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The Economics of Payments

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In this paper we provide a survey of the payment literature in a unified framework. The environment is a variant of the Lagos and Wright (2005) model of monetary exchange, where some trades occur in bilateral meetings while others occur in more or less decentralized markets. We use this basic environment to introduce alternative sets of trading frictions that give rise to different payments instruments and/or payments institutions. We investigate credit economies, monetary economies, and economies in which money and credit coexist. We also study alternative assets, such as foreign exchange, capital (equity), and government liabilities, which can be used as payment instruments in conjunction with money. We introduce banks as deposit-taking institutions whose liabilities circulate in the economy. We also provide an extension in which the process of the settlement of debt for money is modeled and the potential social costs of settlement are characterized. Finally, we investigate government policy responses to the social costs introduced by various trading frictions.

POLICY DISCUSSION PAPERS

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Introduction

Economics is all about exchange, but exchange need not be seamless. How else can one explain the existence of the myriad assets and institutions—domestic currency, bank deposits, bonds, capital, large value payments systems, such as Fedwire and CHIPS, foreign exchange and the foreign exchange market, to name but a few—whose main purpose is to facilitate trade? In the real world, trade between agents is not conducted in a frictionless environment, as, for example, it is in an Arrow–Debreu economy. Instead, real resources must be used in order for exchange to take place at all and, as a result, people will attempt to design instruments of exchange that will economize on resource use. The precise instrument or institution that one might use will depend upon the obstacles or *frictions* that agents face in a particular trade. So, in our view, the existence of trading frictions implies that agents will use some sort of instrument, asset, or institution to facilitate trade, and the instrument, asset, or institution that is actually used will depend upon the nature of the trading friction(s) that agents face.

There does not really exist a well-defined literature on the economics of payments. There are, of course, comprehensive literatures on credit, money, foreign currencies, banking, and so on. But, by and large, these literatures have evolved independently of one another and may have quite different focuses. For example, in the banking literature, there are large bodies of works on bank runs and optimal lending contracts, but little time has been spent on banks' liabilities as a medium of exchange. An implication of this independent development is that the economic environments in these various literatures are not necessarily comparable. Indeed, even within a literature there is considerable variation in the specification of economic environments. This lack of comparability or a common model environment is problematic if one is interested in understanding, for example, why one set of payment instruments might emerge in one situation and not another. In this *Policy Discussion Paper*, a common economic environment is used, one that models exchange between agents. Within this common economic environment, alternative sets of trading frictions are introduced, where different sets of frictions may give rise to different payments instruments and/or payments institutions. The benefit of our approach is that one will be able to associate the particular trading frictions with payment instruments. Government policy can be thought of in terms of attempting to counteract these trading frictions, either directly or indirectly.

The paper is organized as follows. We begin by describing the common model environment that will be used throughout the paper; it is a variation of the Lagos–Wright model of exchange. We then proceed to consider alternative sets of trading frictions that give rise to different payments instruments. First we investigate the use of only credit in exchange, then the use of only money. In reality, both money and credit are used to facilitate exchange, so we next combine the trading environments of the previous sec-

tions to explain the coexistence of money and credit in trade. Although money seems to be central to exchange, in practice money as well as other assets are used in trade. We next study alternative assets, such as foreign exchange, capital (equity) and government liabilities that can be used as payments instruments in conjunction with money. Banks are intermediaries and have long been considered facilitators of trade, so we proceed to show how a deposit-taking institution, whose liabilities circulate in the economy, can be a useful institution of trade. In practice, virtually all debt is settled with some form of money. But the very act of settlement can introduce additional frictions into a trading environment. So we model the process of settlement of debt for money and characterize the potential social costs of settlement. Finally, we investigate government policy responses to the social costs introduced by the various trading frictions that have been discussed in previous sections.

The Basic Environment

We consider a simple model to describe different payments methods to carry out trades. The *benchmark* model is as follows: Time is discrete and continues forever. Each period is divided into two subperiods, *day* and *night*. During the day, trades occur in *decentralized* markets according to a bilateral matching process. Each agent meets a trading partner with probability σ. During the day, some agents can produce but do not want to consume, while other agents want to consume but cannot produce. We call the former agents *buyers* and the latter *sellers*, and the measures of buyers and sellers are normalized to one. This generates a simple double-coincidence-of-wants problem in the decentralized market. The double-coincidence-of-wants problem can be exacerbated by, for example, having sellers produce different kinds of goods and having buyers wanting to consume only certain types of goods. All this can be captured by the parameter σ. Below, we will be explicit in terms of the lack of double coincidence problem. We will call the good that is produced and traded during the day the *search good*, since buyers and sellers are randomly matched and trade is decentralized.

Exactly how production and trade are organized at night will depend upon the issue that is under investigation. What can be said about the night market is that, in general, it will be characterized by fewer frictions than those that plague the morning market. At night, all agents can produce and consume. The good that is produced and consumed at night will be called the *general good*.

Search goods can only be produced during the day and general goods can only be produced at night. All goods, whether produced in the day or at night, are nonstorable, so a search good cannot be carried over to the night and a general good cannot be carried over to the next day.

The period utility functions for buyers and sellers are given by

(1) $U^b(q,x) = u(q) + x,$

(2) $U^s(q,x) = -c(q) + x,$

where q is the quantity of the search good consumed and produced during the day, and x is the net consumption—the difference between what is consumed and produced— of the general good at night. We assume $u'(q) > 0,\ u''(q) < 0,\ u(0) = c(0) = c'(0) = 0,$ $c'(q) > 0,\ c''(q) > 0,$ and $c(\bar{q}) = u(\bar{q})$ for some $\bar{q} > 0.$ All agents discount between the night and the next day at rate $r = \beta^{-1} - 1,$ where $\beta \in (0,1).$ Lifetime utility for agent $i, i \in \{\text{buyer}, \text{seller}\},$ at date j is given by $\Sigma_{t=j}^{\infty} \beta^{t-j} U^i(q,x).$ Let q^* denote the efficient (static) level of production and consumption of the search good, where q^* is the solution to $u'(q^*) = c'(q^*).$ Note that the linear specification goods produced and consumed at night implies that there is no benefit associated with producing the general good for one's own consumption.

We define a *trade match* to be a match between a buyer and seller during the day, where the buyer wants the good that the seller can produce, the seller is actually able to produce the good, and the buyer has the resources to pay for the good. We assume that agents always have the option to "exit" or not participate in any particular market or markets, but can always return.

Generally speaking, we adopt the benchmark model specification in all of the discussions that follow. Without exception, the day market, with its bilateral matching of agents and its lack of a double-coincidence-of-wants problem, will be common to all of the environments discussed below. At times, however, we may depart slightly from the specification of our benchmark model. When this does happen, we will be very clear in explaining both how and why we are modifying the benchmark model.

Pure Credit Economies

In this section, we consider environments with the following characteristics: First, a match between a buyer and a seller that is formed during the day is maintained at night. The fact that agents are matched for the entire period allows them to make promises or "write" debt contracts during the day, which can be settled at night. Second, there are no frictions or costs associated with settling debt at night: An agent can settle his debt at night by simply producing the general good and transferring it to his creditor. Third, there are no assets, e.g., money, that agents can use for trade purposes. So agents can only trade by using credit arrangements. An environment that satisfies these characteristics will be referred to as a *credit economy*. We are interested in characterizing the set of allocations that can be implemented as an equilibrium in a credit economy.

The extent to which a particular allocation can be implemented in a credit economy depends on the degree of commitment that agents possess, as well as methods available for punishing a debtor who reneges on his obligations. We will consider three related environments in which credit arrangements can be sustained. We first assume that agents have the ability to commit to repay their debts. We then consider an environment where agents cannot commit but there exists a public recordkeeping device that can monitor agents' production levels. Finally, we assume that agents are able to form long-term part-nerships, where trading relationships can be sustained by reputations.

Credit with Commitment

We first consider an economy where buyers can commit to repay their debts. Since com-mitment may be interpreted as a rather strong assumption, we will limit the extent to which agents (and, in particular, buyers) are able to commit. We assume that in the day market, buyers can promise to undertake future actions, but only for the subsequent night market. This assumption implies, among other things, that private debt will not cir-culate across periods.

We will describe the set of allocations that are feasible in the sense that they are in ac-cordance with agents' willingness to trade. We restrict the set of allocations to those that are symmetric across matches and that are constant across time. To find these allocations, we assume the following simple trading mechanism: When a match is formed during the day, the buyer and seller must consider implementing the allocation (q,y), where q is the quantity of the search good produced by the seller for the buyer in the day and y is the amount of the general good that the buyer promises to produce and deliver to the seller at night. When a buyer and seller are in a trade match, they must decide simultaneously whether to accept or reject allocation (q,y). This allocation is implemented only if both agents accept it.

The sequence of events within a typical period is as follows: At the very beginning of the period, all agents are unmatched. During the day, each agent finds a trading partner with probability σ. The buyer and seller in a trade match decide simultaneously whether to accept or reject the proposed allocation (q,y). If either player rejects the proposed allocation, the match is dissolved; otherwise, the seller produces q units of the search good for the buyer during the day and the buyer produces y units of the general good for the seller at night. At the end of the period, all matches are terminated. Without loss, unmatched agents and matched agents who rejected the proposed allocation simply exit the night market and re-enter the day market in the next period.

The value function for a buyer evaluated at the beginning of the day market is

(3) $$V^b = \sigma\left[u(q) - y\right] + \beta V^b,$$

assuming, of course, that both the buyer and seller accept allocation (q,y). According to

(3), the buyer meets a seller with probability σ and, in the event that he meets a seller, he consumes q units of the search good and produces y units of the general good. Note that there is no state variable that characterizes the lifetime expected utility of a buyer since agents hold no assets at the beginning of the period, and the agents' trading histories are irrelevant. In addition, since we focus on stationary allocations, time indexes are suppressed on variables and functions. The value function of a seller evaluated at the beginning of the day market is

$$(4) \qquad V^s = \sigma\left[-c(q)+y\right]+\beta V^s.$$

Equation (4) has an interpretation similar to (3), except for the fact that during the day sellers produce (and buyers consume) the search good and at night sellers consume (and buyers produce) the general good.

