$\log\left(R'\right) = c_0 + c_1 \log(K) + c_2 \log(\phi) + c_3 \log(R) + c_4 \log\left(\phi'\right) \tag{28}$$

The problem to be solved is the following:

$$\tilde{v}\left(i, k, K, \phi, R; \phi'\right) = \max_{k'} u\left(c\left(k'\right)\right) + \beta_i v\left(i, k', K', \phi', R'\right)$$

The algorithm proceeds in the following manner:

1. Let $\left\{a_j^n, b_j^n, c_j^n\right\}_{j=0}^{4}$ and $v^n\left(i, k', K', \phi', R'\right)$ be the current guess for the continuation function v and the coefficients to the laws of motion on the n^{th} iteration.

2. Construct grids for k, K, ϕ, R, and ϕ'.

3. For each type i, loop over every combination of K, ϕ, R, and ϕ'.

   (a) To find the value of the continuation function, forecast K' and R' using equations above (26)-(28).

   (b) Linearly interpolate v^n in the K' and R' directions. Fit cubic splines to v^n in k' direction to approximate both the value function and its first derivative. Let these approximations be $\varpi\left(i, k', \phi'\right)$ and $\varpi_{k'}\left(i, k', \phi'\right)$ respectively.

[35] Corbae, D'Erasmo, and Kuruscu (2008) also use a similar method.

(c) Find k'^* such that either

$$u'\left(c\left(k'^*\right)\right) = \beta_i \varpi_{k'}\left(i, k'^*, \phi'\right)$$

is satisfied or $k'^* = 0$.

(d) Step 3(c) returns an array $q\left(i, k, K, \phi, R; \phi'\right)$ and $h\left(i, k, K, \phi, R; \phi'\right)$, the value function and savings decision, respectively, under policy ϕ' for a household of type i given k, K, ϕ, and R.

4. Fit cubic splines to q in the ϕ' direction.[36]

5. Find the value of ϕ' which maximizes the cubic approximation to q. This yields $\theta^*\left(i, k, K, \phi, R\right)$ the preferred tax policy choice of a type i household given the states k, K, ϕ, and R.

6. Using the rules, $h\left(i, k, K, \phi, R; \phi'\right)$ and $\theta^*\left(i, k, K, \phi, R\right)$ simulate the economy for N periods. Choose N such that the difference between k_{N-1} and k_N is small.

   (a) The initial transfer, T_0, is given from the initial distribution, however future transfers must be found along the equilibrium path. Next period's government budget clearing transfer can be found from the current periods h and θ^*.

   (b) At each vote, order $\theta^*\left(i, .\right)$ from lowest to highest. The equilibrium ϕ' is the value of $\theta^*\left(i, .\right)$ which solves

$$\sum_{\{i:\theta^*(i,.)\leq\phi'\}} \psi_i = \sum_{\{i:\theta^*(i,.)\geq\phi'\}} \psi_i = 0.5. \tag{29}$$

   Because the number of types is finite, these sums will never equal 0.5. To deal with this, I take a weighted average of the value of $\theta^*\left(i, .\right)$ which is closest to 0.5 from below and the one that is closest from above.

7. The simulation returns a sequence $\{K_s, \phi_s, R_s\}_{s=0}^{N+1}$. Run OLS on this data to get new values for the coefficients to the laws of motion, a^{new}, b^{new}, and c^{new}.

8. For some λ_1,$\lambda_2 \in (0, 1]$, update the value function and the laws of motion according to $v^{n+1} = \left(1 - \lambda_1\right) q + \lambda_1 v^n$ and $x^{n+1} = \left(1 - \lambda_2\right) x^{new} + \lambda_2 x^n$ where $x = [a; b; c]$.

9. Iterate on 3-8 until the $\left\|v^{n+1} - v^n\right\|^\infty$and $\left\|x^{n+1} - x^n\right\|^\infty$ are less than some tolerance.

[36] Here is where I check for the single-peakedness of the indirect utility function in the policy direction. For each ϕ'_s on the ϕ'-grid, evaluate $q_{diff} = q\left(\blacklozenge, \phi_{s+1}\right) - q\left(\blacklozenge, \phi_s\right)$. If the sign of q_{diff} changes more than once, then single-peakedness is violated. Note that although single peakedness is never violated in any iteration on the value function for the experiments reported in this paper, it is only necessary that single-peakedness be satisfied for the converged value function, saving rules, and laws of motion.

B Initialization Method

The goal is to back out preferences β_i and labor productivity ε_i from household level data on income and wealth by using the steady state Euler equations and the definition of income. In this way, one may calibrate β_i and ε_i so that the long-run distribution from the model closely approximates the data. Since there are 3906 households in the 1992 SCF, it is not computationally feasible to assign a type in the model to every household in the data. The distribution in the data is then "coarsened" by reducing the number of types to 51. In addition to Table 1, Tables 7 and 8 show that key features of the SCF distribution can still be captured by this coarse approximation. The initialization steps used for the numerical exercise are presented below.

1. Let $Y = 1$, $K = 3$, $\delta = 0.05$, and $\alpha = 0.36$.

2. $$N = \frac{(Y - rK)}{w}$$

   and

   $$w = (1-\alpha)\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha-1}},$$

   imply that

   $$r = \alpha\left(\frac{K}{N}\right)^{\alpha-1} - \delta = 0.138.$$

3. Guess the tax function parameters (ξ, ϕ) and the long run transfer, T.

4. Fix a range of income and wealth values over which to place grid points and partition the income interval and the wealth interval into n_y and n_k segments, respectively. While it is permissible to make these grid points evenly spaced, because of the skewness of the data a better approximation can be achieved by bunching more grid points at the lower ends of the intervals. This paper uses the function

   $$z_{i+1} = z_i + \exp\left(c + \frac{d * i}{n}\right)$$

   where c and d are constants and n is the number of grid points. For income set $c = -1.2$ and $d = 9.5$, and for wealth set c is -0.8 and d is 7.2. 30 grid points in each direction are used. For every combination of income and wealth on the grid, define a rectangular box such that the vertices of the box lie at the midpoints between the current grid point and its four neighbors (2 neighbors in the income direction and 2 neighbors in the wealth direction). For example, let $(x_{y,j}, x_{k,m})$ be the combination of the j^{th} income grid point and the m^{th} wealth grid point. The box assigned to this point would have vertices

$$\left(\frac{x_{y,j}-x_{y,j-1}}{2},\frac{x_{k,m}-x_{k,m-1}}{2}\right),\ \left(\frac{x_{y,j}-x_{y,j-1}}{2},\frac{x_{k,m+1}-x_{k,m}}{2}\right)$$
$$,\ \left(\frac{x_{y,j+1}-x_{y,j}}{2},\frac{x_{k,m}-x_{k,m-1}}{2}\right),\ and\ \left(\frac{x_{y,j+1}-x_{y,j}}{2},\frac{x_{k,m+1}-x_{k,m}}{2}\right).$$

Add the population weights from the SCF data of each household whose income and wealth fall inside the box and assign that weight to the grid point.

5. Normalize the type weights so that $\sum_{i=1}^{n_y * n_k}\psi_i = 1$. To reduce computational load for the model, if $\psi_i < 1.0 * 10^{-8}$ reset $\psi_i = 0$. Let the number of types with non-zero weight be $n_t \leq n_y * n_k$. Normalize the grid points for income and wealth such that $\sum_{i=1}^{n_t}\psi_i x_{y,i} = 1$. $\sum_{i=1}^{n_t}\psi_i x_{k,i} = 3$.

6. Because wealth in the data may be composed of many types of assets each yielding a different return, while the model has only one asset, it is possible for some wealth levels in the data to imply negative income at r. To avoid this, these observations are removed. 62 households are eliminated by this condition.

7. Check that $\sum_{i=1}^{n_t}\psi_i\frac{\tau(y_i)}{y_i} = 0.132$, $\sum_{i=1}^{n_t}\psi_i\tau_y(y_i) = 0.227$, and that $\bar{T}+G = \sum_{i=1}^{n_t}\psi_i\tau(y_i)$. If so then go to step 8, else update $[\xi,\phi,T]]$, and return to step 4.

