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This paper uses a small open economy model to address two outstanding issues in monetary policy: (1) what restrictions on the policy rule ensure that the central bank does not introduce real indeterminacy into the economy, and (2) what is the optimal long run rate of inflation. The small open economy model provides unique insights on both fronts. In the case of determinacy issues, the model's simplicity makes the analysis remarkably transparent. As for long run inflation rates, a small open economy takes as given the foreign nominal interest rate. To the extent that this rate distorts domestic behavior, there is a role for positive domestic nominal rates (in contrast to Friedman's celebrated optimum quantity of money). This motivation arises naturally in the setting of a small open economy.

Optimal Monetary Policy in a Small Open Economy: A General Equilibrium Analysis

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Abstract: This paper uses a small open economy model to address two outstanding issues in monetary policy: (1) what restrictions on the policy rule ensure that the central bank does not introduce real indeterminacy into the economy, and (2) what is the optimal long run rate of inflation. The small open economy model provides unique insights on both fronts. In the case of determinacy issues, the model's simplicity makes the analysis remarkably transparent. As for long run inflation rates, a small open economy takes as given the foreign nominal interest rate. To the extent that this rate distorts domestic behavior, there is a role for positive domestic nominal rates (in contrast to Friedman's celebrated optimum quantity of money). This motivation arises naturally in the setting of a small open economy.

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I. Introduction.

The two central issues in monetary policy are separated by time horizon: (1) the short run: what is the appropriate monetary policy across the business cycle? and (2) the long run: what is the optimal long run rate of inflation? This paper explores these classic issues from the vantage point of a small open economy. This "smallness" opens up both possibilities and pitfalls for the policy-maker as several important variables (eg., the foreign interest rate, the real exchange rate, the worldwide real rate of interest) are now exogenous from the perspective of the home country.

The short run focus of monetary policy is to determine how best to respond to real shocks buffeting the economy. These real shocks include fiscal disturbances, fluctuations in multifactor productivity, and movements in real exchange rates. The central banker must make decisions in real time in which the data problems are severe. In particular, it is typically unlikely that the central bank can respond directly to fundamental shocks, but instead must respond to movements in endogenous variables. For example, the central bank will not immediately observe movements in productivity, but instead must respond to movements in the rate of inflation.

These informational restrictions imply that the central bank must use a fairly simple rule, where a rule is a reaction function linking movements in the nominal rate to movements in endogenous variables (eg., the celebrated Taylor (1993) rule). The last several years of research has included numerous theoretical analyses of the welfare advantages of different policy rules. Papers include Carlstrom and Fuerst (1995), Ireland (1997), King and Wolman (1996), and Rotemberg and Woodford (1999). These papers

posit structural models of the real economy and the monetary transmission mechanism. A principle conclusion of this line of research is that the welfare gain of being at the first-best rule are small in comparison to the second or even tenth best. For example, in a limited participation model, Carlstrom and Fuerst (1995) report that the welfare gain of switching from a money growth peg (a seemingly disastrous policy in the posited model) to an interest rate peg (the second best policy) are quite small—representing 0.017% of the steady state capital stock.

As a consequence of these small welfare numbers, the focus of recent research has shifted to a related question—how to avoid doing harm. Papers include Clarida, Gali and Gertler (1997), King and Kerr (1996), Benhabib et al. (1998), Carlstrom and Fuerst (1998,1999ab), Christiano and Gust (1999). By following a rule in which the central bank responds to endogenous variables, the central bank may introduce real indeterminacy and sunspot equilibria into an otherwise determinate economy. These sunspot fluctuations are welfare-reducing and can potentially be quite large. Hence, an important focus of this paper is to isolate the conditions sufficient to ensure that the monetary policy rule does not introduce sunspot equilibria into the economy.

The paper's analysis is conducted in the context of a fully articulated general equilibrium model of a small open economy. The underlying real model is essentially that of Mendoza (1991). Money is introduced by assuming that it is required to facilitate certain transactions. The paper considers both a flexible price economy in which anticipated inflation effects are paramount, and a limited participation model [eg., Lucas (1990), Christiano, Eichenbaum and Evans (1997), Fuerst (1992)] in which unanticipated

money supply shocks have real effects.[1] The "smallness" assumption manifests itself in the assumptions of perfect capital markets, and an exogenous worldwide real rate of interest and real exchange rate. From a theoretical point of view, the "smallness" assumption is particularly attractive, as many of the results that are obscured in a closed economy are more readily apparent here. Additionally, there are additional shocks (eg., the real exchange rate) that are not relevant to a closed economy.

The next two sections of the paper address the issue of real indeterminacy in a small open economy with flexible prices (section II) and with limited participation (section III). A principle conclusion is that to avoid real indeterminacy the central bank must respond aggressively to *past* movements in inflation. It is well known that basing policy responses on market expectations can generate non-uniqueness of equilibria. For this reason "looking forwards" tends to create indeterminacy because monetary policy is driven by an endogenous variable that in turn depends on policy.[2]

The final section of the paper turns to the long run issue of monetary policy—what is the optimal long run rate of inflation? By positing a Taylor rule in Sections II and III, we assume that it is not optimal for the central bank to follow Milton Friedman's (1956) advice and engineer a long run deflation that will peg the nominal rate of interest to zero. There is an enormous literature concerning the robustness of Friedman's optimum quantity of money result (eg., Woodford (1990)). This paper provides a novel explanation for a positive long run rate of inflation that is unique to a small open

[1] Open economy limited participation models include Baier (1997), Grilli and Roubini (1992), and Schlagenhauff and Wrase (1995).
[2] Carlstrom and Fuerst (1999b) demonstrate that these results on indeterminacy are robust to a closed economy model with a more elaborate production technology.

economy. This "smallness" implies that the domestic country takes the foreign nominal rate of interest as given. To the extent that this foreign rate distorts domestic behavior, there is a rationale for positive domestic nominal rates.