Since agents are able to commit, the only relevant constraints are buyers' and sellers' *participation constraints*, which are evaluated at the time that a match is formed. The participation constraints indicate whether agents are willing to participate in a given mechanism or to go along with a given allocation. These constraints are

$$(5) \qquad u(q) - y + \beta V^b \geq \beta V^b,$$

$$(6) \qquad -c(q) + y + \beta V^s \geq \beta V^s.$$

According to (5), a buyer will accept allocation (q,y) if the lifetime utility associated with acceptance, the left-hand side of (5), exceeds the lifetime utility associated with rejection, the right-hand side of (5), or if his surplus from the trade, $u(q) - y$, is non-negative. Condition (6) has a similar interpretation for the seller. From (5) and (6), the set of *incentive-feasible* allocations, $\mathcal{A}^C$, is given by

$$(7) \qquad \mathcal{A}^C = \left\{(q,y)\in \mathbb{R}_+^2 : c(q) \leq y \leq u(q)\right\}.$$

From (7) it is easy to check that $\{q^*\}\times[c(q^*),u(q^*)]\subseteq \mathcal{A}^C$. This implies that the efficient level of production and consumption of the search good, q^*, is incentive-feasible for any values of β and σ. In presence of commitment, the intertemporal nature of the trades and any associated moral hazard considerations are irrelevant.

Hence, the gains from trade will be maximized if agents produce and consume q^* units of the search good in the day. And the level of output for the general good, y, will determine how the gains from trade are split between the buyer and seller. But since any allocation in $\mathcal{A}^C$ is incentive-feasible, which of these allocations will be implemented? Can agents somehow agree to produce q^* units of the search good? If so, how is y determined? One way to address these questions is to impose a bargaining procedure for bilateral matches and to characterize the outcome of the bargaining procedure. For

example, we can assume that the allocation (q,y) is determined in accordance with the generalized Nash bargaining solution, where the buyer's bargaining power is $\theta \in (0,1)$. The generalized Nash bargaining solution maximizes a weighted geometric mean of the buyer's surplus, $u(q) - y$, and the seller's surplus, $-c(q) + y$, from trade, where the weights are given by the agents' bargaining power. The generalized Nash bargaining solution is given by the solution to

$$(8) \qquad \max_{q,y} \left[u(q) - y \right]^{\theta} \left[y - c(q) \right]^{1-\theta}.$$

The solution to (8) is $q = q^*$ and $y = (1 - \theta)u(q^*) + \theta c(q^*)$. Note that the allocation is efficient for any buyer's bargaining power θ in the sense that the level of the search production is always at the efficient level q^*. Furthermore, as one varies θ over $[0,1]$, the set of generalized Nash bargaining solutions is given by $\{q^*\} \times [c(q^*), u(q^*)]$; as θ increases, the buyer's share of the surplus increases and the seller's share decreases.

Credit with Public Recordkeeping

In this section, we relax the assumption of commitment. In order to sustain trade in a credit economy when agents cannot commit, they must experience some sort of negative consequence if they do not deliver on their promises. The punishment that we impose is that an agent can no longer use credit if he fails to pay back his debt obligation. Furthermore, we will consider global punishments, in the sense that the entire economy reverts to autarky if at least one agent behaves in an opportunistic manner.

When there is a large number of agents in the economy, there must be some sort of public recordkeeping of agents' trades if punishments are to be feasible and effective. We assume that there exists a public-record device that provides all agents in the economy with the list of quantities of the search and general goods that were produced and traded during the period. In particular, the pair (q,y) is recorded for all agents in a trade match, and this information is made available at the end of each period, i.e., at the end of each night. Note that the public record lists only quantities, not the names of the agents who produced the quantities. It is for this reason that any deviation from proposed play will result in a global punishment. If names were associated with quantities, then nonglobal (personalized) punishments would be possible. It turns out that very little is changed if nonglobal punishments are possible, and we discuss the implications of nonglobal punishments at the end of this section.

The chronology of events is as follows: At the beginning of the day market, buyers and sellers are randomly matched and meet a trading partner with probability σ. In each trade match, an allocation (q,y) is proposed and agents simultaneously accept or reject trade at those terms. If the allocation is accepted, then the seller produces q units of the search good for the buyer. At night, the buyer chooses to either produce y units of the

general good for the seller or to renege on his promise and produce nothing. At the end of the night, a list of all the pairs of the day and night production levels for agents who were in a trade match is publicly observed. Based on this list, agents decide whether or not to enter the morning search market or exit the day and night markets forever to live in autarky.

We restrict our attention to symmetric stationary allocations (q, y) that are incentive-feasible. Incentive-feasibility now implies that not only do the buyer and the seller agree to allocation (q, y), as before, but that the buyer is willing to repay his debt when it is his turn to produce. We assume that agents choose autarky at the end of the night whenever an allocation from any trade match is different from the proposed allocation (q, y). (Indeed, having all agents revert to autarky is an equilibrium outcome in this situation.) During the day, matched sellers and buyers agree to a trade (q, y) if

(9) $-c(q) + y + \beta V^s \geq 0,$

(10) $u(q) - y + \beta V^b \geq 0.$

Condition (9), which is the seller's participation constraint, says that a seller prefers allocation (q, y) plus the continuation value of participating in future day and night markets, βV^s, to autarky at the time when the match is formed. The seller compares the payoff associated with acceptance to that of autarky because if the seller rejects the proposal, a $(0, 0)$ trade will be recorded and such a trade will trigger global autarky. The condition (10) has an interpretation similar to (9) but for the buyer; i.e., the buyer prefers to go along with the suggested trade (q, y) rather than go to autarky. Note that the participation constraints (9) and (10) differ from the participation constraints when agents could commit, (5) and (6), in that now agents go to autarky if they do not accept the proposed allocation (q, y).

Since the buyer produces the general good after he consumes the search good, we now need to check that the buyer is, in fact, willing to produce. The buyer will have an incentive to produce the general good for the seller if

(11) $-y + \beta V^b \geq 0.$

The left-hand side of inequality (11) is the buyer's payoff if he repays his debt by producing y units of output for the seller and the right-hand side is his continuation payoff of zero if he defaults (since the economy reverts to autarky). Clearly, if the buyer's incentive constraint (11) is satisfied, then so is his participation constraint (10).

Equations (3) and (4) still represent the beginning-of-period value functions for the buyer and seller, respectively. Using these Bellman equations, the seller's participation constraint (9) and the buyer's incentive constraint (11) can be re-expressed, respectively, as

(12) $-c(q) + y \geq 0$

(13) $\dfrac{\sigma[u(q) - y]}{r} \geq y,$

where $r = \beta^{-1} - 1$. Condition (12) simply says that the seller has to get some of the surplus from the trade match to be willing to participate in the trade. Note that this participation condition does not depend on discount factors or matching probabilities. Condition (13) is the incentive constraint for the buyer to repay his debt. The left-hand side of (13) is the buyer's expected payoff beginning next period if he pays back his debt; it is the discount sum of the expected surplus from trade in future periods. This term depends on the frequency of trades, σ, and on the discount rate, r. The right-hand side of (13) is the buyer's (lifetime) gain if he does not produce the general good for the seller. Not surprising, because a necessary—but not sufficient—condition for inequality (13) to hold is that the buyer's surplus from the trade is positive, i.e., $u(q) - y \geq 0$.

The set of incentive-feasible allocations, $\mathcal{A}^{PR}$, when agents cannot commit but when public recordkeeping is available, is obtained by combining inequalities (12) and (13), i.e.,

(14) $\mathcal{A}^{PR} = \left\{ (q, y) \in \mathbb{R}_+^2 : c(q) \leq y \leq \dfrac{\sigma}{r + \sigma} u(q) \right\}.$

The set of incentive-feasible allocations, $\mathcal{A}^{PR}$, is smaller than the set of incentive-feasible allocations when agents can commit, $\mathcal{A}^C$. This is a consequence of the additional buyer incentive constraint, (11), which must be imposed when buyers are unable to commit to repay their debts. Note that the set $\mathcal{A}^{PR}$ expands as the frequency of trades, σ, increases or as agents become more patient, i.e., when r decreases and when $r \to 0$, $\mathcal{A}^{PR} \to \mathcal{A}^C$. The efficient production and consumption level of the search good, q^*, is incentive-feasible if

(15) $c(q^*) \leq \dfrac{\sigma}{r + \sigma} u(q^*).$

The right-hand side of (15) is increasing in σ and decreasing in r. Suppose that inequality (15) holds for a particular σ and r. Then if σ decreases, the probability of finding a future match decreases and, hence, the buyer has a greater incentive not to produce y since the benefit of avoiding autarky has now been reduced. If σ falls sufficiently, then the buyer will not, for sure, produce any general good for the seller and, therefore, the efficient level of production and consumption of the search good is not incentive-feasible. Similarly, if buyers discount the future more heavily, i.e., if β decreases or if r increases, the buyer will have a greater incentive not to produce y since he cares more about his current payoff than future payoffs. Once again, if r increases sufficiently, the efficient level of production and consumption of the search good is not incentive-feasible.

Another way to think about inequality (15) is as follows. For each level of search friction in the day market, $\sigma \in (0,1]$, there exists a threshold for the discount factor, $\bar{\beta}(\sigma)$, such that if $\beta \geq \bar{\beta}(\sigma)$, then the efficient allocation (q^*, y^*) is incentive-feasible. This threshold $\bar{\beta}(\sigma)$ is a decreasing function of σ, which means that the efficient level of production and consumption of the search good, q^*, is easier to sustain when there are few frictions in the day market. If, however, $\beta(\sigma) \leq \bar{\beta}$, incentive-feasible allocations are such that the output level of the search good, q, is inefficiently low, i.e., $q < q^*$.