8. Iterating on steps $(5-7)$ returns vectors $\{k_i\}_{i=1}^{n_t}$ and $\{y_i\}_{i=1}^{n_t}$. For each i, solve for β_i and ε_i which solve

$$\beta_i = \left[\left(1-\tau_y(y_i)\right)r+1\right]^{-1} \tag{30}$$
$$\varepsilon_i = \frac{y_i - rk_i}{w\bar{h}} \tag{31}$$

C Accuracy of Approximations to the Laws of Motion

This appendix details a test for the accuracy of the approximations to the laws of motion, (26)-(28). Because the coefficients to these approximations come from running OLS on a deterministic path and because 0.80 is the only realization of progressivity along this path, it seems reasonable to ask how well households evaluate the future effects on capital and government revenue from choosing alternative ϕ' values. To address this question, divide the ϕ'-grid into 20 evenly-spaced points. For each of these 20 alternative ϕ' points, impose that ϕ' value as the first realization of ϕ' and simulate the economy using the decision rules solved from the model.

Then estimate the coefficients for the laws of motion implied by that path. If the approximation used in the model is close then these coefficients should not change much.

The coefficients changed the most when ϕ' is initially fixed to be 0.2. Table 9 compares the coefficient values from the laws of motion in the model equilibrium to those implied by this counterfactual case.[37] The final columns give the maximum absolute percentage error in any period and the average percentage error from using the model laws of motion to forecast the K', ϕ', and R' on the alternative path. By either measure, the errors are very small. These results are interpreted to mean that (26)-(28) do a sufficiently good job of approximating the paths of K', ϕ', and R' in response to a one-period deviation in policy.

[37] The full results are available from the author upon request.