II. A Flexible Price Model.

The economy consists of households, firms and financial intermediaries, the latter accepts deposits from households and provides loans to firms. The firms borrow cash to finance their wage bill. The households consume a single good that is produced abroad, while the firms produce a single good that is sold abroad at real exchange rate e_t. The economy is small in that: (1) the real exchange rate is taken as given, and (2) households' foreign asset accumulation earns a constant real rate of return r.

Households are infinitely lived, with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - L_t)$$

where E_0 denotes the expectation operator conditional on time-0 information, $\beta \in (0,1)$ is the personal discount factor, c_t is time-t consumption, and L_t is time-t labor supply. To purchase consumption goods, households are subject to the following cash-in-advance constraint:

$$P_t c_t \leq M_t + P_t w_t L_t - N_t$$

where P_t is the price level, M_t denotes beginning-of-period cash balances, w_t denotes the real wage, and N_t denotes the household's choice of one-period bank deposits. These deposits earn nominal rate R_t that is paid out at the end of the period. The household's intertemporal budget constraint is given by:

$$M_{t+1}=M_t+P_t(w_tL_t+rA_t)-P_tc_t-P_t(A_{t+1}-A_t)+\Pi_t+N_t(R_t-1).$$

A_t denotes the household's investment in foreign assets that earn the constant real rate of return r, with $\beta(1+r) = 1$. Note that we are assuming that asset accumulation occurs at the household level and that cash in advance is not needed to finance its purchase. Π denotes the profit flow from firms and financial intermediaries.

Firms in this economy produce an export good using a production function employing domestic labor:

$$y_t = e_t\theta_t f(H_t)$$

where θ_t is a measure of aggregate productivity, e_t is the real exchange rate, and H_t denotes hired labor. One can imagine that f is constant returns or that land is an additional fixed factor in the production function. In the former case there are no profits to distribute, while in the latter case the profit flow to equity owners is simply the rents on land. To finance its wage bill the firm must acquire cash and does so by borrowing cash short term from the financial intermediary at (gross) rate R_t.

The intermediary in turn has two sources of cash, the cash deposited by households and the new cash injected into the economy by the central bank. Hence, the loan constraint is:

$$P_tw_tH_t \le N_t + M_t^s(G_t-1)$$

where G_t denotes the (gross) money supply growth rate, $G_t \equiv M^s_{t+1}/M^s_t$. Note that monetary injections are carried out as lump sum transfers to the financial intermediary.

We restrict our attention to equilibria with strictly positive nominal interest rates so that the two cash constraints are binding. A recursive competitive equilibrium is given by stationary decision rules that satisfy these two binding cash constraints and the following:

$$E_t\left\{\frac{P_t U_c(t+1)}{P_{t+1}}\right\} = \beta E_t\left\{(1+r)\frac{P_{t+1}U_c(t+2)}{P_{t+2}}\right\} \tag{1}$$

$$\frac{U_L(t)}{U_c(t)} = \frac{e_t\theta_t f'(L_t)}{R_t} \tag{2}$$

$$\left\{\frac{U_c(t)}{P_t}\right\} = \beta E_t\left\{R_t \frac{U_c(t+1)}{P_{t+1}}\right\} \tag{3}$$

$$c_t + A_{t+1} = e_t\theta_t f(L_t) + (1+r)A_t \tag{4}$$

The social resource constraint (4) comes from imposing market-clearing and netting out the firm and intermediary profits from the household's intertemporal budget constraint.

Note that using (3) we can rewrite (1) as

$$\left\{\frac{U_c(t)}{R_t}\right\} = \beta E_t\left\{(1+r)\frac{U_c(t+1)}{R_{t+1}}\right\}. \tag{5}$$

The system given by (2), (4), and (5) is isomorphic to Mendoza's (1991) corresponding real business cycle (RBC) economy except, because of the CIA constraint, it is distorted by a consumption tax rate of $(1+tc_t) = R_t$. Because of this implicit consumption tax, the rate of return on foreign assets does not equal the usual consumption Euler equation

(equation (5)), and the marginal rate of substitution is not equal to the marginal product of labor (equation (2)).

This public finance interpretation is key in what follows. For example, Carlstrom and Fuerst's (1995) result that a constant nominal interest rate is preferred to a variable one is a manifestation of the standard result that constant taxes are better than fluctuating ones (holding the mean distortion fixed).[3] However, as demonstrated by Carlstrom and Fuerst (1995) in a calibrated model, these welfare gains are quite small, and so our attention is shifted to issues of indeterminacy.

Below we will consider monetary policy rules in which the nominal rate (consumption tax) is endogenous, responding to movements in the economy. In a real economy, Schmitt-Grohe and Uribe (1997) demonstrate that the endogenous tax rate movements implied by a balanced budget rule can lead to real indeterminacy. An important issue below is whether interest rate operating procedures can have the same effect.

The small economy assumption makes stability analysis particularly transparent. Substituting equation (3) into (1) yields

$$\frac{R_{t+1}}{\pi_{t+1}} = (1+r). \tag{6}$$

As mentioned above this relationship is not the standard Fisherian relationship. The above nominal interest rate is the rate between t+1 and t+2 *not* between t and t+1. The inflation rate, however, is between t and t+1 ($\pi_{t+1} \equiv P_{t+1}/P_t$). The reason for this distortion

[3] Such an interest rate smoothing policy is exactly the typical central bank policy over the seasonal cycle.

is because of the CIA constraint is on consumption but not investment. It is because of this distortion that there is the potential for indeterminacy.