Instead of characterizing the set of all incentive-feasible allocations one can, alternatively, focus on the allocation that would be suggested by a bargaining solution; for instance, the generalized Nash bargaining solution. The bargaining outcome, (q,y), in a trade match now needs to be restricted to take into account the condition under which the buyer has an incentive to repay his debt. When agents are unable to commit, but there is a public recordkeeping device, the generalized Nash bargaining solution is given by

$$(16) \qquad \max_{q,y} \left[u(q) - y \right]^{\theta} \left[y - c(q) \right]^{1-\theta},$$

$$(17) \qquad \text{s.t.} \quad -y + \beta V^b \geq 0.$$

The solution to this problem is given by $q = q^*$ and $y = (1-\theta)u(q^*) + \theta c(q^*) \equiv y^*$ if $\beta V^b \geq y^*$; and $\theta u'(q)[y - c(q)] = c'(q)(1-\theta)[u(q) - y]$ and $y = \beta V^b$ otherwise. When $\beta V^b \geq y^*$, the generalized Nash bargaining solution here corresponds exactly to the generalized Nash bargaining solution when agents are able to commit. Otherwise, the bargaining solutions will differ. When the buyer lacks commitment, he has an "outside option" of not producing the general good for the seller. If the proposed bargaining outcome (q^*, y^*) does not provide the buyer with sufficient surplus, then the buyer will choose not to produce the general good for the seller, and this happens when $\beta V^b < y^*$. In such a situation the unconstrained generalized Nash bargaining solution (q^*, y^*) will not be incentive-feasible and the constrained generalized Nash bargaining solution (q,y) will be characterized by $q < q^*$ and $y < y^*$. When agents are unable to commit, it can be shown that (q^*, y^*) is the generalized Nash bargaining solution if

$$(18) \qquad c(q^*) \leq \left[\frac{\theta(\sigma + r) - r}{\theta(r + \sigma)} \right] u(q^*).$$

Inequality (18) implies that a necessary condition for (q^*, y^*) to be the generalized Nash bargaining solution is that the term in the square brackets on the left-hand side of the inequality is greater than zero, or that $\theta > r/(r + \sigma)$. Hence, buyers must have sufficiently high bargaining power in order to obtain the efficient consumption and production of the search good. Note also that conditions (15) and (18) coincide whenever $\theta = 1$, i.e., when buyers have all the bargaining power.

We conclude this section with a brief discussion of punishments. First, we have assumed that if agents in a trade match do not accept the proposed offer, then the economy will forever revert to autarky in the next period. Suppose instead that we assume that only outputs produced from accepted proposals become part of the public record—i.e., agents could, without impunity, reject the proposed offer. Now, the agents' participation constraints are given by (5) and (6), instead of (9) and (10); for this case, it can be shown that all of the above results still go through. This, perhaps, should not be so surprising because as long as agents receive some surplus from trade, they will always accept the proposed offer. Second, we have assumed that if an agent defects from proposed play, then the economy will revert to global autarky forever. This is because an agent defecting from equilibrium play is not identified by other agents in the economy, except his trading partner. Since the trading partner is of measure zero, the defector could escape individual punishment even if his trading partner and subsequent trading partners of that trading partner could credibly pass on the message that a particular agent defected from equilibrium play. If, however, in addition to the list of outputs that are observed at the end of the night market, the name associated with each output is observed, then it is possible to support credit arrangements through *individual* punishments; that is, the above credit arrangements can be sustained without having to revert to global autarky in the event of a defection from a proposed allocation.

Credit with Reputation

Any credit arrangement necessitates some degree of cooperation between a buyer and a seller, or a debtor and a creditor. As is well known, cooperation is more easily attainable when agents repeatedly interact with one another. Intuitively, with repeated interactions, agents are able to develop reputations for behaving appropriately. In this section, we rule out the existence of both commitment and public recordkeeping but introduce the notion of reputation by allowing pairs of agents to repeatedly interact with one another via a *long-term partnership*. We do this by assuming that agents who are in a trade match during the day can form a partnership that can be maintained beyond the current period. That is, agents can continue their trade match or partnership into the next period (day) if they so desire.

We allow for both the creation and destruction of a partnership. At the end of each period, an existing partnership is destroyed with some probability $\lambda \in (0,1)$. One can justify the exogenous destruction of a partnership by supposing that sellers produce different types of goods and buyers only value a subset of these goods. A match destruction can be interpreted as an event where the buyer receives a preference shock (at night) with the result that he no longer wants to consume the good that the seller produces. In this case, the partnership is no longer viable and agents split apart. More generally, agents can choose to terminate a partnership at the end of any period. For example, the seller

may choose to dissolve the partnership at the end of the period if the buyer does not deliver on his promise to produce the general good. This sort of termination is important because it provides the seller with a punishment vehicle—namely, the destruction of the value asset of an enduring match or partnership—which is required in order to make a partnership viable in the first place.

The chronology of events is as follows. At the beginning of the day, unmatched buyers and sellers participate in a random matching process. Each unmatched agent finds a partner with probability σ; that is, with probability σ the buyer is matched with a seller whose search good he desires to consume. In each match, an allocation (q,y) is proposed, which agents can either accept or reject. If both agents accept the offer, the seller produces q units of the search good for the buyer in the day. At night, the buyer chooses whether or not to produce y units of the general good for the seller. At the end of the night, agents decide simultaneously to stay together or to split apart. If either or both agents decide to destroy the partnership, then (in equilibrium) their best response will be to enter the random matching process at the beginning of the day in order to find a new trading partner. A partnership, which is basically a continuation of a trade match, can only be formed during the random matching process at the beginning of the day.

We will characterize the set of symmetric stationary equilibrium allocations for this economy. Let e_t denote the measure of new trade matches and existing partnerships during the day of period t. Assuming that buyers do not renege on their promises, the law of motion for e_t is

(19) $\qquad e_{t+1} = (1-\lambda)e_t + \sigma[1-(1-\lambda)e_t].$

According to (19), if there are e_t partnerships in period t, a fraction $(1-\lambda)$ of them will be maintained in period $t+1$. Among the $1-(1-\lambda)e_t$ agents who are unmatched at the beginning of $t+1$, a fraction σ find new partners. In the steady state, $e_{t+1} = e_t = \bar{e}$ which, from (19), implies that

(20) $\qquad \bar{e} = \dfrac{\sigma}{\sigma + \lambda(1-\sigma)}.$

Let V_m^b be the value function of a buyer who is in a partnership at the beginning of a period and V_u^b the value function of a buyer who is not. Then, assuming that the buyer does not defect,

(21) $\qquad V_m^b = u(q) - y + \lambda\beta V_u^b + (1-\lambda)\beta V_m^b,$

(22) $\qquad V_u^b = \sigma V_m^b + (1-\sigma)\beta V_u^b.$

According to (21), the buyer receives q units of search goods in the day and produces y units of general goods at night. The partnership is exogenously destroyed with probability λ, in which case both the buyer and the seller go to the random matching process at the beginning of the day of the next period to find a new partner. According to (22), an unmatched buyer finds a seller who produces a search good that he desires to consume with probability σ.

Let V_m^s be the value function of a seller who is in a partnership at the beginning of the period and V_u^s the value function of a seller who is not. Then,

$$(23) \qquad V_m^s = -c(q) + y + \lambda \beta V_u^s + (1-\lambda)\beta V_m^s,$$

$$(24) \qquad V_u^s = \sigma V_m^s + (1-\sigma)\beta V_u^s.$$

According to (23), the seller produces q units of the search good during the day and consumes y units of the general good at night. With probability λ the partnership is dissolved, in which case the seller enters the random matching process at the beginning the next period. According to (24), the seller is matched with a buyer who likes the search good that the seller produces with probability σ.

For (q,y) to be an equilibrium allocation, three sets of conditions have to be satisfied. First, agents who enter the day search market unmatched, and subsequently become matched, will accept the proposed allocation (q,y) if the following (participation) constraints hold,

$$(25) \qquad -c(q) + y + \lambda \beta V_u^s + (1-\lambda)\beta V_m^s \geq \beta V_u^s,$$

$$(26) \qquad u(q) - y + \lambda \beta V_u^b + (1-\lambda)\beta V_m^b \geq \beta V_u^b,$$

If the seller and buyer accept the allocation (q,y), then their expected payoffs are given by the left-hand sides of (25) and (26), respectively. If, however, the allocation is rejected, the continuation payoffs are given by the right-hand sides of (25) and (26), respectively. Second, if the buyer does not receive a preference shock, then a matched buyer and the seller will agree to continue their partnership if

$$(27) \qquad -c(q) + y + \lambda \beta V_u^s + (1-\lambda)\beta V_m^s \geq V_u^s,$$

$$(28) \qquad u(q) - y + \lambda \beta V_u^b + (1-\lambda)\beta V_m^b \geq V_u^b.$$

If the seller and the buyer choose to continue the partnership, their payoffs at the beginning of the subsequent period are given by the left-hand sides of (27) and (28), respectively. If the seller and buyer choose to dissolve the partnership, the expected payoffs at the beginning of the subsequent period are given by the right-hand sides of (27) and (28),

respectively. Clearly, if inequalities (27) and (28) hold, then inequalities (25) and (26), respectively, hold as well. Finally, a buyer in a partnership must be willing to produce the general good for the seller at night. This requires that

$$(29) \qquad -y + \lambda \beta V_u^b + (1-\lambda)\beta V_m^b \geq \beta V_u^b.$$

If the buyer produces y units of the general good, then his expected payoff is given by the left-hand side of (29). If, however, he deviates and does not produce, then the partnership will be dissolved at the end of the period and the buyer starts the next day search market seeking a new match; the utility associated with this outcome is given by the right-hand side of (29).