Figure 1: Marginal tax schedules across countries

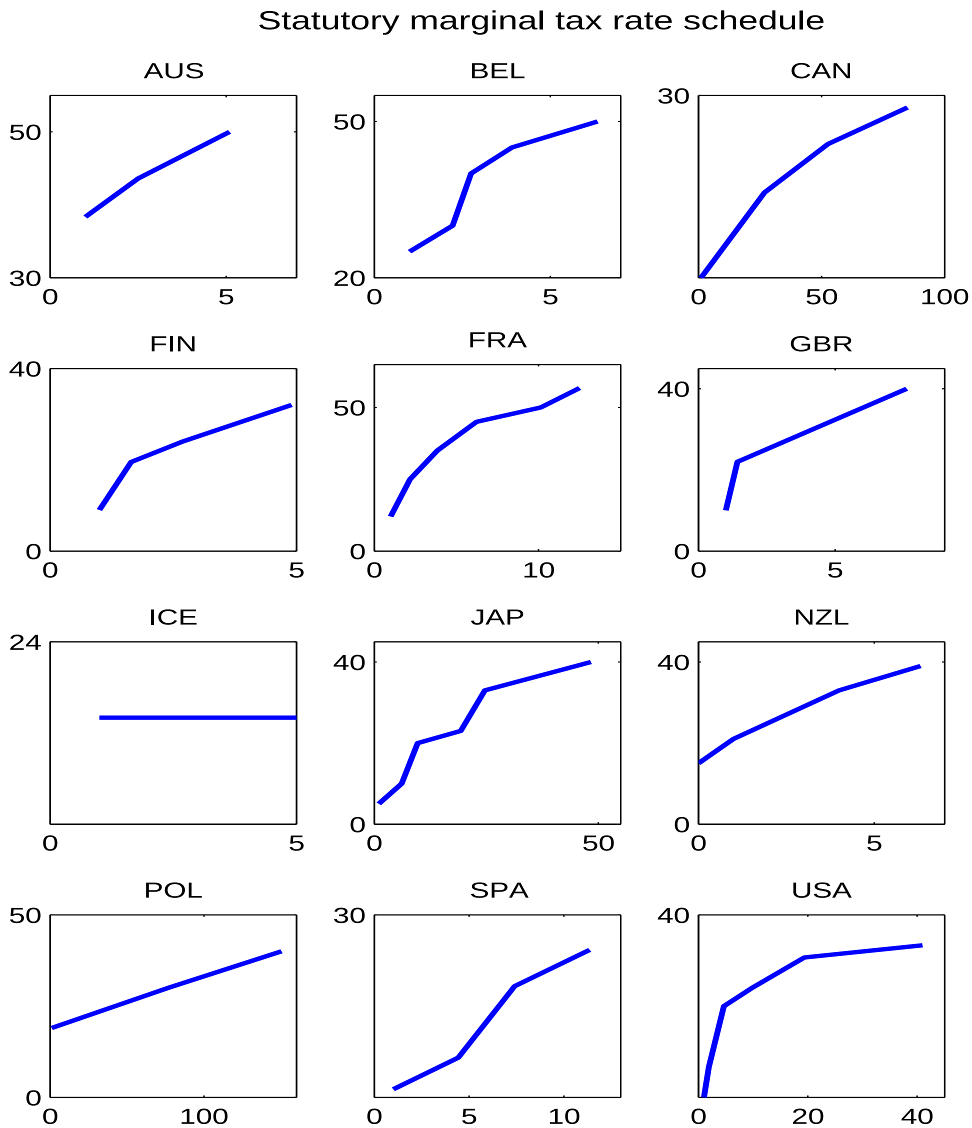

Figure 2: Failure of single-peakedness

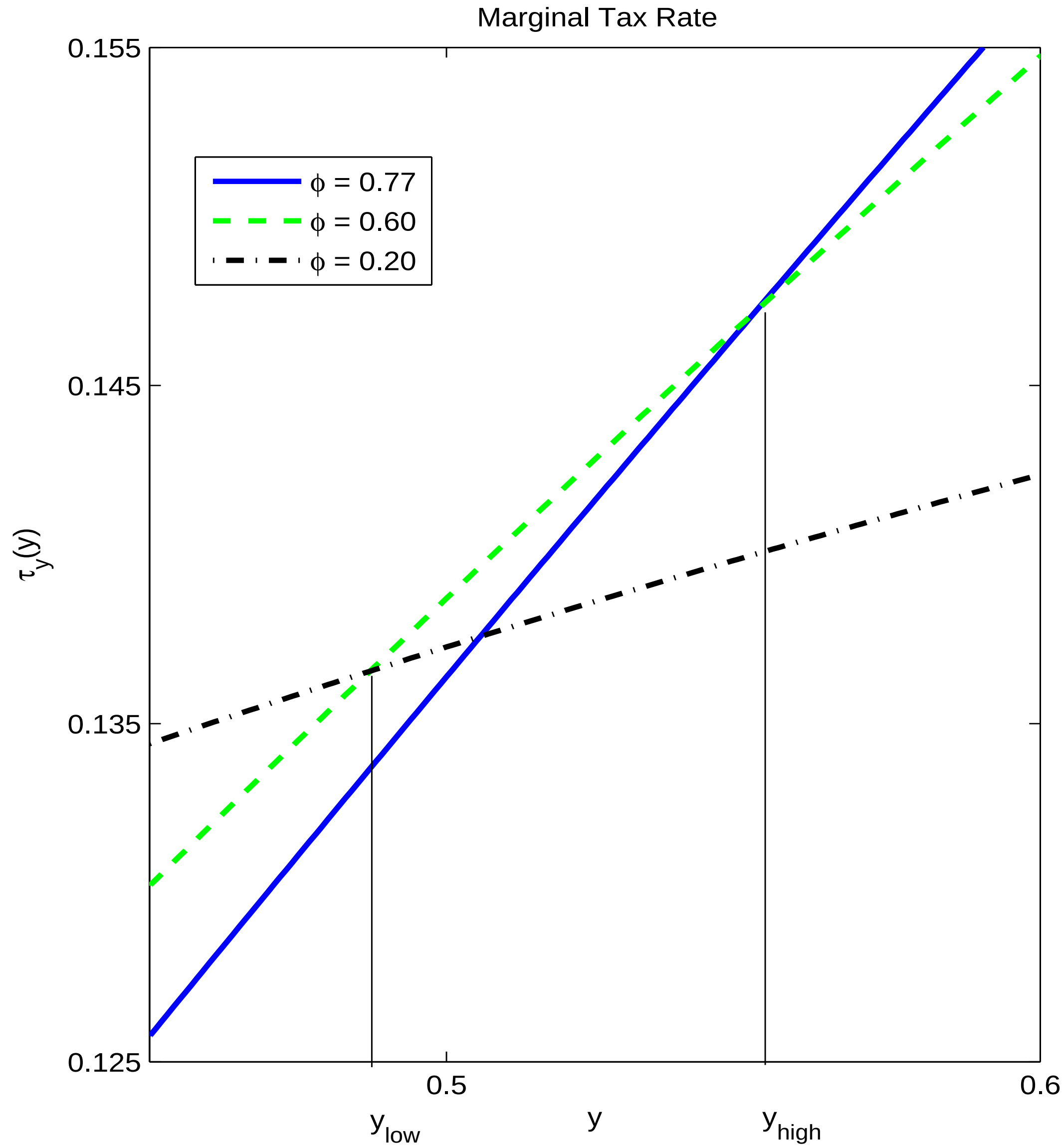

Figure 3: Marginal tax rate

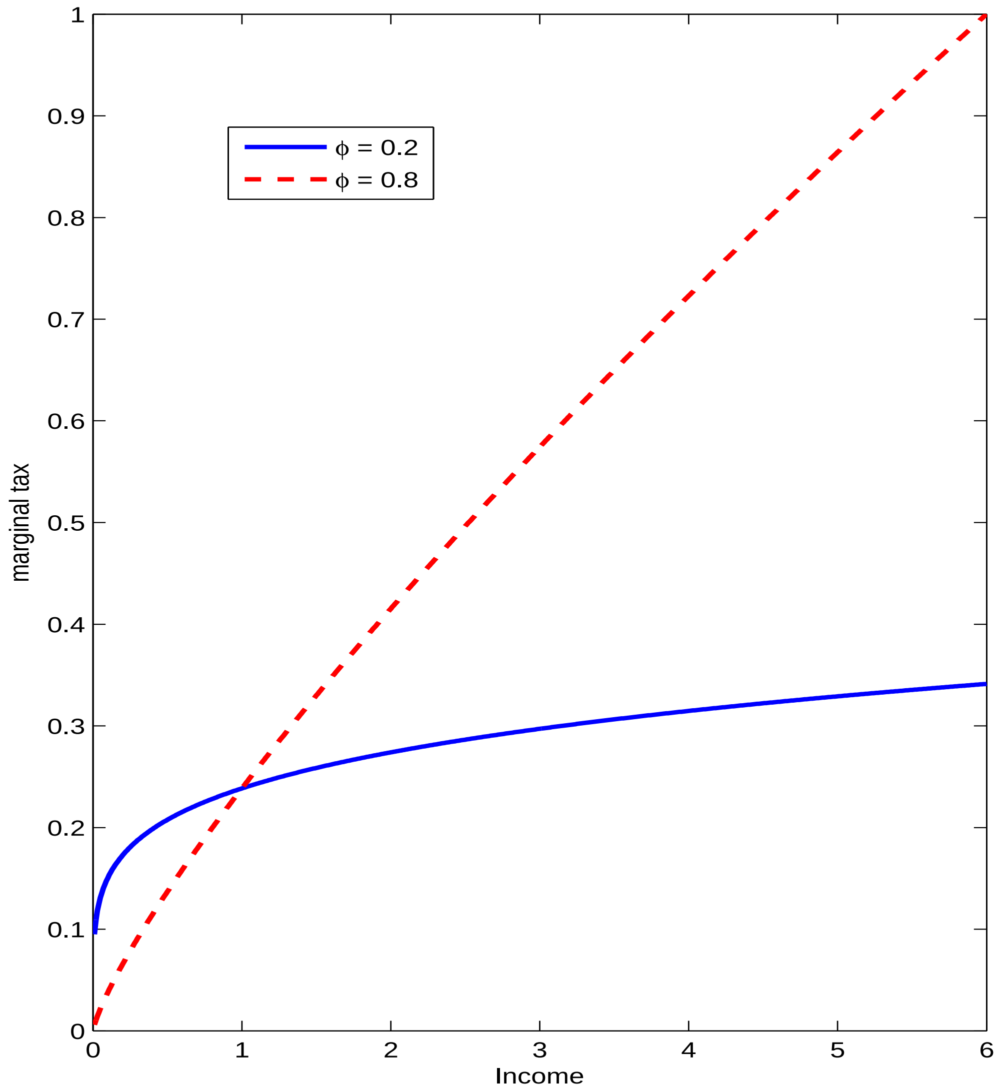