Proposition 1: Suppose that monetary policy is given by the forward-looking interest rate rule given by:

$$R_t = R_{ss}\left[\prod_{n=0}^{\infty}\left(\frac{\pi_{t+n+1}}{\pi_{ss}}\right)^{\lambda^n}\right]^{\tau(1-\lambda)}, \text{ where } \tau \geq 0,\ R_{ss} = \frac{\pi_{ss}}{\beta}, \text{ and } \lambda \in [0,1). \tag{7}$$

Then in the flexible-price model there is real determinacy if and only if $0 \leq \tau < 1$. In any event, there is always nominal indeterminacy as π_t is free.

Proof: Since (6) starts at t+1 we scroll (7) ahead one period. Taking logs of (7) (expressing R and the π's as log deviations) we have

$$\tilde{R}_{t+1} = (1-\lambda)\tau\sum_{n=0}^{\infty}\lambda^n\tilde{\pi}_{t+n+2}\ . \tag{8}$$

Exploiting the recursion in (8) we have

$$\lambda\tilde{R}_{t+2} = \tilde{R}_{t+1} - (1-\lambda)\tau\tilde{\pi}_{t+2}\,. \tag{9}$$

Using equation (6), this then implies that

$$\lambda\tilde{\pi}_{t+2} = \tilde{\pi}_{t+1} - (1-\lambda)\tau\tilde{\pi}_{t+2}\,, \tag{10}$$

or

$$\tilde{\pi}_{t+2} = \left(\frac{1}{\lambda + (1-\lambda)\tau}\right)\tilde{\pi}_{t+1} \qquad . \tag{11}$$

For real determinacy we need this mapping to be explosive. Hence, there is real determinacy if and only if $0 \leq \tau < 1$. If this is the case then π_{t+j} is determined for $j \geq 1$.

From (7) this pins down R_{t+j} for $j \geq 0$ so that (1)-(4) uniquely pin down the inflation rate and real behavior. Since (11) starts with π_{t+1}, the initial price level is free, π_t is free. QED

An immediate corollary to the above theorem is that an interest rate peg ($\tau = 0$) delivers real determinacy, but like the other rules, has nominal indeterminacy.

Under the policy rule given by (8) (the linearized version of (7)), the monetary authority responds with an elasticity of τ to a geometric weighted-average of all future expected inflation rates, where more weight is placed on the near-future. A special case of such a rule is where $\lambda = 0$ and all the weight is placed on the inflation rate between t and t+1.

$$R_t = R_{ss}\left(\frac{\pi_{t+1}}{\pi_{ss}}\right)^{\tau}.$$

Under this policy rule, increases in expected inflation increase the nominal rate. But for active policies ($\tau > 1$), these nominal rate increases are also associated with increases in the real rate of interest. Thus, we have an implicit consumption tax (the nominal rate) correlated tightly with expected consumption growth (the real rate). The self-fulfilling circle goes something like this. An increase in expected inflation increases investment demand lowering current consumption. The decline in current consumption increases the real rate of interest; with $\tau > 1$, the nominal rate (consumption tax) rises sharply with this real rate movement; this tax movement implies that the initial increase in expected inflation and decline in current consumption are rational.

The fact that π_t is free for all values of τ is just a manifestation of nominal indeterminacy, i.e., there is nothing to pin down the initial growth rate of money. This innocent remark has some interesting implications:

Proposition 2: Suppose that monetary policy is given by a forward-looking rule that includes the current inflation rate,

$$R_t = R_{ss}\left[\prod_{n=0}^{\infty}\left(\frac{\pi_{t+n}}{\pi_{ss}}\right)^{\lambda^n}\right]^{\tau(1-\lambda)}, \text{ where } \tau \geq 0, R_{ss} = \frac{\pi_{ss}}{\beta}, \text{ and } \lambda \in [0,1). \qquad (12)$$

Then in the flexible price model there is real indeterminacy for all values of $\tau \neq 0$.

Proof: The proof proceeds as before leading to the following difference equation

$$\tilde{\pi}_{t+2} = \frac{1-(1-\lambda)\tau}{\lambda}\tilde{\pi}_{t+1}.$$

Under appropriate conditions this is explosive so that π_{t+j} is pinned down for $j \geq 1$. But even in this case π_t is free and hence so is R_t. Thus we have real indeterminacy. Once again if $\tau = 0$, we have an interest rate peg and there is no real indeterminacy. QED

The nominal indeterminacy from before is now real. Before π_t being free had no real affect but now given our interest rate rule R_t is now free. Since R_t acts like a tax on consumption the fact that R_t is not pinned down implies that real behavior is indeterminate. The reason for this indeterminacy is because our policy rule is responding to endogenous variables. This is why an interest rate peg ($\tau = 0$) is determinate.

This discussion suggests that the central bank should look further backwards so that it only responds to exogenous variables. Remarkably, by looking backwards the conditions for determinacy are (almost) entirely flipped on their head from when the Taylor Rule is forward looking.

Proposition 3: Suppose that monetary policy is given by the backward-looking interest rate rule given by

$$R_t = R_{ss}\left[\prod_{n=0}^{\infty}\left(\frac{\pi_{t-n-1}}{\pi_{ss}}\right)^{\lambda^n}\right]^{\tau(1-\lambda)}, \text{where } \tau \geq 0, R_{ss} = \frac{\pi_{ss}}{\beta}, \text{and} \lambda \in [0,1). \quad (13)$$

Then in the flexible-price model there is real determinacy if and only if $\tau = 0$ or $\tau > 1$. In the case of $\tau > 1$, there is also nominal determinacy.