The set of incentive-feasible allocations that can be sustained by reputations, which can be inferred from the value functions (21)–(24) and constraints (27)–(29), is given by

$$(30) \qquad \mathcal{A}^R = \{(q,y) : c(q) \leq y \leq \beta(1-\lambda)(1-\sigma)u(q)\}.$$

Note that inequalities (27) and (28) require that $c(q) \leq y \leq u(q)$ in order for both the seller and the buyer to continue with the partnership. The buyer's incentive-compatibility condition, (29), however, generates the endogenous borrowing constraint $y \leq \beta(1-\lambda)(1-\sigma)u(q)$. This endogenous borrowing constraint indicates that the maximum amount the buyer can promise to repay at night depends on the buyer's tastes, β, the stability of the match, λ, and market frictions, σ. The buyer can promise to repay more: (i) the more patient he is, i.e., the higher is β; (ii) the more stable are his preferences, i.e., the lower is λ; (iii) the greater is the matching friction, i.e., the lower is σ; and (iv) the higher is his consumption, q, the next day.

Note that from (30), when $q > 0$, the set of incentive-feasible allocations, $\mathcal{A}^R$, is empty when all matches are destroyed at the end of a period, i.e., when $\lambda = 1$, or when an agent can find a partner in the day market with certainty, i.e., when $\sigma = 1$. The existence of credit relationships relies on the threat of termination, but such a threat has bite only if matches, for exogenous reasons, are not destroyed with high probability or if it is difficult to create a new trade match.

From (30), the efficient production and consumption level of the search good, q^*, is implementable if and only if

$$(31) \qquad c(q^*) \leq \beta(1-\lambda)(1-\sigma)u(q^*).$$

Agents are able to trade the quantity q^* through long-term partnerships if the average duration of a long-term partnership is high, i.e., if λ is low, and if the matching frictions are severe, i.e., if σ is low.

Related Literature

Pairwise credit in a search-theoretic model was first introduced by Diamond (1987a,b, 1990). The environment described by Diamond is similar to that in one of his earlier paper (Diamond, 1982), where agents are matched bilaterally and trade indivisible goods. The number of trades is given by a matching function that exhibits increasing returns to scale. Diamond allows agents to use lotteries in order to endogenize terms of trade. As in our setup credit is repaid with goods. The punishment for not repaying a loan is permanent autarky.

Kocherlakota (1998a,b) describes credit arrangements in different environments, including a search-matching model based on a public record of individual transactions. He uses mechanism design to characterize the set of symmetric, stationary, incentive-feasible allocations. Kocherlakota and Wallace (1998) extends the model to consider the case where the public record of individual transactions is updated after a probabilistic lag. They establish that society's welfare increases as the frequency with which the public record is updated increases. As pointed out by Wallace (2000), this is the first model that formalizes the idea according to which technological advances in the payment system improve welfare. The model by Kocherlakota and Wallace has been extended by Shi (2001) to discuss how the degree of advancement of the credit system affects specialization.

Most search-theoretic models of the labor market assume long-term partnerships. A canonical model is provided by Pissarides (2000). However, in these economies, trades do not involve credit and are free of moral hazard considerations. Corbae and Ritter (2004) consider an economy with pairwise meetings where agents can form long-term partnerships to sustain credit arrangements. A related model of reciprocal exchange is also presented by Kranton (1996).

Aiyagari and Williamson (1999) consider a random-matching model in which agents receive random endowments that are private information. Exchange is motivated by risk-sharing. The social planner designs the optimal dynamic contract. It acts as a financial intermediary that opens accounts for the different agents. Optimal allocations have several features similar to those of real-world credit arrangements, including credit balances and credit limits.

Pure Monetary Economies

In the previous section, credit arrangements could be sustained because agents could commit to repay their debt, observe a record of other agents' trades, or were able to form enduring relationships. In this section, we assume that none of these options is possible. As a result, trade cannot be mediated by credit, and some alternative payments instrument must emerge if trade is ever to occur. The payments instrument that we consider here is fiat money, which is a durable but intrinsically useless object. Fiat money is essen-

tial to the economy in the sense that its existence allows buyers and sellers to trade with one another; the introduction of fiat money generates (desirable) outcomes that would not be possible in its absence.

The economic environment is the benchmark model. During the day, buyers and sellers are randomly matched; in a trade match, the seller produces the search good for the buyer. We will consider two versions of the night market. The first version enables us to compare money economies with credit economies. In this version, buyers and sellers who are matched in the day can continue their relationship at night; at the end of the period, all matches are dissolved. This setup is similar to that of the credit economies in the previous section. The second version of the night market is a more "standard" setup for models of money. In this version, matches are destroyed at the end of the day, and at night there exists a competitive market where all agents can trade money for the general good.

Money Is Memory

How does a monetary economy compare with a credit economy in terms of the set of allocations that can be implemented? To address this question, we structure a monetary economy to mirror a credit economy when public recordkeeping is possible.

At the beginning of time, each buyer is endowed with one unit of an indivisible and durable object, fiat money, that is intrinsically useless. The sequence of events in a typical period is as follows: At the beginning of the day, buyers and sellers are matched pairwise and at random. In each trade match, an allocation (q, d^{am}) is proposed where q is the amount of search good that the seller produces, and d^{am} is the transfer of one unit of money from the buyer to the seller. The allocation can depend on the money balances of the buyer and seller in the match. The buyer and the seller in a trade match simultaneously accept or reject the offer. If one of the agents rejects the offer, then the match is dissolved; otherwise, the trade takes place and agents remain matched into the night. At night, an allocation (y, d^{pm}) is proposed where y is the amount of the general good that is produced by the buyer for the seller and d^{pm} is the money transfer from the seller to the buyer. The matched agents decide simultaneously on whether to accept or reject this allocation. If it is accepted by both agents, then it is implemented. At the end of the period, all matches are dissolved.

We will consider allocations such that all buyers at the beginning of each period hold one unit of money. This guarantees that whenever a buyer is matched with a seller, he is able to trade. If matched buyers and sellers follow their equilibrium strategies of accepting the proposed (incentive-feasible) allocations and producing output for one unit of money, then buyers will, in fact, always begin each period with one unit of money in hand and sellers with no units of money. The mechanism that we consider is such that if a matched buyer does not hold one unit of money at the beginning of the period, or, if the seller holds more than zero units of money, then the proposed allocation will be

$(q,d^{\mathrm{am}}) = (0,0)$; and if the seller does not hold exactly one unit of money at night, then the proposed allocation is $(y,d^{\mathrm{pm}}) = (0,0)$. If any one of these circumstances arises, it necessarily implies that in the past, an agent departed from equilibrium play. Hence, an agent's money holdings reveal if he behaved according to the proposed allocation in the past.

The value function of a buyer holding one unit of money at the beginning of the day is

$$(32) \qquad V^b(1) = \sigma\left[u(q) - y\right] + \beta V^b(1).$$

According to (32), the buyer meets a seller with probability σ. If this event occurs, then the buyer will consume q units of the search good in exchange for his unit of money. At night the buyer gets his unit of money back in exchange for y units of the general good. If the buyer does not hold money, the mechanism proposes no trade and, therefore, $V^b(0) = 0$. (Alternatively, the mechanism could allow the buyer to get his money back if it proposes an allocation that has $y = \beta V^b(1)$, which implies that $V^b(0) = 0$).

The value function of a seller holding zero units of money at the beginning of a period is

$$(33) \qquad V^s(0) = \sigma\left[-c(q) + y\right] + \beta V^s(0).$$

According to (33), a seller meets a buyer with probability σ. In the event a trade match occurs, the seller produces q units of the search good for the buyer in exchange for one unit of money. At night the seller gives the unit of money back to the buyer in exchange for y units of the general good. If the seller holds a different amount than zero at the beginning of a period, he cannot trade and, therefore, $V^s(m) = 0$ for $m > 0$, i.e., in this situation, the mechanism will propose the offer $(0,0)$.

For the allocations $\{(q,1),(y,1)\}$ to be incentive-feasible, agents must be willing to participate in a trade. This requires

$$(34) \qquad u(q) - y + \beta V^b(1) \geq \beta V^b(1),$$

$$(35) \qquad -c(q) + y + \beta V^s(0) \geq \beta V^s(0),$$

$$(36) \qquad -y + \beta V^b(1) \geq \beta V^b(0),$$

$$(37) \qquad y + \beta V^s(0) \geq \beta V^s(1).$$

Conditions (34) and (35) require the buyer and the seller, respectively, to accept the trade during the day, while conditions (36) and (37) require the buyer and the seller, respectively, to accept the trade at night. Conditions (34) and (35) imply that $c(q) \leq y \leq u(q)$, i.e., the buyer and seller surpluses from trade must be non-negative. Clearly, condition (37) is

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satisfied whenever (35) holds. Finally, condition (36), in conjunction with the value function (32), implies that

$$(38) \qquad y \le \frac{\sigma}{r + \sigma} u(q).$$

Hence, the set of implementable incentive-feasible allocations in this monetary economy, $\mathcal{A}^M$, is

$$(39) \qquad \mathcal{A}^M = \left\{ (q, y) \in \mathbb{R}_+^2 : c(q) \le y \le \frac{\sigma}{r + \sigma} u(q) \right\}.$$

The set of incentive-feasible allocations in the monetary economy is identical to the set of incentive-feasible allocations in a credit economy with a public record, i.e., $\mathcal{A}^M = \mathcal{A}^{PR}$, where the set $\mathcal{A}^{PR}$ is described in (14). In this sense, money is equivalent to a public recordkeeping mechanism, i.e., *money is memory*. The explanation for this result is as follows. The money balances of an agent are a state variable that conveys some information about his past trading behavior. The money balances of an agent indicate whether he has defected from a given allocation by not producing for his trading partner when it was his turn to produce. Money here can also be interpreted as a license to consume or as collateral. The buyer who transfers his unit of money to the seller knows that he won't be able to consume in future periods if he does not get his unit of money back at night.