Figure 4: Tax rate and factor price effects for low income household

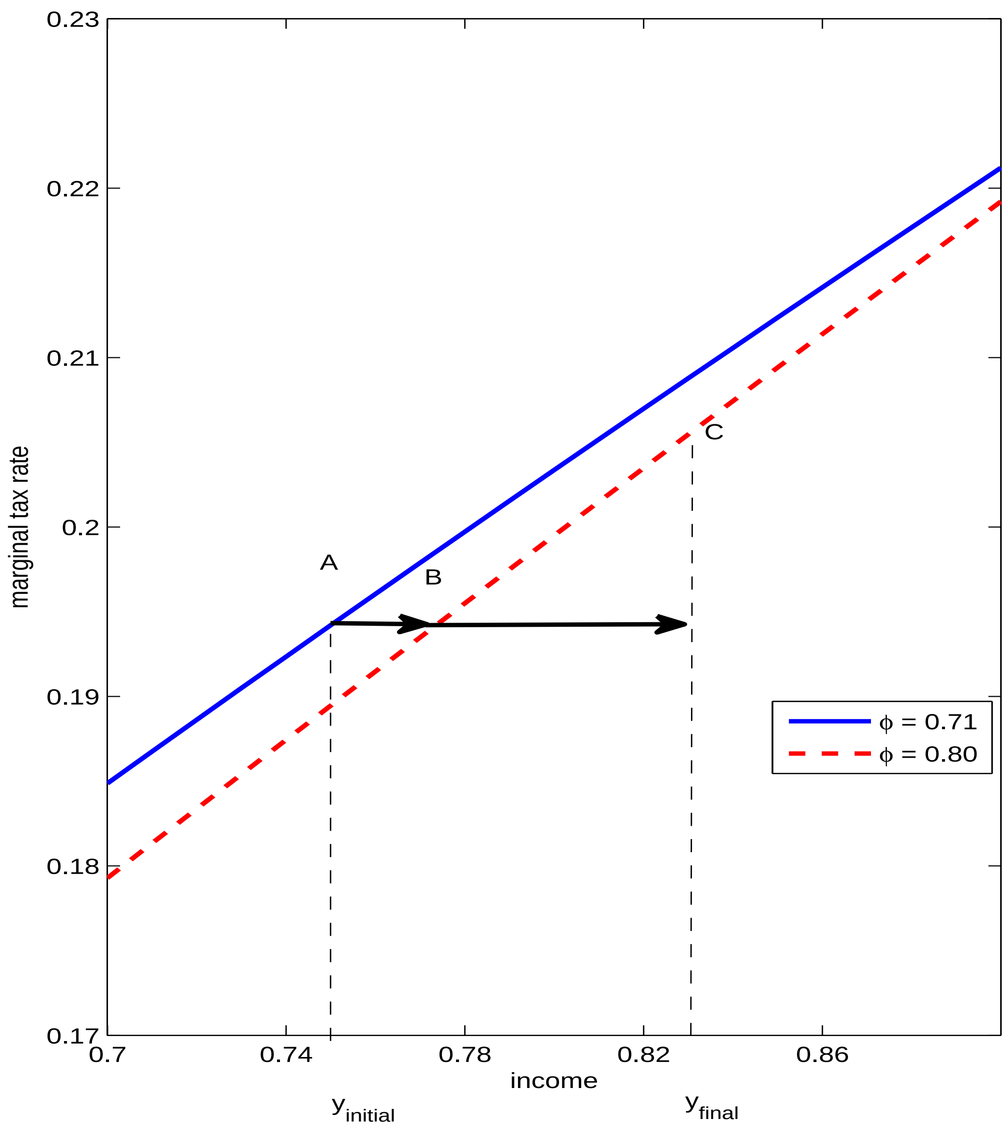

Figure 5: Tax rate and factor price effects for high income household

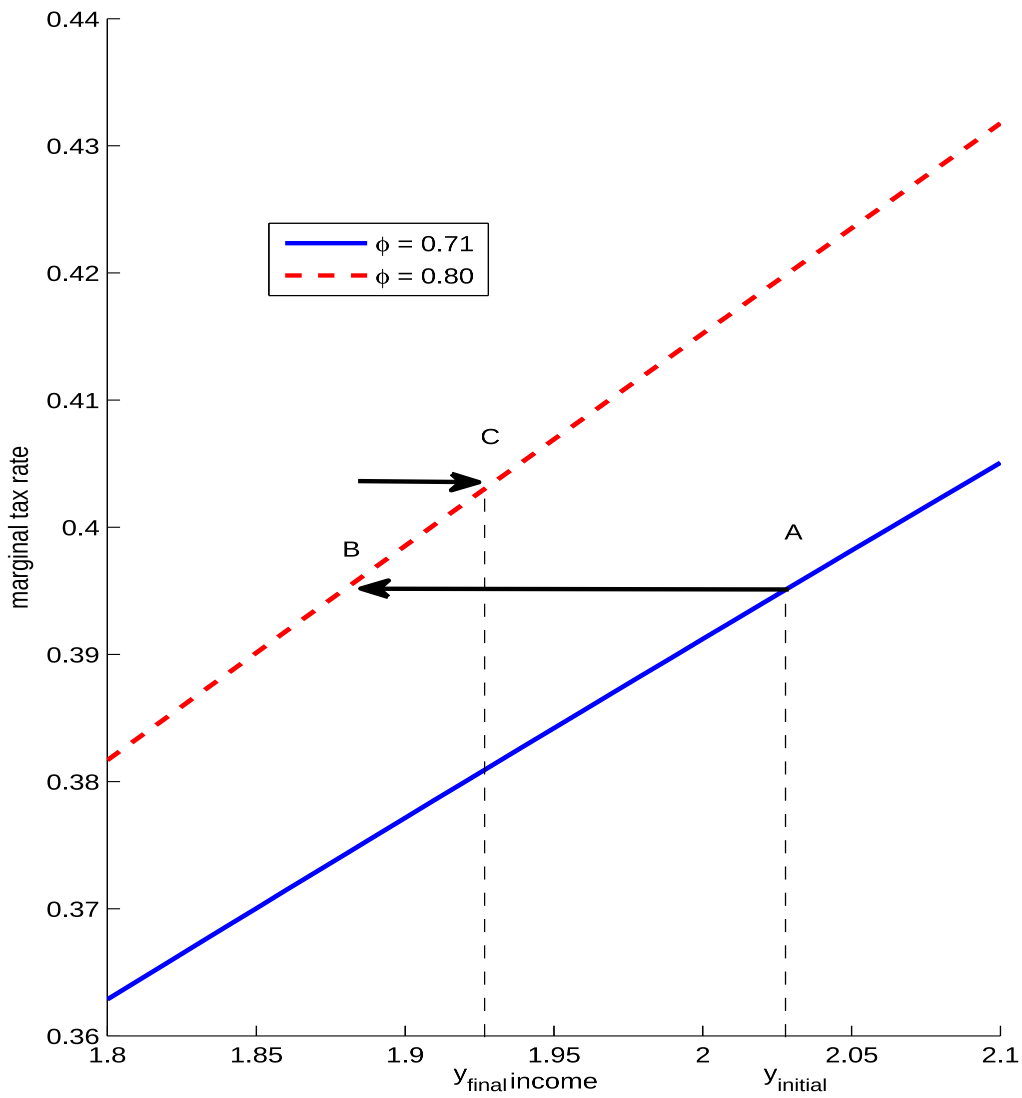

Figure 6: CDF of income