Proof: Following the strategy pursued in Proposition 1, we are led to the difference equation

$$\tilde{R}_{t+2} = [\lambda + (1-\lambda)\tau]\tilde{R}_{t+1}$$

For real determinacy we thus need $\tau > 1$, which then implies π_{t+j} is pinned down for $j \geq 0$. QED

An interest rate peg is determinate but like before has nominal indeterminacy. Notice, however, that if the monetary authority responds aggressively to past inflations ($\tau > 1$), initial inflation and hence the initial money stock, is pinned down. This result is a general equilibrium generalization of McCallum's (1981) earlier result. He argued that because an interest rate peg suffered from nominal indeterminacy the monetary authority needed a nominal anchor, which could be accomplished by responding to a nominal variable. Proposition (3) confirms this but shows that merely responding to a nominal variable, like past inflation, is not enough. The monetary authority has to aggressively respond to past inflation to ensure nominal determinacy.

For completeness, we consider a backward looking rule in which the central bank also considers the current inflation rate.

Proposition 4: Suppose that monetary policy is given by a backward-looking interest rate rule that also includes the current inflation rate:

$$R_t = R_{ss}\left[\prod_{n=0}^{\infty}\left(\frac{\pi_{t-n}}{\pi_{ss}}\right)^{\lambda^n}\right]^{\tau(1-\lambda)}, \text{ where } \tau \geq 0, R_{ss} = \frac{\pi_{ss}}{\beta}, \text{ and } \lambda \in [0,1). \qquad (14)$$

Then in the flexible-price model there is real determinacy if and only if $\tau = 0$ or $1 < \tau < (1+\lambda)/(1-\lambda)$. In the case of $1 < \tau < (1+\lambda)/(1-\lambda)$, there is also nominal determinacy.

Proof: Following the strategy pursued in Proposition 3, we are led to the difference equation

$$\tilde{R}_{t+1} = \left[\frac{\lambda}{1-(1-\lambda)\tau}\right]\tilde{R}_t.$$

For determinacy, the term in brackets must be outside the unit circle. There are two regions to consider. First if $\tau(1-\lambda) < 1$ the bracketed term exceeds unity whenever $\tau > 1$. If $\tau(1-\lambda) > 1$, we must consider the other side of the unit circle. Now for determinacy we must have

$$\tau < \frac{1+\lambda}{1-\lambda}.$$

Combining the two regions we have the region noted in the Proposition. QED

With an active policy looking backwards can render the economy determinate. Yet the more weight that is placed on the distant past (the larger is λ) the bounds for determinacy shrink.

Although at this stage nominal indeterminacy is merely a nuisance, its presence becomes critical in the next section when we consider how the presence of a particular type of nominal rigidity, namely sluggish portfolios, affects the above results.

III. A Limited Participation Model.

A natural criticism of the previous analysis is that it was conducted in a monetary model in which only anticipated inflation has real effects. For example, under most of the policies considered there is nominal indeterminacy in the flexible-price model implying that an iid shock to the money supply has no effect on any real variables.[4] This section extends the analysis to a more compelling model of monetary non-neutrality in which iid shocks have real effects.

The model we choose to examine is the limited participation model [eg., Lucas (1990), Christiano and Eichenbaum (1992), Fuerst (1992)]. There are at least two reasons for this choice. First, in contrast to sticky price models there has been very little work on indeterminacy issues in this type of model.[5] Second, there is a compelling empirical reason to consider such a model. A well-known empirical phenomenon is that in

[4] This exception is a backward-looking Taylor rule when $\tau > 1$. Otherwise the fact that initial money growth is free implies that iid shocks to money growth will also achieve the interest rate directive.

[5] A notable exception is Christiano and Gust (1998). An important difference between their analysis and the model presented here is that they assume that investment purchases are subject to a cash-in-advance constraint, while this paper makes the more standard assumption that investment is a credit good.

response to a positive monetary policy innovation (a movement downward in the nominal interest rate), aggregate real behavior displays a hump-shaped time profile, while prices rise slowly to a higher level. A hump-shaped consumption profile implies an increase in the real rate of interest, while an upward movement in prices implies that expected inflation cannot fall. But then we are left with a puzzle—how can nominal interest rates fall? The obvious answer is that over some time horizon the standard Fisherian interest rate determination is broken, ie., equation (3) must not hold over some time interval.

This is exactly the assumption made in limited participation models. In particular, these models assume that the (nominal) portfolio choice (N_t) is made one period in advance so that (3) is replaced with

$$E_{t-1}\left\{\frac{U_c(t)}{P_t}\right\} = \beta\, E_{t-1}\left\{R_t\,\frac{U_c(t+1)}{P_{t+1}}\right\}.$$

The positive implications of these models are well-known. For example, suppose that

$$U(c_t, L_t) = \ln(c_t) - \frac{L_t^{1+\tau}}{1+\tau}.$$

Then straightforward calculations imply

$$L_t = \left(\frac{N_t + G_t - 1}{G_t}\right)^{\frac{1}{1+\tau}}.$$

With N_t predetermined, this implies that monetary shocks increase employment and domestic output, while driving down the domestic nominal interest rate. From a policy perspective, the more important observation is that employment does not respond to productivity or real exchange rate shocks because these shocks drive the nominal interest rate upwards. This sluggishness makes a clear case for monetary policy—vary money

growth to accommodate these shocks. Carlstrom and Fuerst (1995) use this as a starting point to make the case for interest rate pegging in this model. Once again, the welfare gains are small.

Because of this we turn to stability analysis. The attractive feature of the limited participation model is that it breaks the rigid Fisherian interest rate determination. But for the stability properties of interest rate operating procedures, this turns out to be somewhat disastrous since the interest rate is no longer linked to underlying real variables such as consumption growth and expected inflation.