Indivisible Money and Currency Shortage

We now assume that matches are dissolved at the end of the day. As well, we will also impose a specific pricing mechanism for both search goods and general goods. During the day, the terms of trade in bilateral matches are determined according to a bargaining process. We assume that the buyer has all of the bargaining power and makes a take-it-or-leave-it offer to the seller. At night, we assume that the market for general goods is competitive, where price-taking agents can buy and sell units of money in exchange for general goods at the market-clearing price, ϕ, where ϕ represents how many general goods can be purchased at night for one unit of money. As is typically the case for Walrasian markets, the way in which agents meet or trades actually take place is not made explicit. Since agents trade with "the market," they are anonymous to one another; i.e., agents who purchase goods do not know which agents produced them, and agents who receive money do not know which agent previously held the money. This anonymity precludes the use of credit arrangements in the general goods market.

Regarding the quantity of money in the economy, we assume that $M < 1$. This implies that not all buyers can be endowed with money: There is a currency shortage. The reason that we investigate this case is twofold. First, currency shortages were common until the

mid-ninteenth century and they can be captured by this assumption. Second, this assumption was used in the early search-theoretic literature of money for tractability purposes.

The value function for a buyer holding m units of money at the beginning of the period satisfies

$$(40) \qquad V^b(m) = \sigma \max_{(q,d) \in O(m)} \left\{ u(q) + W^b(m-d) \right\} + (1-\sigma)W^b(m),$$

where $O(m)$ is the set of offers that are acceptable by sellers and that are feasible given the money balances of the buyer, and W^b is the value function of a buyer at the beginning of the night. According to (40), a buyer with money at the beginning of the day is randomly matched with a seller with probability σ. In a trade match, he consumes q units of the search good and delivers d units of money to the seller, where both q and d are chosen optimally in the set of acceptable offers. The value function of a buyer in the Walrasian market at the beginning of the night satisfies

$$(41) \qquad W^b(m) = \max_{\hat{m},x,y} \left\{ x - y + \beta V^b(\hat{m}) \right\}$$

$$(42) \qquad \text{s.t. } x + \phi\hat{m} = y + \phi m.$$

The budget constraint (42) simply says that the buyer finances his end-of-period money balances, $\hat{m}$, and consumption, x, with production of the general good, y, and with money balances brought into the night, m. Substituting $x - y$ from the budget identity (42) into the maximand of (41) we obtain

$$(43) \qquad W^b(m) = \phi m + \max_{\hat{m}} \{ -\phi\hat{m} + \beta V^b(\hat{m}) \}.$$

Note that equation (43) tells us that the buyer's choice of end-of-period money balances, $\hat{m}$, is independent of the money balances brought in from the day, m. This comes from the linearity of the utility function that eliminates wealth effects. Furthermore, it is straightforward to see that $W^b(m) = \phi m + W^b(0)$.

Sellers spend all of their money balances at night in the general goods market since they do not require money in order to trade in the morning search market, and it is costly to hold money balances in the morning search market. Hence, the value function for a seller at the beginning of the period is given by

$$(44) \qquad V^s = \sigma \int [-c(q) + \phi d] dF(q,d) + \beta V^s,$$

where $F(q,d)$ is the distribution of offers made by buyers, which depends on the distribution of money balances across buyers. According to the value function (44), the seller

is matched to a buyer with probability σ, and the offer (q,d) is a random draw from the distribution $F(q,d)$.

A seller in a trade match will accept the offer (q,d) if it satisfies $-c(q)+\phi d+\beta V^s \geq \beta V^s$. Consequently, the set of offers the buyer with m units of money can make, $O(m)$, is given by

$$O(m) = \{(q,d): -c(q)+\phi d \geq 0, d \leq m\}.$$

The buyer will extract all of the surplus in the trade match, which implies that the buyer's offer (q,d) will satisfy

(45) $\quad -c(q)+\phi d = 0.$

Equations (44) and (45) imply that $V^s = 0$, and equations (40) and (45) may be rearranged to read

$$V^b(m) = \sigma \max_{d \in \{0,\ldots,m\}} \left[u \circ c^{-1}(\phi d) - \phi d \right] + W^b(m).$$

Substituting this expression for $V^b(m)$ into the buyer's value function at the beginning of the night, equation (43), the buyer's problem, can be re-expressed more compactly as

(46) $\quad \max_{\hat{m}} \left\{ -r\phi\hat{m} + \sigma \max_{d \in \{0,\ldots,\hat{m}\}} \left[u \circ c^{-1}(\phi d) - \phi d \right] \right\},$

where $r = (1-\beta)/\beta$. Equation (46) has a simple interpretation. The buyer faces a trade-off when determining his money holdings: There is a cost associated with holding real balances, which is equal to agents' rate of time preference; the cost of bringing $\hat{m}$ balances into the next period is $r\phi\hat{m}$. But there is also a benefit associated with holding real balances, which is equal to the expected surplus that is obtained in the search market; the expected surplus is given by $\sigma[u(q)-\phi d]$. Since $r > 0$, it is easy to check from (46) that buyers will not hold more money than they expect to spend if they are matched in the search market; this implies that $d = \hat{m}$ and $c(q) = \phi\hat{m}$.

If money were perfectly divisible, the buyer's maximization problem (46) would have a unique solution. However, because money is indivisible, this solution may not be attainable. It can be checked that (46) has, at most, two solutions which are two consecutive integers. Furthermore, since M, the quantity of money per buyer, is less than one, market-clearing implies that buyers must be indifferent between holding 0 or 1 unit of money. Therefore, $\hat{m} = 1$ and, from problem (46),

(47) $\quad r\phi = \sigma\{u(q)-\phi\}.$

The seller's participation constraint (45) implies that $c(q) = \phi$ and, therefore,

$$(48) \qquad c(q) = \frac{\sigma}{r + \sigma} u(q).$$

An equilibrium in this environment is the quantity of search goods produced, q, that satisfies (48). Given our assumptions about c and u, it is easy to check that there exists a unique $q > 0$ that satisfies (48).

Let us turn to a comparative static analysis of the purchasing power of money as fundamentals change. From (48), $\partial q / \partial \sigma > 0$ and $\partial q / \partial r < 0$. As the matching probability σ increases, a buyer has a higher chance to trade during the day, which makes money more valuable. As a consequence, the quantities traded increase. As the rate of time preference, r, increases, agents become more impatient, and the cost of holding money increases. As a consequence, the value of money falls, and agents trade less. Note that the purchasing power of money is independent of M, the quantity of money in the economy.

If we measure social welfare by the sum of utilities of buyers and sellers, then $W = \sigma M[u(q) - c(q)]$. Here money is not neutral and an increase in M, for $M \in (0,1)$, raises welfare by increasing the number of trades in the economy. Hence, a change in M has no effect on the *intensive margin*, which is the quantity produced in a particular trade match, but affects the *extensive margin*, which is the number of trade matches. We will see later how this extensive margin result disappears when money is assumed to be perfectly divisible.

Depending on the precise functional form of u and c and the values of β and σ, the equilibrium value of the amount of search goods produced, q, can be greater than, less than, or equal to the efficient level, q^*. Hence, a problem associated with the indivisibility of money is that sometimes the amount of search goods produced, q, is greater than what is socially efficient, q^*. This can easily be seen from (48), if we assume that $r \approx 0$.

Indivisible Money and Lotteries

When money is indivisible, there are circumstances under which the buyer could be made better off if he could somehow give up only a fraction of his money balances to the seller; this happens when $q > q^*$. Before turning to the case of perfectly divisible money, we show that there is a way to achieve this outcome *when money is indivisible* by introducing lotteries over the outcome (q,d), $d \in \{0,1\}$. Since there is no (social) benefit associated with having a lottery over output, the take-it-or-leave-it offer by the buyer of the search good can be described by (q, τ), where q is the amount of the search good produced by the seller, and τ is the probability that the buyer surrenders his unit of money to the seller.

Consider a trade match between a buyer and a seller. As above, equilibrium in the general goods market will require that buyers hold either zero or one unit of money at the

end of the period. In a trade match, the take-it-or-leave-it offer that the buyer makes to the seller, (q, τ), solves the problem

(49) $\quad \max_{q, \tau}[u(q) - \tau\phi] \quad$ s.t. $\quad -c(q) + \tau\phi \geq 0, \quad$ and $\quad 0 \leq \tau \leq 1.$

The solution is $q = q^*$ and $\tau = c(q^*)/\phi$ if $\tau \leq 1$; otherwise $\tau = 1$ and $q < q^*$ satisfies (48). The solution to the buyer's choice of money balances to bring into the next period—where the problem is now described by (46), except that ϕd is replaced by $\tau\phi d$—is given by

(50) $\quad r\phi = \sigma[u(q) - \tau\phi],$

which is a modified version of equation (47). An expression for τ can be obtained by substituting the seller's participation constraint, $-c(q) + \tau\phi = 0$, into (50) and rearranging, i.e.,

(51) $\quad \tau = \min\left\{\dfrac{rc(q^*)}{\sigma[u(q^*) - c(q^*)]}, 1\right\}.$

Hence, $q = q^*$ iff $\tau \leq 1$ and, from equation (51), $\tau \leq 1$ iff $c(q^*) \leq \sigma u(q^*)/(r + \sigma)$. So if allocation $(q^*, c(q^*))$ is incentive-feasible in the "money is memory" environment (see the section on Money is Memory and the definition of $\mathcal{A}^M$ in [39]) or in a credit environment with public recordkeeping (see the section on Credit with Public Recordkeeping and the definition of $\mathcal{A}^{PR}$ in [14]), then it can be implemented as an equilibrium in a monetary economy with indivisible money by a take-it-or-leave-it offer when buyers can use lotteries, i.e., if $(q^*, c(q^*)) \in \mathcal{A}^M = \mathcal{A}^{PR}$, then $\tau \leq 1$. If $\tau = 1$, then the level of search good production, q, is given by equation (48), and $q \leq q^*$.