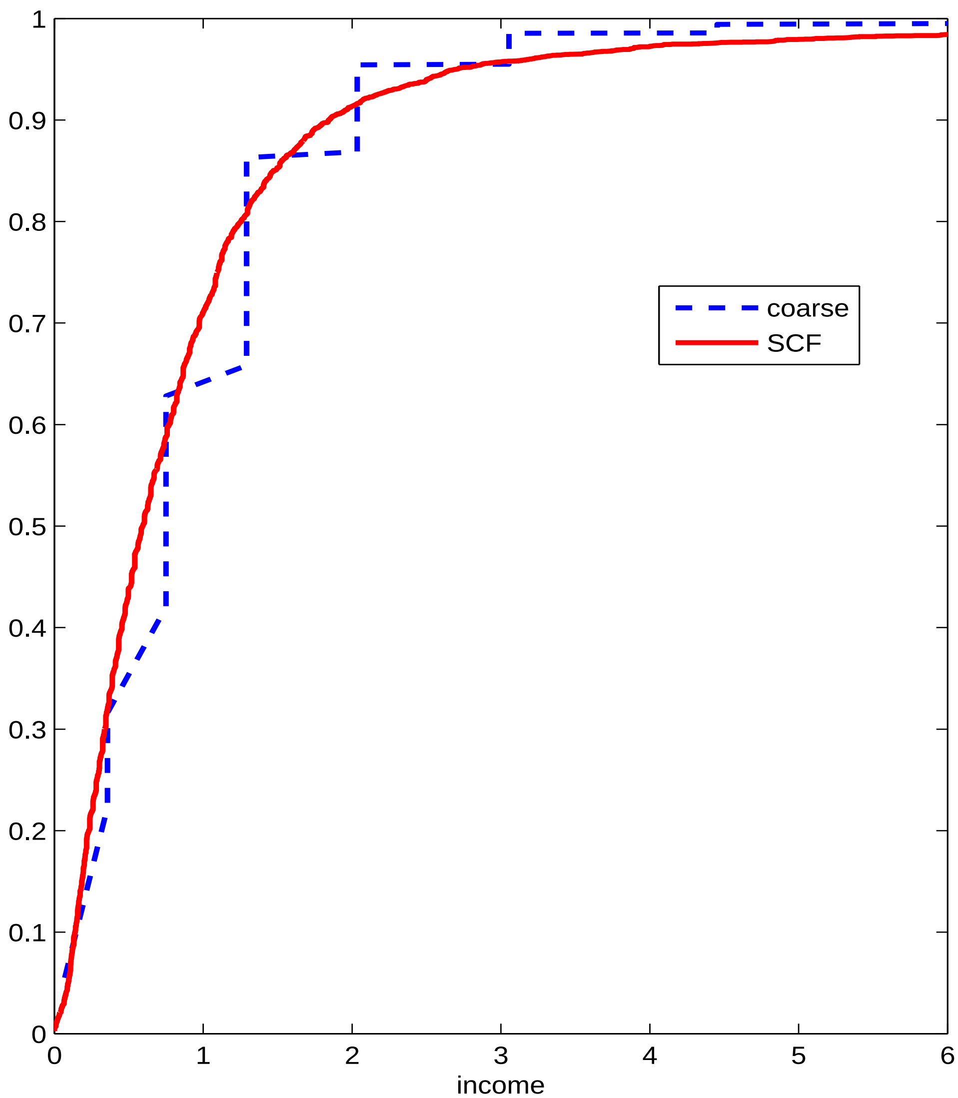

Figure 7: CDF of wealth

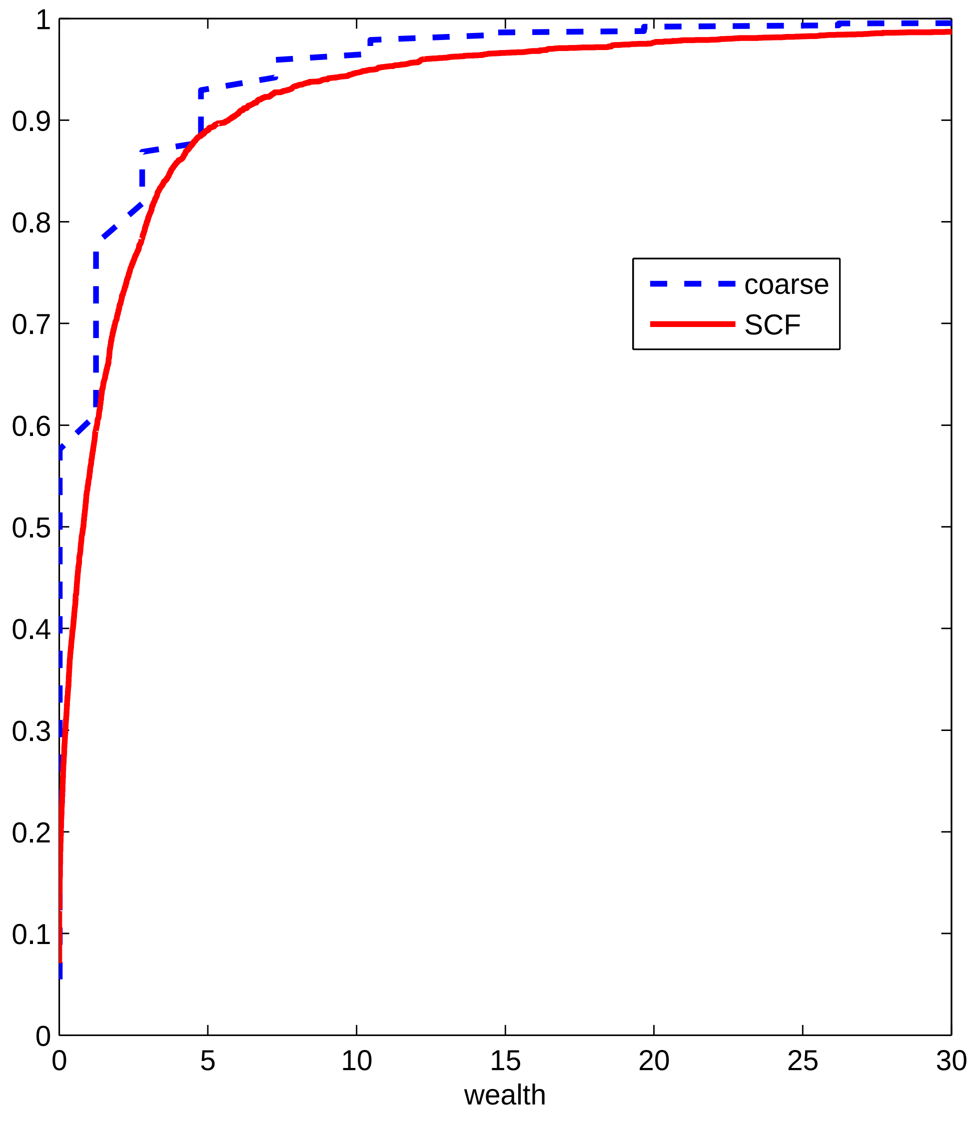

Figure 8: Marginal tax function in calibration

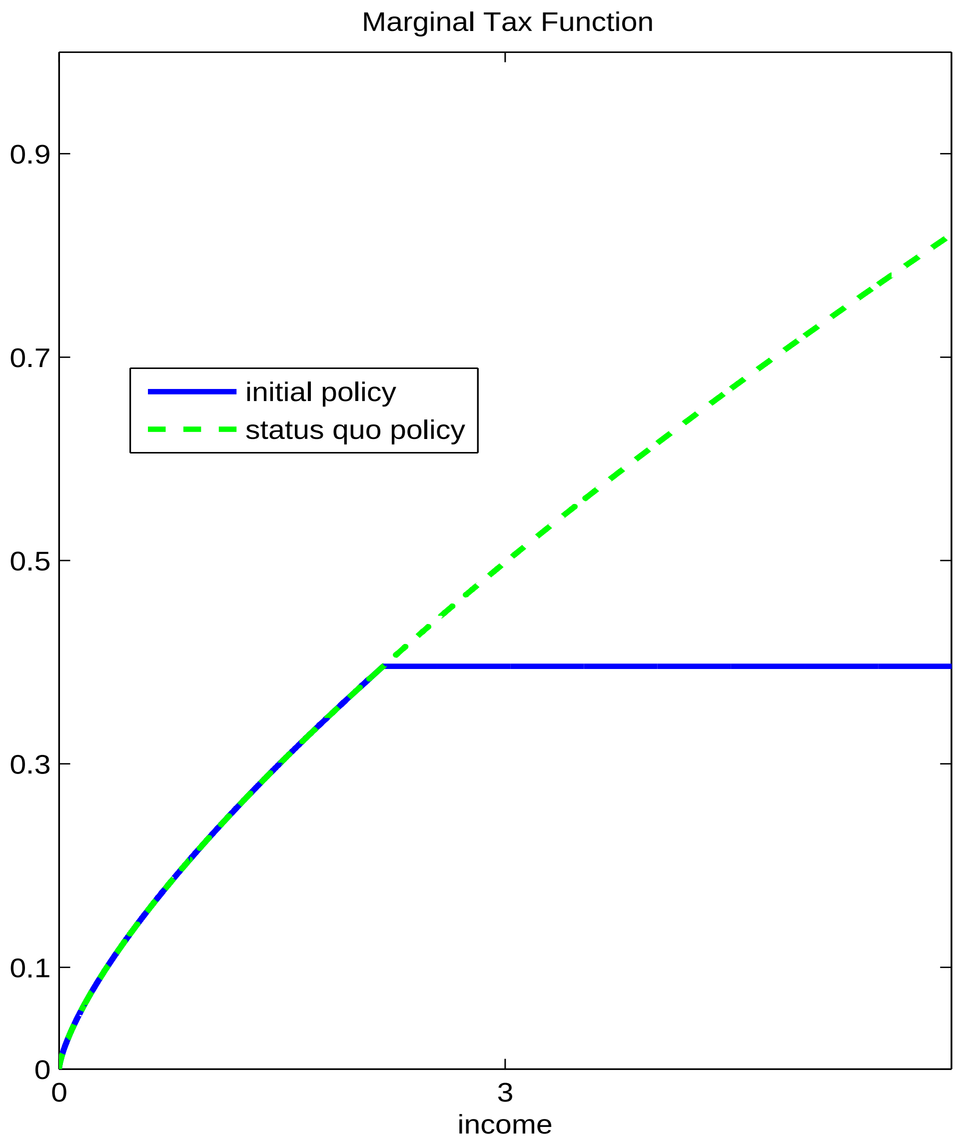

Figure 9: Transition path

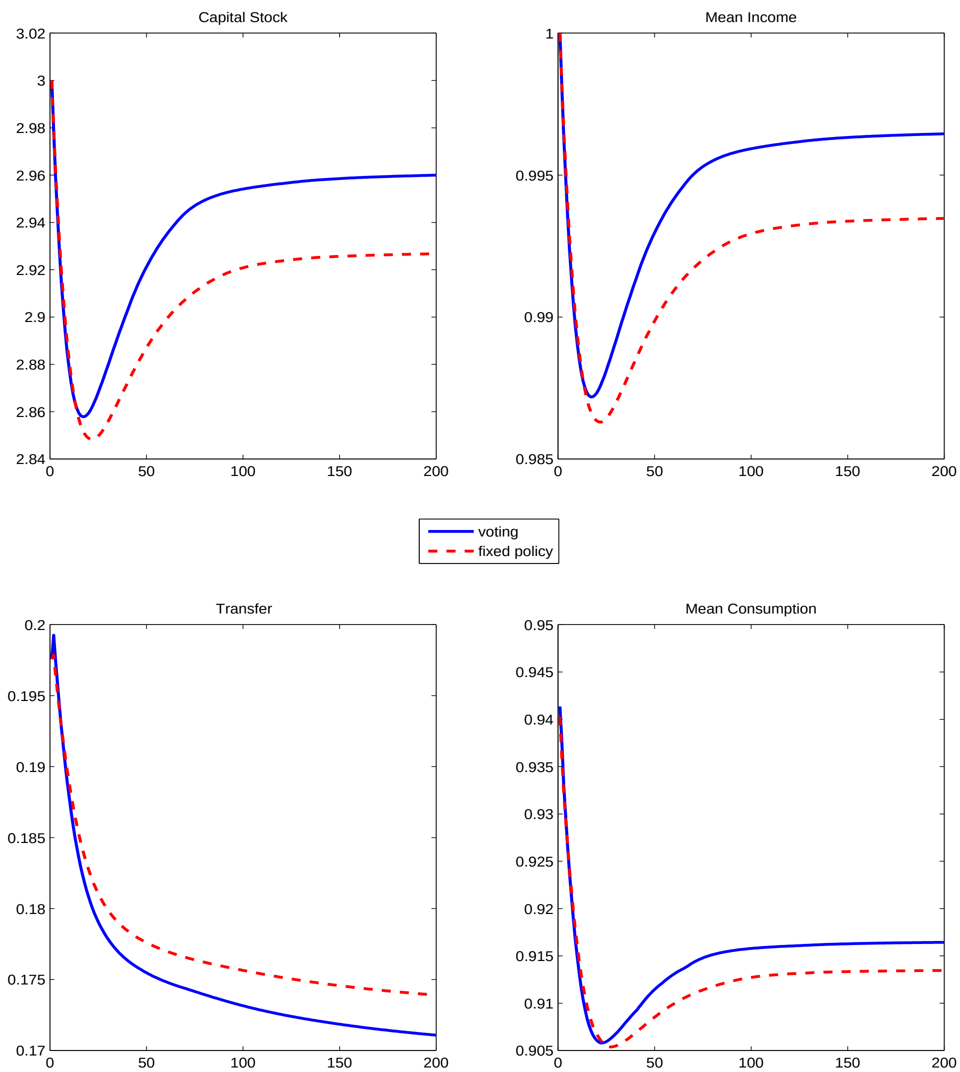