After one period portfolios can adjust implying that for stability analysis we must replace (3) with its counterpart scrolled forward:

$$\left\{\frac{U_c(t+1)}{P_{t+1}}\right\} = \beta\left\{R_{t+1}\frac{U_c(t+2)}{P_{t+2}}\right\} \quad . \tag{15}$$

Combining this with the asset accumulation equation (1) we have

$$\frac{R_{t+1}}{\pi_{t+1}} = (1+r). \tag{16}$$

$$\left\{\frac{U_c(t+1)}{\pi_{t+1}}\right\} = \left\{\frac{U_c(t+2)}{\pi_{t+2}}\right\} \quad . \tag{17}$$

For stability analysis, the system of equations includes equations (16)-(17), the labor equation (2), and the resource constraint (4).

Equation (16) is familiar from before. Under suitable conditions on the interest rate operating procedure it will pin down inflation and interest rate dynamics. The novelty is in (17)—the limited participation constraint implies that there is little to pin

down the initial $U_c(t)$ and thus the initial asset accumulation decision, even if the initial nominal interest rate is determined.

For any hopes of determinacy we need an extra restriction. The limited participation constraint provides one: N_t is a pre-determined nominal variable. Combining the two binding cash constraints, and using the household's labor choice decision we have:

$$M_t - N_t = P_t\left[c_t - \frac{U_L(t)L_t}{U_c(t)}\right] \tag{18}$$

Since M_t and N_t are predetermined, this provides an additional restriction (in the flexible price model N_t is free and (18) does not restrict real behavior). In particular, if a policy rule pins down the current price level P_t, then the additional restriction in (18) will pin down the initial consumption-asset-accumulation decision. Hence, the only way of achieving determinacy is if the interest rate operating procedure pins down the current $\pi_t \equiv P_t / P_{t-1}$. This immediately implies the following:

Proposition 5: Suppose that monetary policy is given by either a forward-looking (7) or a current/forward-looking (12) Taylor rule. Then in the limited participation model there is real indeterminacy for all values of τ.

And its corollary:

Proposition 6: Suppose that monetary policy is given by a backward-looking Taylor rule (13). Then in the limited participation model there is real determinacy if and only if $\tau > 1$. Suppose that monetary policy is given by the current/backward-looking Taylor rule given in (14). Then there is real determinacy if and only if $1 < \tau < (1+\lambda)/(1-\lambda)$.

Notice that a forward-looking Taylor rule and an interest rate peg both suffer from real indeterminacy. The reason is because they both had nominal indeterminacy in the corresponding flexible price economy.[6] An active backward-looking Taylor rule, however, pins down the price level. Since portfolios are pre-determined in nominal terms, this price level determination pins down an extra real variable.

IV. A Flexible Price Model with Two Currencies.

This section changes the nature of the analysis from the short-run (policy across the business cycle) to the long-run (the long run rate of inflation). Since our focus is on the long run we ignore dynamic issues, and instead concentrate on steady-state analysis.

In terms of modeling, this section builds on the earlier model by assuming that the home country agent has preferences over both the imported and the exported good and that the imported good must be paid for with foreign currency accumulated in advance.[7] Hence, we have a model with two cash constraints, and thus two implicit consumption taxes. The imported good is taxed at the foreign interest rate, while the domestically-produced good is taxed at the domestic interest rate. In Section II, domestic currency could be used for all transactions so that there was only one implicit tax—the domestic interest rate. But now there are two taxes, one of which the domestic central bank takes as exogenous. In this environment it may not be optimal for the home country to follow Friedman's dictum and drive the domestic nominal interest rate to zero.

[6] Carlstrom and Fuerst (1995) show that an interest rate peg suffers from real indeterminacy in a closed-economy limited participation model.

Household preferences over the imported good (good two), the domestically produced good (good one), and work effort are given by

$$\sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}, L_t). \tag{20}$$

To finance its consumption purchases the household uses cash accumulated in advance and thus faces the following cash constraints:

$$P_{1t} c_{1t} \leq M_{1t} + W_t L_t - N_{1t} \tag{21}$$

$$P_{2t} c_{2t} \leq M_{2t} - N_{2t} \tag{22}$$

where M_{it} denotes holdings of the home (i = 1) and foreign (i = 2) currency, P_{it} denotes the nominal price of goods i = 1,2, W_t denotes the nominal wage expressed in home currency, and N_{it} denotes the bank deposits denominated in the two currencies i = 1,2. The intertemporal constraint (expressed in home currency) is thus given by:

$$M_{1t+1} + s_t M_{2t+1} = s_t M_{2t} + M_{1t} + W_t L_t - P_{1t} c_{1t} - s_t P_{2t} c_{2t} + \Pi_t + N_{1t}(R_{1t} - 1) + s_t N_{2t}(R_{2t} - 1) + s_t X_t$$

where R_{it} denotes the nominal interest rate denominated in the two currencies i = 1,2, s_t is the end of period nominal exchange rate, and X_t is a currency transfer from the foreign government. Since this is a small open economy, we need to exogenously impose an equilibrium condition on domestically-owned bank deposits of foreign currency. For simplicity we assume that these are in zero net supply so that one equilibrium condition imposed below will be $N_{2t} = 0$. Similarly, since our focus is on steady-state issues we abstract from real foreign asset accumulation (A_t). This is without loss of generality.

The first order conditions to the household's problem include:

[7] This more complicated modeling environment would not affect the earlier conditions for real determinacy.

$$\frac{U_2(t)}{U_1(t)} = \frac{s_t P_{2t} R_{2t}}{P_{1t} R_{1t}} \tag{23}$$

$$\frac{U_L(t)}{U_1(t)} = \frac{W_t}{P_{1t}}. \tag{24}$$

As for the firm, it produces good 1 using the production technology from before, and subject to the need to borrow domestic currency to finance its wage bill. In addition, it imports good 2 at an exogenous terms of trade. (As before, the imported good is the numeraire, so that e_t is the real price of good 1 in terms of good 2.) The firm then sells the two goods at the respective prices P_{it}. Since the firm does not distribute dividends until the end of the period, it maximizes its end of period profits. Hence, purchasing power parity is in terms of the end-of-period exchange rate:

$$e_t = \frac{P_{1t}}{s_t P_{2t}}. \tag{25}$$

Interest rate parity is given by

$$\hat{s}_t = \frac{s_t R_{2t}}{R_{1t}}$$

where $\hat{s}_t$ is the beginning of period or "spot" exchange rate. The fact that (25) does not hold with the spot exchange rate, but instead with the end-of-period exchange rate, is a manifestation of the cash-in-advance distortion.