Divisible Money

We now turn to the case where money is perfectly divisible when at night the market for general goods is competitive. Since there is no technological constraint that prevents the stock of money from being divided evenly across buyers, there can be no shortage of currency. In this environment, the buyer's value functions satisfy (40) and (41), and his choice of money balances is still given by (46), but now $d \in [0, \hat{m}]$. Since money is costly to hold, it will be the case that $d = \hat{m}$. Recognizing that the seller's participation constraint will bind, i.e., $c(q) = \phi m$, it will be convenient to rewrite the buyer's choice of money balances problem (46) as

(52) $\quad \max_{\hat{m}}\{-r\phi\hat{m} + \sigma\{u[q(\hat{m})] - c[q(\hat{m})]\}\}.$

The solution to problem (52) when money is perfectly divisible is given by

(53) $$\frac{u'(q)}{c'(q)} = 1 + \frac{r}{\sigma},$$

since, from the seller's participation constraint, $dq/dm = \phi/c'(q)$. It can be checked that there is a unique q satisfying (53) and it is such that $\partial q/\partial\sigma > 0$ and $\partial q/\partial r < 0$. So, the comparative static results are similar to those obtained in the model with indivisible money. The term r/σ is a measure of the cost of holding real balances: this is the product of the rate at which agents depreciate future utility, times the average number of periods for a match to occur. As this cost increases, buyers reduce their real balances and output falls. As the rate of time preference approaches 0, q tends to q^*. Finally, M is neutral; it affects neither the quantity of search goods produced, q, nor the frequency of trades, σ.

Fiat objects, be they divisible or indivisible, can be valued in an environment where there is a double-coincidence-of-wants problem and agents cannot use credit. The value of money depends on the fundamentals of the economy, including agents' rate of time preference, r, and the extent of the search-matching frictions, σ. When money is perfectly divisible, the number of trades is maximized but the quantities traded are too low when $r > 0$. When money is indivisible and scarce, the number of trades is too low since not all buyers can be endowed with money. However, the quantities traded can be too high if agents are unable to use lotteries.

Related Literature

Using a mechanism design approach, Kocherlakota (1998a,b) has established that the technological role of money is to act as a societal memory that gives agents access to certain aspects of the trading histories of their trading partners. As a corollary, imperfect knowledge of individual histories is necessary for money to play an essential role in the economy (Wallace, 2000). The recordkeeping role of money was emphasized by Ostroy (1973), Ostroy and Starr (1974, 1990) and Townsend (1987, 1989), among others.

Diamond (1984) was the first to introduce fiat money into a search-theoretic model of bilateral trade. However, money was not essential in Diamond's environment since all matches were double-coincidence-of-wants matches. Kiyotaki and Wright (1989, 1991, 1993) have introduced a double-coincidence-of-wants problem into a search-theoretic environment to explain the emergence of a medium of exchange and the essentiality of fiat money. These models were based on important restrictions: money and goods were indivisible and agents could hold at most one unit of an object. Shi (1995) and Trejos and Wright (1995) have relaxed the assumption of indivisible goods to endogenize prices. Wallace and Zhou (1997) have used a related framework to explain currency shortages. Berentsen, Molico, and Wright (2002) have introduced lotteries. The assumption of indi-

visible money and its implications for the efficiency of monetary exchange are discussed in Berentsen and Rocheteau (2002). Search models with divisible money include Shi (1997), Green and Zhou (1998), and Lagos and Wright (2004). The formalization adopted in this section follows the one in Lagos and Wright.

Alternative models of monetary exchange are surveyed in Wallace (1980) and Townsend (1980).

Coexistence of Money and Credit

Actual economies differ from the pure credit and the pure monetary economies described in the previous sections in that many modes of payments coexist. For example, some trades are conducted through credit arrangements, while other trades are based on monetary exchange. Why do different means of payment coexist? How does the presence of monetary exchange affect the use and the availability of credit? How does the availability of credit affect the value of money? We address these questions below.

One way to explain the coexistence of monetary exchange and credit arrangements is to introduce some heterogeneity among agents and/or trading matches. For example, some agents may have the ability to commit to repay their debt or to have their trading histories publicly observable, while others don't. The former set of agents will be able to trade using credit arrangements, while the latter set of agents will need to use money. In this section, we explain the coexistence of money and credit by introducing heterogenous matches: Some matches will be short-lived and last only one period, while other matches will be longer-lived and can be productive for many periods. The use of credit will not be incentive-feasible in short-lived matches, since the buyer will always default on repaying his obligation at night. In contrast, the buyer's behavior in a long-lived match is disciplined by reputation considerations that will trigger the dissolution of a valuable relationship following a default.

In this section, we extend the long-term partnership environment described in the section on Credit with Reputation by allowing the possibility of short-term partnerships to arise. When unmatched agents arrive at the beginning of a period, with probability σ_ℓ they find a long-term trade match and, with probability σ_s they can enter into a short-term trade match. We assume that $0 < \sigma = \sigma_\ell + \sigma_s < 1$. A short-term match is destroyed with a probability of one at the end of the day, and a long-term match will be destroyed exogenously, with probability $\lambda < 1$ during the night.

The timing of events is as follows: Buyers enter the day market either attached, i.e., in a long-term trade match (or partnership), or unattached. Unattached buyers and sellers participate in a random matching process. Note that since there is the same number of buyers and sellers, there is also the same number of unattached buyers and unattached sellers. After the matching process is completed, all matched sellers, i.e., those in either a long-term or short-term relationship, produce the search good for buyers. All short-term

matches are destroyed at the end of the day. The night begins with buyers *who are in a long-term partnership* producing the general good for sellers. Buyers then realize a preference shock, which is followed by the opening of a competitive general goods market, where the general good is exchanged for money. As a result of the preference shock, a fraction, λ, of long-term trade matches are dissolved.

We assume a specific pricing mechanism for traded goods. For search goods, buyers make take-it-or-leave-it offers to sellers; for general goods, the market is competitive, where one unit of money trades for ϕ units of the general good.

The aim of this section is to present an environment where money and credit can coexist and to see how these two payments systems may affect one another. To this end, we will focus on a particular class of equilibria that exhibit two features. First, money is valued but is only used in short-term trade matches. Second, the buyer's incentive-compatibility constraint in long-term matches—that the buyer is willing to produce the general good for the seller—is not binding. This latter assumption implies that the buyer in a long-term trade match can obtain the efficient quantity of the search good, q^*, without having to use any money. At the end of this section, we will briefly discuss the implications of relaxing these assumptions.

Value Functions

The Bellman equations for the value functions for a buyer assume that the equilibrium has money being used in only short-term trade matches. Consider first a buyer who enters the day market unattached, with m units of money. The value function for such a buyer is

$$(54) \qquad V_u^b(m) = \sigma_\ell V_\ell^b(m) + \sigma_s V_s^b(m) + (1 - \sigma_\ell - \sigma_s) W_u^b(m),$$

The buyer finds a long-term trade match that has value $V_\ell^b(m)$ with probability σ_ℓ; with probability σ_s, he finds a short-term match whose value is $V_s^b(m)$. The buyer remains unattached with probability $1 - \sigma_\ell - \sigma_s$ and enters the night market with m units of money with value $W_u^b(m)$.

Consider first a buyer holding m units of money who is in a short-term trade match. The buyer makes a take-it-or-leave-it offer (q_s, d_s) to the seller, where q_s is the amount of the search good that the seller produces and d_s is the amount of money transferred from the buyer to the seller. Both the seller's output and the money transfer will depend on the money holdings of the buyer. The utility to the buyer in a short-term trade match, evaluated in the day, is

$$(55) \qquad V_s^b(m) = u[q_s(m)] + W_u^b[m - d_s(m)].$$

The buyer will consume q_s units of the search good in the day and will enter the com-

petitive general goods market with $m - d_s$ units of money. The value of the (now un-matched) buyer at night satisfies

$$(56) \qquad W_u^b(m) = \max_{\hat{m}}\{\phi(m - \hat{m}) + \beta V_u^b(\hat{m})\}.$$

The buyer acquires $\hat{m} - m$ units of money at the price ϕ in terms of general goods at the competitive market, in order to readjust his balances to the desired level $\hat{m}$. Recall that $x - y = \phi(m - \hat{m})$ and that $W_u^b(m)$ is linear in m, i.e., $W_u^b(m) = \phi m + W_u^b(0)$.

Consider now a buyer who enters the period in a long-term partnership. The buyer consumes q_ℓ units of the search good, which is assumed to be independent of money balances, m, that he might hold (since we focus on equilibria where the buyer's incentive-compatibility constraint in long-term matches is nonbinding). The value function for such a buyer at the beginning of the period is

$$(57) \qquad V_\ell^b(m) = u(q_\ell) + W_\ell^b(m, y_\ell),$$

where $W_\ell^b(m, y_\ell)$ is the value of the matched buyer at night holding m units of money and a promise to produce y_ℓ units of the general good for his partner in the trade match. The value function of the matched buyer at night satisfies

$$(58) \qquad W_\ell^b(m, y_\ell) = -y_\ell + (1 - \lambda)\{\phi m + \beta V_\ell^b(0)\} + \lambda \max_{\hat{m}}\{\phi(m - \hat{m}) + \beta V_u^b(\hat{m})\}.$$

At the beginning of the night, the buyer fulfills his promise and produces y_ℓ units of the general good for the seller. If the trade match is not exogenously destroyed, the buyer consumes his real balances, ϕm, since money is not needed in a long-term relationship. Alternatively, if the partnership breaks up at night, an event that occurs with probability λ, the buyer has the opportunity to readjust his money balances in the competitive general goods market before he proceeds to the next period in search of a new trading partner. In this case, the buyer will choose to bring $\hat{m}$ money balances into the next period. Note from (58) that $W_\ell^b(m, y_\ell)$ is linear in both m and y_ℓ, i.e., $W_\ell^b(m, y_\ell) = -y_\ell + \phi m + W_\ell^b(0,0)$.