Figure 10: Wealth over transition for two household types

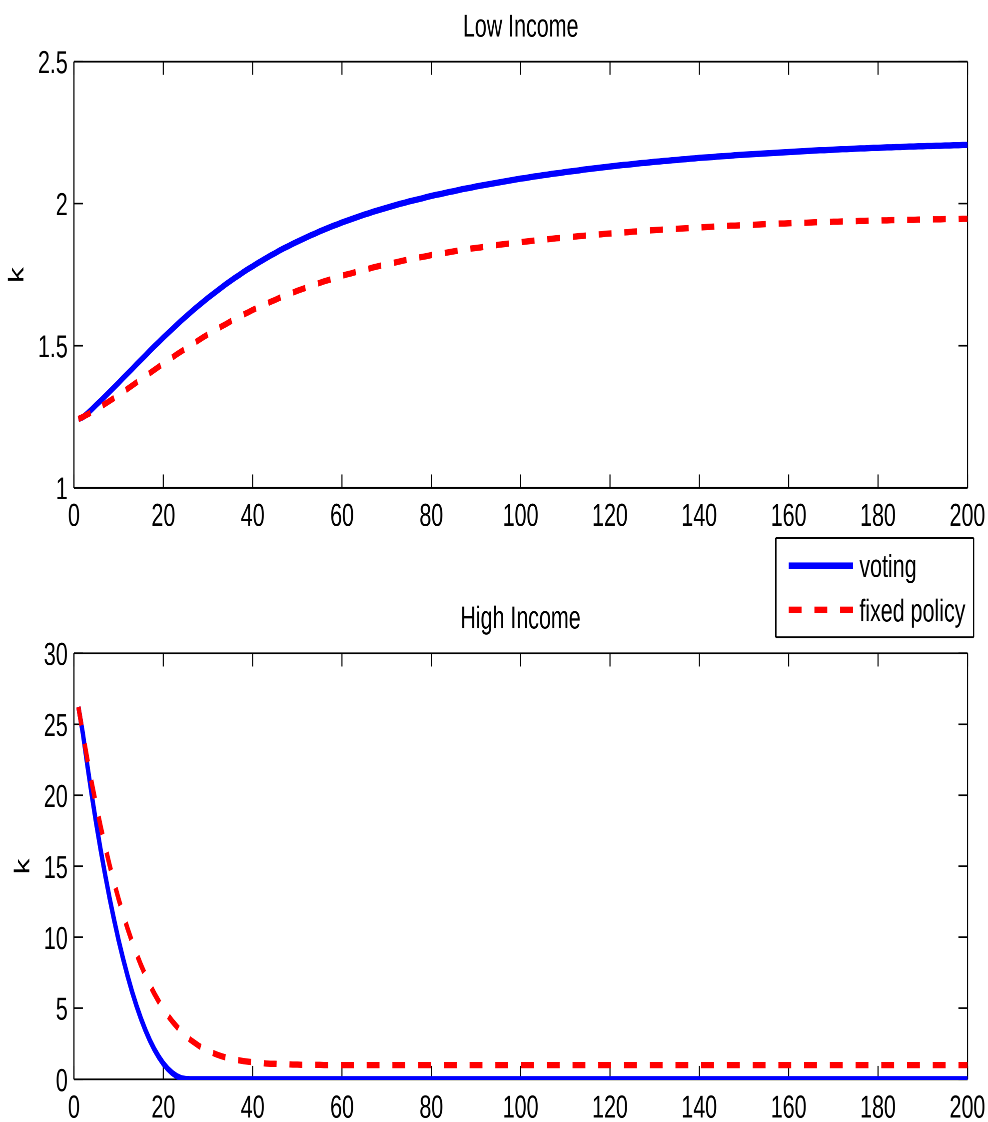

Figure 11: Preferred policy by income

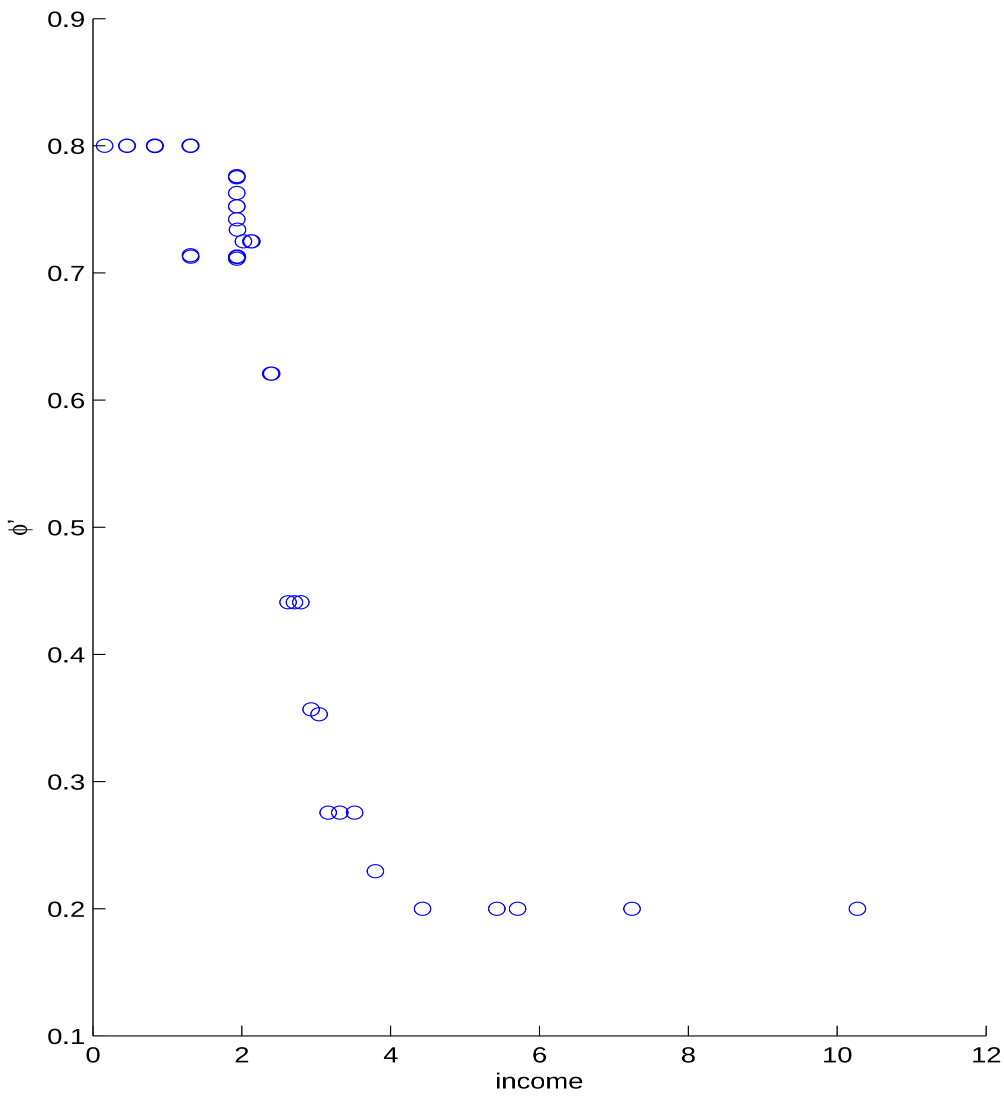

Figure 12: Preferred policy by labor productivity for several income levels