Substituting the firm's optimization conditions into the household first order conditions yields

$$\frac{U_L(t)}{U_1(t)} = \frac{\theta_t f'(L_t)}{R_{1t}} \tag{26}$$

$$\frac{U_1(t)}{U_2(t)} = \frac{e_t R_{1t}}{R_{2t}} \tag{27}$$

As noted earlier, this is now a model with two implicit consumption taxes, both of which are manifested in (26)-(27). The home-produced good (good one) is taxed at the domestic nominal rate, while the imported good (good two) is taxed at the foreign rate of interest. For a given foreign interest rate it is clear that the central bank faces a second-best problem. Setting the domestic rate to match the foreign rate may alleviate one distortion (equation (27)) but exacerbates another (equation (26)).

The social resource constraint comes from imposing the equilibrium decision-rules and market-clearing conditions on the household's intertemporal constraint. As previously, we assume that only domestic households hold domestic currency. Hence, we need only adjust the earlier resource constraint (4) for domestically held foreign cash balances:

$$\frac{M^a_{2t+1} - M^a_{2t}}{P_{2t}} + e_t c_{1t} + c_{2t} = e_t \theta_t f(L_t) + \frac{X_t}{P_{2t}} \tag{28}$$

where M^a_{2t} denotes per capita foreign cash balances held by the representative domestic household (in the earlier model domestic households held no foreign cash).

In the steady-state, domestic labor supply and purchases of the two goods are constant. Since we are uninterested in the wealth effects coming from exogenous foreign price movements, we assume that

$$X_t \equiv (P_{2t+1} - P_{2t})c_2 \tag{29}$$

where c_2 is the steady-state level of the imported good.[8] The household takes this transfer as exogenous so that it does not affect marginal decision-making, but is only felt at the aggregate resource level. In particular, with the transfer given by (29), the binding CIA constraint implies that foreign money holdings drop out of the resource constraint so that the steady-state resource constraint is given by

$$ec_1 + c_2 = e\theta\, f(L). \tag{30}$$

The *steady-state welfare problem* is to maximize (20) subject to (26), (27) and (30).

We must utilize numerical methods to make any headway. Hence, assume the following functional forms:

$$U(c_1, c_2, L) = \frac{(c_1^{1-\delta} + \mu c_2^{1-\delta})^{\frac{1-\sigma}{1-\delta}}}{1-\sigma} - \frac{\nu L^{1+\eta}}{1+\eta}$$

$$f(L) = L^{\alpha}$$

For our benchmark calibration, we set to one the intertemporal elasticity of substitution, $(1/\sigma)$, and the elasticity of substitution between the two consumption goods

[8] Thus this assumption is in the spirit of Lucas (1982) who assumes that domestic and foreign households are insured against "seigniorage risk". Without this assumption the analysis would proceed with an inflation tax term in the aggregate resource constraint (30).

$(1/\delta)$; the labor supply elasticity, $(1/\eta)$, to 0.333; and the labor share to $\alpha = 2/3$. We also normalize $\theta = 1$, $e = 1$, and $\mu = 1$. The latter implies an import share of $[c_2/(c_2 + c_1)] = 0.47$. The foreign interest rate is set at $R_2 = 1.06$. We then chose ν so that with $R_1 = 1.06$, we have $L = 1/3$.

Figures 1-3 present the results for differing values of the elasticity of substitution between the two consumption goods $(1/\delta)$, differing values of the labor supply elasticity $(1/\eta)$, and differing import shares (variations in μ). Across all three figures all other parameters are held constant except for the one being analyzed.

The figures reveal standard second-best intuition. When there are two distorted margins, the benevolent planner will choose to more heavily distort the margin that is less elastic or less important. The two margins here are the labor-leisure choice (26) and the good 1 vs. good 2 choice (27). The first distortion is eliminated with a zero nominal rate (a gross rate of unity) while the second distortion is eliminated by matching the foreign interest rate.

Figures 1-3 have the qualitative shape that one would expect. As $1/\delta$ goes to zero, and demands become highly inelastic, the distortionary impact of divergent nominal interest rates declines. Hence, the optimal response is to lower the distortion on the labor margin by lowering the domestic interest rate. As labor supply becomes highly inelastic and $1/\tau$ goes to zero, just the opposite is true, and the optimal response is to more closely match the foreign rate. Figure 3 reveals that as the importance of the foreign sector rises, the cost of this distortion rises. Hence, as the foreign share rises, the optimal response is to move closer to the foreign rate. Notice that except under extreme parameter values, the optimal domestic rate is significantly different from Friedman's zero.

Before closing, it is instructive to compare this steady-state analysis to the corresponding dynamic Ramsey problem (eg., Chari, Christiano, and Kehoe (1996)). In the present context, the key issue is the form of the aggregate resource constraint. For the household budget constraint to collapse into the form given by (30), one needs to preclude an initial and one-time flight from the foreign currency (a possibility that, by assumption, cannot arise in a steady-state analysis). That is, if the Ramsey planner began with the level of foreign cash balances given by the steady-state problem, he would encourage a one-time drop in these currency holdings by setting the domestic nominal rate to zero. This lower domestic rate would lead the household to decrease its consumption of foreign goods and thus lower its holdings of foreign currency. For all future periods this flight would leave the domestic household at a period-by-period utility level lower than the utility level from the steady-state problem (as consumption of the imported good would be forever lower). To rule out this type of behavior, we must replace (29) with the stronger condition

$$X_t \equiv (P_{2t+1}c_{2t+1} - P_{2t}c_{2t}). \qquad (31)$$

This transfer scheme imposes an implicit tax on currency flight. Once again the domestic household takes this transfer as exogenous and thus it does not enter private marginal conditions. However, the Ramsey planner is affected by these transfers and they lead to an aggregate resource constraint like (30). Hence, under the assumption (31), the steady-state analysis and the dynamic Ramsey problem are equivalent.