Terms of Trade

We first examine the amount traded in the case where buyers and sellers are in a short-term partnership. The buyer makes a take-it-or-leave-it offer, (q_s, d_s), such that $d_s \leq m$ and the seller will accept as long as $-c(q_s) + \phi d_s \geq 0$. As described in the section on Divisible Money, the offer will be characterized by $q_s(m) = q^*$ and $\phi d = c(q^*)$ if $\phi m \geq c(q^*)$; otherwise $q_s = c^{-1}(\phi m)$.

The optimal choice of money balances for a buyer who is not in a long-term relationship satisfies

$$(59) \qquad \max_{\hat{m}}\{-r\phi\hat{m} + \sigma_s\{u[q_s(\hat{m})] - c[q_s(\hat{m})]\}\},$$

which is identical to problem (52) in the section on Divisible Money and leads to the familiar first-order condition,

(60) $$\frac{u'(q_s)}{c'(q_s)} = 1 + \frac{r}{\sigma_s}.$$

For buyers and sellers who are in a long-term partnership, we focus on equilibria where the incentive-compatibility constraint, $W_\ell^b(m, y_\ell) \geq W_u^b(m)$, does not bind. When this constraint does not bind, a buyer in a long-term relationship proposes (q, y) such that

$$\max_{q,y}[u(q) - y] \quad \text{s.t.} \quad -c(q) - y \geq 0,$$

which gives $q_\ell = q^*$ and $y_\ell = c(q^*)$.

Equilibrium

We consider a steady-state equilibrium where money is only used in short-term matches. For an allocation $(q_s, q_\ell, y_s, y_\ell)$ to be an equilibrium, we need to check that the buyer in a long-term partnership is willing to repay y_ℓ at night. The incentive-compatibility constraint requires $W_\ell^b(m, y_\ell) \geq W_u^b(m)$, or, from the linearity of the value functions,

(61) $$W_\ell^b(0, y_\ell) \geq W_u^b(0).$$

With the help of equations (54)–(58), and after some rearranging, inequality (61) can be rewritten as

(62) $$c(q^*) \leq (1 - \lambda)\beta\{(1 - \sigma_\ell)u(q^*) - \sigma_s[u(q_s) - c(q_s)]\},$$

where q_s satisfies equation (60). If inequality (62) holds, then there exists an equilibrium where buyers and sellers who are in a long-term relationship consume and produce $q_\ell = q^*$ units of the search good and $y_\ell = c(q^*)$ units of the general good and use credit arrangements to implement these trades. Buyers and sellers in short-term partnerships trade q_s units of the search good for $y_s = c(q_s)$ units of real balances.

Perhaps not surprisingly, when $\sigma_s = 0$, the incentive condition (62) is identical to the one obtained in a model where money was absent and trade in long-term relationships was supported by reputation (see the definition of $\mathcal{A}^R$ given by [30] in the section on Credit with Reputation). So when allocation $(q^*, c(q^*)) \in \mathcal{A}^R$, this allocation can be implemented in the long-term relationship—in a world where credit and money coexist—as long as σ_s is sufficiently small. If the frequency of short-term matches, σ_s, increases, then, from (60), agents will increase their real balance holdings; as a result the incentive-constraint (62) becomes more difficult to satisfy. Hence, the availability of mon-

etary exchange in the presence of a long-term partnership increases the attractiveness of defaulting on promised performance.

In this section, we have only described one type of equilibrium, where money is not needed in long-term trade matches. There exist other kinds of equilibria when the incentive-constraint (61) binds. In those equilibria, agents will use money in both short-term and long-term trade matches, but fewer money balances will be needed in long-term relationships. In these kinds of equilibria, money can be used to weaken the buyer's incentive-compatibility constraint. Hence, payment in long-term trade matches be will a combination of money and a promise to repay output in the future.

Finally, an interesting extension of this model would consist of introducing money growth in order to investigate how inflation affects the buyer's incentive-compatibility constraint, that is, in a long-term relationship. We conjecture the following: As inflation increases, the cost of holding real balances is higher and, as a consequence, the amount of search good traded in a short-term trade match, q_s, decreases. Hence, buyers obtain a smaller surplus in short-term trade matches which raises the cost of defaulting in long-term trade matches. If the incentive-compatibility constraint (61) is binding, inflation would relax this constraint, although it would also reduce the quantities traded in short-term matches. Depending on the relative magnitudes, it is possible that a mild inflation can deliver a better outcome than a zero inflation. If, however, one allows for negative money growth rates, i.e., deflation, then the cost of holding real balances can be driven to zero, which implies that all trades can be conducted with money and this generates the efficient allocation.

Related Literature

Shi (1996) has constructed a search-theoretic environment where fiat money and credit can coexist, even though money is dominated by credit in the rate of return. A credit trade occurs when two agents are matched and the buyer in the match does not have money. Collateral is used to make the repayment incentive-compatible, and debt is repaid with money. In this approach, monetary exchange is superior to credit in the sense that monetary exchange allows agents to trade faster. Li (2001) extended Shi's model to allow private debt to circulate and to investigate various government policies including open-market operations and public-debt policies.

Townsend (1989) investigates the optimal trading mechanism in an economy with different locations, where some agents stay in the same location while other agents move from one location to another. The optimal arrangement implies the coexistence of currency and credit. Currency is used between strangers, i.e., agents whose histories are not known to one another, and credit is used among agents who know their histories. Kocherlakota and Wallace (1998) consider a random-matching economy with a public record of all past transactions that is updated only infrequently. They show that in this

economy there are roles for both monetary transactions and some form of credit. Jin and Temzelides (2004) consider a search-theoretic model with local and faraway trades. There is recordkeeping at the local level so that agents in local meetings can trade with credit. In contrast, agents from different neighborhoods need to trade with money.

Corbae and Ritter (2004) consider a model of long-term and short-term partnerships similar to the one presented in this section. It is shown that the presence of money weakens incentive-compatibility conditions. Aiygari and Williamson (2000) construct a dynamic risk-sharing model where agents can enter into a long-term relationship with a financial intermediary. They introduce a transaction role for money by assuming random limited participation in the financial market. In each period, agents can defect from their long-term contracts and trade in a competitive money market in each succeeding period. Aiyagari and Williamson show that the value of this outside option depends on monetary policy.

Alternative Media of Exchange

We have examined economies where credit and fiat money are used as means of payment. Even though, in practice, fiat money plays a singular role in facilitating transactions, a large variety of assets and commodities can be and are used as means of payment. For example, real commodities, such as gold and silver, capital or claims on capital, government securities, and foreign currencies, to name but a few, have been known to be used to purchase goods. A number of interesting questions naturally arise. Is money useful when capital can be used as a medium of exchange? Is it useful to have several currencies and how are exchange rates determined? Can money and interest-bearing bonds co-exist?

We will now study economies where agents can choose among different media of exchange: money and something else. We will draw conclusions regarding the possibility and the need of having different payment instruments simultaneously circulating.

Money and Capital

A direct way for a buyer in our model to obtain the search good would be to give the seller what he values: the general good. This direct method of payment, however, is technically infeasible in the benchmark model since it is assumed that goods fully depreciate at the end of the subperiod in which they are produced. The general good is produced at night and cannot carried over to the next period to pay for a search good. In what follows, we relax the assumption that all goods are perishable.

The economic environment is that of the benchmark model, except that the general good can be stored. Trade matches are destroyed at the end of the day, and the market for the general good is competitive. We consider an economy where agents have access to a storage technology that enables them to carry over the general good from one period to

the next. The storage technology is linear, which means that each unit of stored general good generates $A \geq 0$ units of the general good in the following period. More specifically, if a unit of the general good is stored at night, then it turns into A units of the general good the next morning; the A units can be held over the day and consumed (or stored) in the subsequent night. The case where $A = 0$ corresponds to the assumption that goods are perishable, while the case where $A = 1$ corresponds to a pure storage technology. If $A > 1$, then the storage technology is productive, whereas if $A < 1$, the storage technology is characterized by depreciation. For convenience, we will refer to a good that is stored as *capital*.

Consider a buyer holding a portfolio (m,k) of money and capital at the beginning of the day. This portfolio implies that the buyer stored k/A units of the general good from the previous night. We denote (q,d_m,d_k) as the terms of trade in a bilateral match where q is the amount of the search good that the buyer receives from the seller, d_m is the transfer of money from the buyer to the seller, and d_k is the transfer of capital from the buyer to the seller. The buyer's lifetime expected utility satisfies

$$(63) \qquad V^b(m,k) = \sigma\{u[q(m,k)] + W^b(m - d_m(m,k), k - d_k(m,k))\} + (1-\sigma)W^b(m,k).$$

According to (63), a buyer who meets a seller consumes q units of the search good and transfers d_m units of money and d_k units of capital to the seller. The terms of trade (q,d_m,d_k) depend on the buyer's initial portfolio. The utility of the buyer at night obeys

$$(64) \qquad W^b(m,k) = \phi m + k + \max_{\hat{m},\hat{k}}\left\{-\phi\hat{m} - \hat{k} + \beta V^b(\hat{m}, A\hat{k})\right\},$$

since, from the buyer's budget constraint, $x - y = -\phi m - \phi\hat{m} + k - \hat{k}$. Note that the value function for the buyer at the beginning of the next period depends upon Ak units of goods.