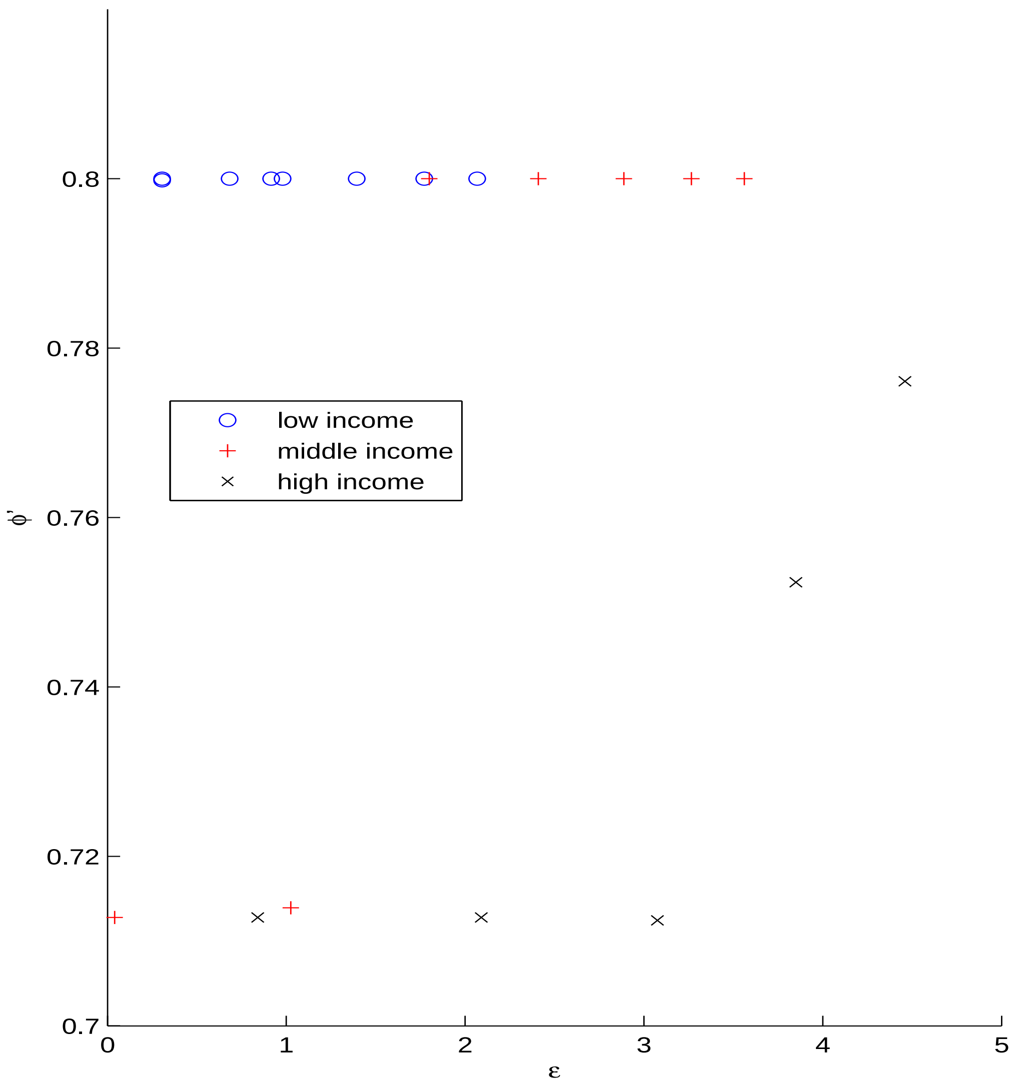

Figure 13: Preferred progressivity across wealth by ε

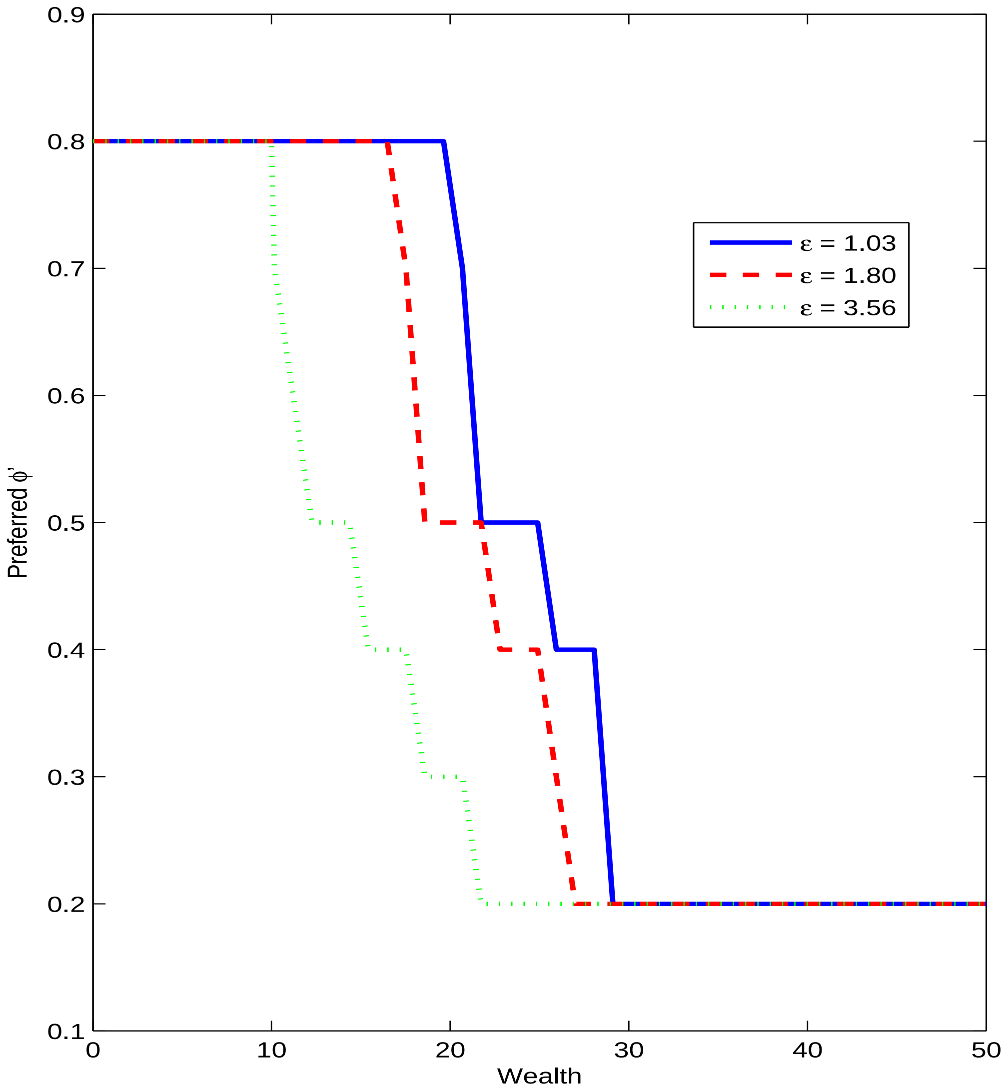

Figure 14: Absolute difference in net tax bill under preferred policy and best alternative policy