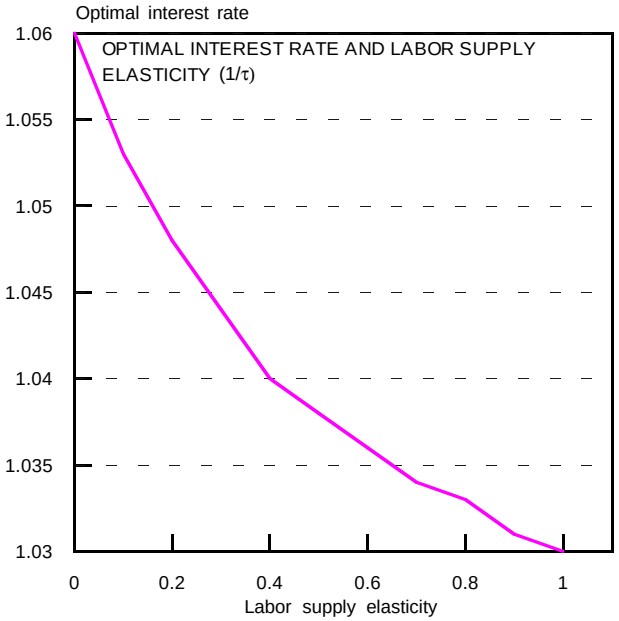
V. Conclusion.

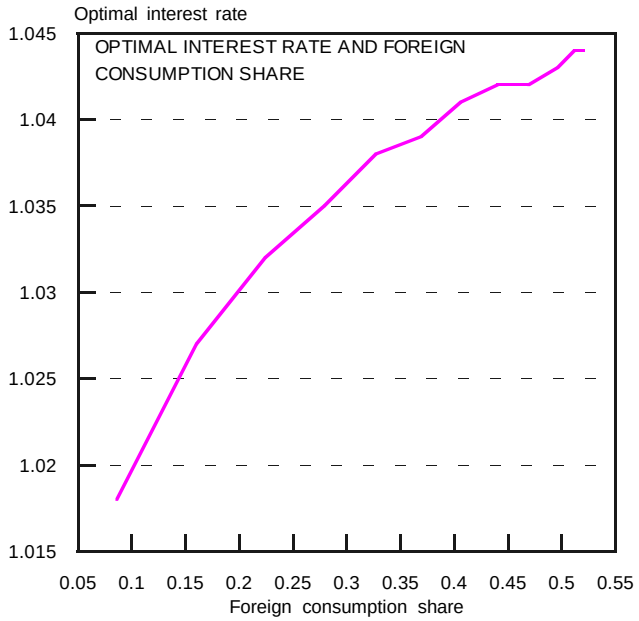
This paper has utilized a standard open economy model to address two classic questions in monetary policy: (1) what is the appropriate monetary policy across the business cycle and (2) what is the optimal long run rate of inflation?

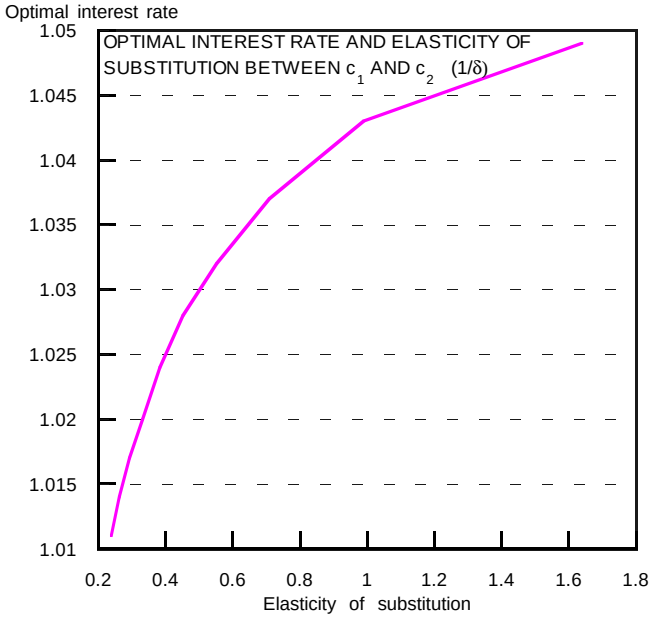
Recent research suggests that an important issue for the first question is what policy restrictions ensure that the central bank's policy rule does not introduce real indeterminacy and sunspot fluctuations into an otherwise stable economy. The message is clear: the central bank should respond aggressively to lagged inflation rates. From the standpoint of indeterminacy issues, a policy that targets current and future expected inflation is disastrous. Responding "passively" to only future expected inflation or targeting the nominal interest rate may avoid real indeterminacy in a flexible price model. However, these policies are disastrous when the economy is subject to a nominal rigidity such as that implied by limited participation. In terms of exchange rates, to the extent that a pegged exchange rate implies that the domestic nominal interest rate is given exogenously by the foreign country, then a pegged exchange rate is also subject to real indeterminacy and sunspot fluctuations.

As for the second question concerning the long run rate of inflation, a small open economy faces an exogenous foreign interest rate. Almost certainly this rate distorts domestic behavior. Hence, the central bank immediately faces a second-best problem. One distortion (a positive foreign rate) can be worse than two distortions (a positive domestic and foreign interest rate). This is a novel reason to stay clear of Friedman's zero nominal rate.

This paper's general equilibrium or structural approach to policy evaluation has many advantages including an obvious welfare criterion (lifetime utility of the typical agent) and a clear articulation of what parameters are policy invariant. General equilibrium analysis forces one to be specific, and as always the devil is in the details. An example illustrates this point. King and Wolman (1996) analyze a sticky-price general equilbrium model and conclude that strict price level targeting is the optimal monetary policy. In contrast, the results of Carlstrom and Fuerst (1998) imply that a plausible alteration in their modeling of money demand implies that an aggressive price level target is actually destabilizing and introduces sunspot equilibria into the economy. Which conclusion is correct? One must examine the details, all of which are obscured by a simple reduced form model.


Optimal interest rate
OPTIMAL INTEREST RATE AND LABOR SUPPLY ELASTICITY (1/τ)
1.06
1.055
1.05
1.045
1.04
1.035
1.03
0
0.2
0.4
0.6
0.8
1
Labor supply elasticity


Optimal interest rate
OPTIMAL INTEREST RATE AND FOREIGN CONSUMPTION SHARE
1.045
1.04
1.035
1.03
1.025
1.02
1.015
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
Foreign consumption share


Optimal interest rate
OPTIMAL INTEREST RATE AND ELASTICITY OF SUBSTITUTION BETWEEN c1 AND c2 (1/δ)
1.05
1.045
1.04
1.035
1.03
1.025
1.02
1.015
1.01
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
Elasticity of substitution

References

Baier, Scott L., "Optimal Monetary Policy in a Two Country, Two Currency Limited Participation Model," Notre Dame University Working Paper, May 1997.

Benhabib, J., S. Schmitt-Grohe, M. Uribe, "Monetary Policy and Multiple Equilibria," working paper, June 1998.

Carlstrom, C., and T. Fuerst, "Interest Rate Rules vs. Money Growth Rules: A Welfare Comparison in a Cash-in-Advance Economy," *Journal of Monetary Economics* 36 (1995): 247-267.

Carlstrom, C., and T. Fuerst, "Timing and Real Indeterminacy in Monetary Models," Federal Reserve Bank of Cleveland working paper, 1998.

Carlstrom, C., and T. Fuerst, "Real Indeterminacy in Monetary Models with Nominal Interest Rate Distortions," Federal Reserve Bank of Cleveland working paper, 1999a.

Carlstrom, C., and T. Fuerst, "Forward vs. Backward-Looking Taylor Rules," Federal Reserve Bank of Cleveland working paper, 1999b.

Chari, V.V., Larry Christiano, and Patrick Kehoe, "Optimality of the Friedman Rule in Economies with Distorting Taxes," *Journal of Monetary Economics* 37(2), 1996, 203-223.

Christiano, L., Eichenbaum, M., and Evans, C. The effects of monetary policy shocks: Evidence from the flow of funds, *Review of Economics and Statistics*, **78:** 16-34.

Christiano, Larry, and Christoper J. Gust, "Taylor Rules in a Limited Participation Model," NBER Working Paper 7017, 1999.

Christiano, Larry, Martin Eichenbaum, and Charles Evans, "Sticky Price and Limited Participation Models: A Comparison," *European Economic Review* 41(6), 1997, 1201-1249.

Clarida, Richard, Jordi Gali, and Mark Gertler, "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," New York University manuscript, 1997.

Fuerst, T.S., "Liquidity, Loanable Funds, and Real Activity", *Journal of Monetary Economics*, **29:** 3-24.

Grilli, V., and N. Roubini, "Liquidity and Exchange Rates," *Journal of International Economics* 32, 1992, 339-352.

Ireland, Peter. "A Small, Structural, Quarterly Model for Monetary Policy Evaluation," *Carnegie-Rochester Conference Series on Public Policy* 47, 1997, 83-108.

Kerr, William, and Robert King, "Limits on Interest Rate Rules in the IS-LM Model," *Federal Reserve Bank of Richmond Economic Quarterly*, Spring 1996.

King, Robert G., and Alexander L. Wolman, "Inflation Targeting in a St. Louis Model of the 21st Century," *Federal Reserve Bank of St. Louis Review* 78(3), 83-107, May/June 1996.

Lucas, Robert E., Jr., "Interest Rates and Currency Prices in a Two-Country World," *Journal of Monetary Economics* (10), 1982: 335-359.

Lucas, R.E., Jr., "Liquidity and Interest Rates", *Journal of Economic Theory*, **50:** 237-264.

Mendoza, Enrique G., "Real Business Cycles in a Small Open Economy," *American Economic Review* 81(4), Septebmer 1991, 797-818.

Rotemberg, Julio, and Michael Woodford, "Interest Rate Rules in an Estimated Sticky Price Model," in John Taylor's *Monetary Policy Rules* (The University of Chicago Press: 1999).

Schlagenhauff, D.E., and J. Wrase, "Liquidity and Real Activity in a Simple Open Economy Model," *Journal of Monetary Economics* 35, August 1995, 431-461.

Schmitt-Grohe, Stephanie, and Martin Uribe, "Balanced Budget Rules, Distortionary Taxes, and Aggregate Instability," *Journal of Political Economy* 105(5), October 1997, 976-1000.

Taylor, John B. (1993), "Discretion versus Policy Rules in Practice," *Carnegie-Rochester Series on Public Policy* 39, 195-214.

Woodford, Michael, "The Optimum Quantity of Money," *Handbook of Monetary Economics Volume 2*, ed. B.M. Friedman and F.H. Hahn (1990), 1067-1149.