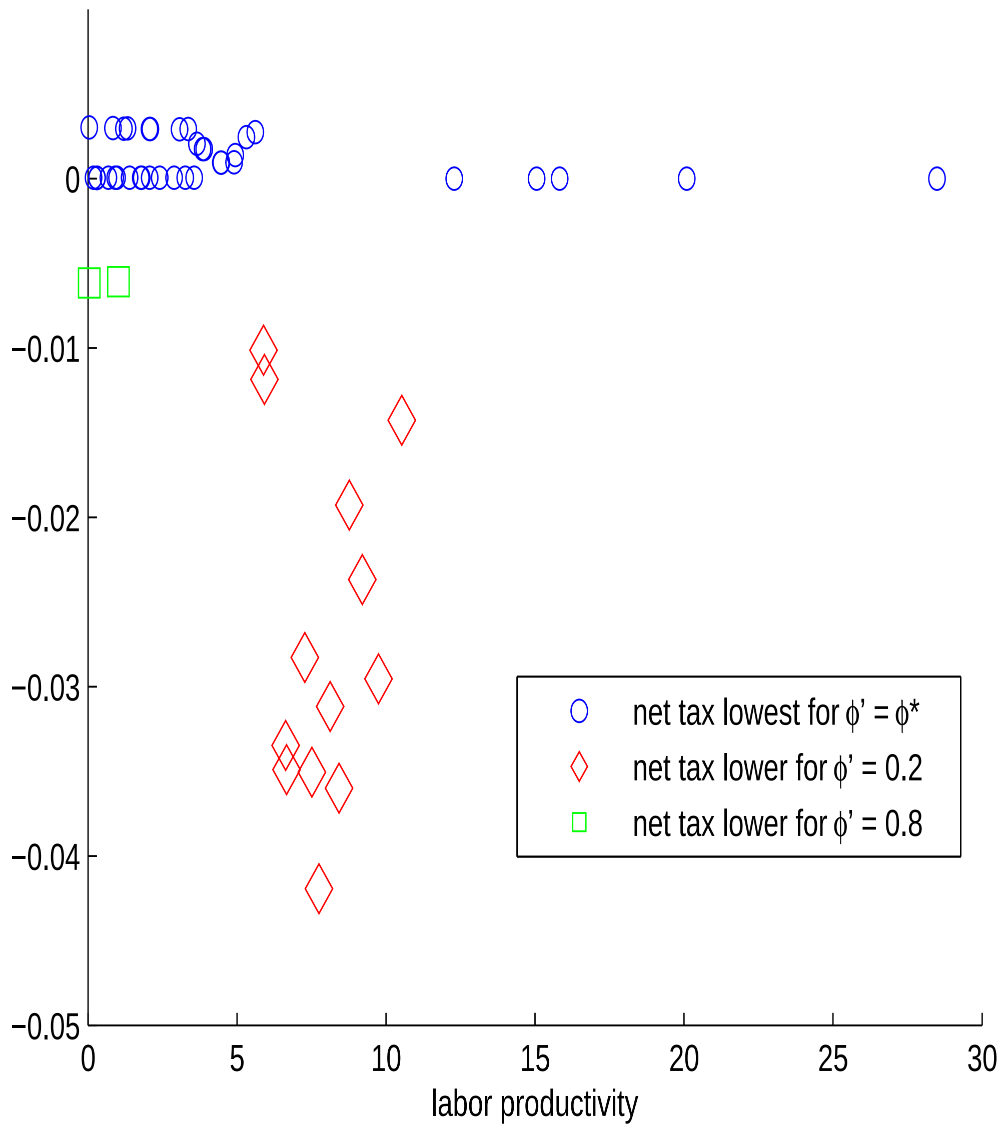

Figure 15: Transition paths of capital under flexible and fixed factor prices

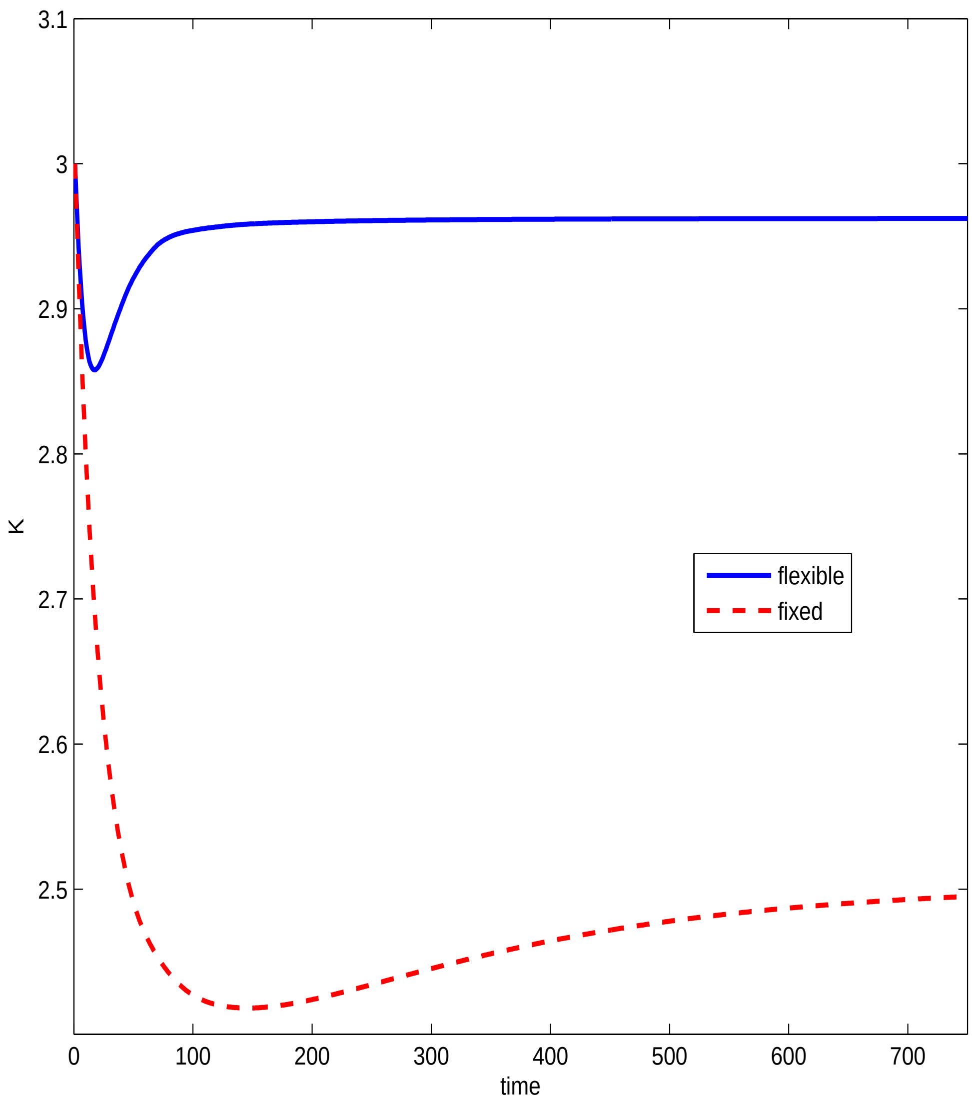

Table 1							
	$\frac{std(y)}{mean(y)}$	$\frac{std(k)}{mean(k)}$	Cov(y,k)	Skew(y)	Skew(k)	Gini(y)	Gini(k)
SCF	1.83	5.96	7.91	47.00	142.03	0.50	0.78
Coarse	0.91	1.97	3.85	4.11	9.52	0.40	0.68

SCF income gini from Weicher (1997). SCF wealth gini from Arthur B. Kennickell, McManus, and Woodburn (1996).

Table 2		
	% Change	
	equil.	fixed ϕ'
ϕ'	12.04	0
K	1.35	0.06
Y	0.35	0.01
C	-0.85	-1.21
T	-12.79	-18.8
avg. tax	-12.3	-11.46
rent	-0.85	-0.03
wage	0.48	0.02

Table 3
Share of Steady State Wealth by Quintile

	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
Initial	0.2	5.8	15.1	12.2	66.8
PE SS	8.6	13.6	19.5	16.4	41.9
FP SS	8.1	12.4	18.1	16.5	44.8

Table 4

	$\frac{std_y}{mean_y}$	$skew_y$	$gini_y$	$\frac{std_k}{mean_k}$	$skew_k$	$gini_k$
Initial	0.91	4.1	0.399	1.97	9.5	0.680
PE SS	0.46	2.9	0.327	23.4	5.2	0.417
FP SS	0.48	2.8	0.335	25.8	5.1	0.433

Table 5										
Percentage changes across initial income distribution by quintile										
	k					y				
Floating r	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
PE SS	$5,410.1$	136.4	32.5	36.5	-36.5	35.6	18.8	8.6	5.8	-13.6
FP SS	$5,001.8$	113.4	20.4	35.6	-32.8	32.9	15.6	5.7	5.5	-12.2
Fixed r	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
PE SS	$3,603.8$	43.1	13.1	-3.7	-78.4	31.8	10.5	4.5	-0.9	-34.1
FP SS	$2,242.0$	3.8	-14.0	-18.7	-77.8	19.8	0.9	-4.8	-4.5	-33.8
	c					$\tau(y) - T$				
Floating r	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
PE SS	22.9	14.2	5.2	4.4	-15.3	-4.2	-5.9	-5.4	33.7	3.6
FP SS	21.4	11.6	2.6	4.1	-13.5	-4.1	-5.9	-5.3	35.3	6.2
Fixed r	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
PE SS	10.4	2.5	-0.5	-3.8	-46.9	-38.1	-53.8	-115.1	38.0	-34.9
FP SS	1.6	-6.1	-8.9	-7.5	-45.4	-42.0	-57.7	-107.7	38.3	-37.6

Table 6: Steady state with fixed prices

	Percentage of Baseline Value
K	84.5
Y	96.0
C	95.6
T	90.2

Table 7

Percentiles of Income Distribution

	5%	25%	50%	75%	95%	99%
SCF	0.09	0.29	0.59	1.09	2.70	8.00
Coarse	0.07	0.36	0.75	1.29	2.03	4.45

Table 8						
Percentiles of Wealth Distribution						
	5%	25%	50%	75%	95%	99%
SCF	-0.2	0.12	0.81	2.36	10.60	38.10
Coarse	0.02	0.02	0.02	1.24	7.27	19.66

Table 9							
	constant	K	ϕ	R	ϕ'	max % error	avg. % error
K'	0.00	0.948	-0.030	-0.029	0.037	1.2	0.004
	0.00	0.948	-0.030	-0.029	0.037		
ϕ'	-0.261	0.00	0.00	0.00	0.00	0.0	0.000
	-0.261	0.00	0.00	0.00	0.00		
R'	0.0	0.00	0.00	0.997	0.021	2.2	0.005
	0.0	0.0	0.0	0.998	0.016		