The terms of trade in bilateral matches are determined by take-it-or-leave-it offers by buyers. An optimal offer for the buyer solves

$$(65) \qquad \max_{q,d_m,d_k} [u(q) - d_k - \phi d_m] \quad \text{s.t.} \quad -c(q) + d_k + \phi d_m \geq 0,$$

$$d_m \leq \mathrm{m}, \quad d_k \leq k.$$

The solution to this bargaining problem is $q(m,k) = q^*$ and $\phi d_m + d_k = c(q^*)$ if $\phi m + k \geq c(q^*)$; otherwise $q = c^{-1}(\phi m + k)$. Notice that it is the real value of the *entire* portfolio, $\phi m + k,$ that is relevant to determining the terms of trade and not the composition of the portfolio.

If we substitute the buyer's beginning-of-period lifetime utility, V^b, given by (63), into buyer's beginning-of-night lifetime utility, W^b, given by (64), recognizing that the buyer

extracts all of the surplus from a trade match, the buyer's choice for the optimal portfolio can be expressed as

$$(66) \qquad \max_{\hat{m},\hat{k}} \left\{ -\left(\frac{1-\beta}{\beta}\right)\phi\hat{m} - \left(\frac{1-\beta A}{\beta A}\right)A\hat{k} + \sigma\left\{u\left[q(\hat{m}, A\hat{k})\right] - c\left[q(\hat{m}, A\hat{k})\right]\right\} \right\}.$$

The expression in the brackets should look familiar: The first two terms represent the cost of carrying money and capital, respectively, into the subsequent period, and the third term represents the expected surplus that is obtained in a trade match. Since the quantity of search goods proposed in the bargain, $q(m,k)$, depends only on the real value of the buyer's portfolio, $\phi m + k$, buyers will be willing to hold both money *and* capital if and only if they both offer the same return, i.e., if and only if $A = 1$. If $A > 1$, capital dominates money in its rate of return and agents will hold only capital goods to make transactions. In this case, the quantity traded satisfies

$$(67) \qquad \frac{u'(q)}{c'(q)} = 1 + \frac{1-\beta A}{\sigma\beta A}.$$

Note that the quantity of search goods traded in bilateral matches increases with the rate of return of capital. As the rate of return of capital approaches the discount rate, i.e., as $A\beta$ approaches one, the quantity traded, q, approaches its efficient value, q^*. If $A < 1$, then capital generates a lower rate of return than money and, as a result, buyers will hold only money for transaction purchases in equilibrium, i.e., buyers will not store any of the general good.

Let us return to the case of a pure storage technology, i.e., $A = 1$, so that money and capital can coexist as a means of payments. The quantities of goods traded will be identical to the quantities traded in the monetary economy studied in the previous section. However, the presence of money is welfare-improving because it frees up real resources for alternative uses: Capital that was previously used as a medium of exchange can now be consumed. This is a standard argument in favor of a fiat money regime rather than a commodity standard.

As it now stands, the model cannot explain how capital and money can coexist if capital dominates money in its rate of return. A crude way to obtain coexistence is to impose costs on the use of capital as a medium of exchange. For example, productive capital may be costly to use as a medium of exchange because it is not as easy to transport as money, or because claims on capital may be easier to forge than fiat money. Hence, if the *net* return to capital—net of transportation or expected forgery costs—equals one, then money and capital can coexist. An alternative approach to obtain coexistence is based on the existence of legal restrictions. Suppose that there exists a government that wants

to promote the use of fiat money. The government can promote the use of fiat money by refusing to accept capital for any transactions with private agents. To the extent that agents want to or have to transact with the government, they will then want to hold some of the (lower rate of return) money. Here, the lower return on money is compensated by its higher "liquidity."

Dual-Currency Payment Systems

Real economies are endowed with many different currencies. We now examine the co-existence of two monies to see whether two currencies can be valued and used in payments. We also want to determine whether there are gains in having more than one currency. Our investigation here is slightly different from that of the previous section. The previous section examined the coexistence of a fiat money and a real object; this section examines the coexistence of two fiat monies.

We now turn to an economy where two fiat monies—called money 1 and money 2—can be used as mediums of exchange. We assume that the stocks of both monies, M_1 and M_2, are constant and that agents are free to use any currency in a trade. The market for general goods is competitive, where one unit of money 1 buys ϕ_1 units of the general good and one unit of money 2 buys ϕ_2 units of the general good.

Consider a buyer holding m_1 units of currency 1 and m_2 units of currency 2. His beginning-of-period value function, $V^b(m_1, m_2)$, satisfies

$$(68) \qquad V^b(m_1, m_2) = \sigma\{u[q(m_1, m_2)] + W^b[m_1 - d_1(m_1, m_2), m_2 - d_2(m_1, m_2)]\}$$

$$+ (1 - \sigma)W^b(m_1, m_2).$$

The interpretation of the value function (68) is similar to that of the value function $V^b(m, k)$ given in (63). The value function of the buyer at night is given by

$$(69) \qquad W^b(m_1, m_2) = \phi_1 m_1 + \phi_2 m_2 + \max_{\hat{m}_1 \hat{m}_2}\left\{-\phi_1 \hat{m}_1 - \phi_2 \hat{m}_2 + \beta V^b(\hat{m}_1, \hat{m}_2)\right\},$$

and the interpretation of this value function is similar to that of $W^b(m, k)$ given in (64).

The terms of trade are determined by take-it-or-leave-it offers by buyers. It is straightforward to show (and should come as no surprise) that the solution to the buyer's bargaining problem is given by $q = q^*$ and $\phi_1 d_1 + \phi_2 d_2 = c(q^*)$ if $\phi_1 m_1 + \phi_2 m_2 \geq c(q^*)$; otherwise $q = c^{-1}(\phi_1 m_1 + \phi_2 m_2)$. As in the previous section, the terms of trade depend only on the real value of the whole portfolio of the buyer. Substituting V^b, given by its expression in (68), into (69), and using the solution for the buyer's bargaining problem, the buyer's optimal portfolio satisfies

$$(70) \qquad \max_{m_1, m_2}\left\{-r(\phi_1 m_1 + \phi_2 m_2) + \sigma\{u[q(m_1 m_2)] - c[q(m_1 m_2)]\}\right\}.$$

Since $q(m_1, m_2)$ depends only on the real value of the buyer's portfolio, $\phi_1 m_1 + \phi_2 m_2$, (70) does not pin down the composition of the portfolio. The buyer is indifferent between holding one currency or another. The buyers-take-all assumption implies that

$$(71) \qquad c(q) = \phi_1 m_1 + \phi_2 m_2;$$

from the first-order condition (70), q satisfies

$$(72) \qquad \frac{u'(q)}{c'(q)} = 1 + \frac{r}{\sigma}.$$

While equation (72) uniquely determines the value of q, equation (71) is effectively left to determine both ϕ_1 and ϕ_2. Equivalently, for any exchange rate ε, there exists a price for money 2, $\phi_2 = c(q)/[\varepsilon M_1 + M_2]$, that solves (71). Consequently, the nominal exchange rate $\varepsilon = \phi_1/\phi_2$ is indeterminate.

The indeterminacy of the exchange rate can be resolved if the government simply imposes a certain exchange rate when trading with private agents. The government can implicitly impose an exchange rate by refusing to accept one of the monies in trades with private agents. Assume, for example, that the government will only accept the first currency. Then buyers will only want to hold the first currency, and the second currency will lose its value, $\phi_2 = 0$. Finally, it turns out that multiple currencies are not useful in this economy since the equilibrium allocation is the same as the one in the single currency economy. That is, agents trade the same quantities q in all matches, irrespective of the number of currencies.

Government Liabilities as Means of Payment

In real economies, financial institutions (e.g., banks) whose liabilities are used by private agents as mediums of exchange (deposit and saving accounts) hold government securities. This phenomenon reflects an intermediation activity on the part of the financial institution consisting of transforming illiquid assets into more liquid ones. We will now describe economies where agents hold government bonds and fiat money. We will investigate whether government bonds that pay interest can be used as a means of payment. This discussion will require that we formalize the notion of illiquid bonds. We will conclude with a discussion on whether illiquid assets have a role to play in the economy.

Consider an economy where agents can use both money and government bonds as mediums of exchange. A one-period government bond is issued at night and is redeemed for one unit of money in the night market of the subsequent period. Government bonds are of the pure discount variety and are perfectly divisible, payable to the bearer, and default-free. (These assumptions make money and bonds close substitutes.) The flow of bonds sold by the government each period is equal to B. The government finances the interest payments on bonds, if any, by lump-sum taxation at the end of each period.

Since matured bonds are exchanged for money one to one, the price of matured bonds, in terms of night goods, is ϕ. Let ω be the price of newly-issued bonds in terms of night commodities. If $\omega < \phi$, newly-issued bonds are sold at a discount for money. The one-period rate of return on newly issued bonds $r_b = \phi / \omega - 1$.

Can newly issued bonds ever be sold at a discount? If bonds were sold at a discount for money, then agents would prefer to sell all their money for bonds, since bonds are as liquid as money but provide a rate of return. (A standard backward-induction argument would generalize this result to the case where the length of the maturity period is more than one period.) So, in equilibrium, money and newly issued bonds are perfect substitutes, i.e., $\omega = \phi$. This result is then similar to the dual-currency economy of the section on Terms of Trade, where the exchange rate between the two currencies is unity and the two nominal assets are traded at par. Hence, interest-bearing government bonds cannot coexist with fiat money unless one assumes some form of illiquidity associated with government bonds.

We introduce an arbitrary restriction on the use of bonds in bilateral meetings during the day in order to generate a form of illiquidity for bonds. A buyer holding a portfolio of b units of bonds can use only a fraction $g \in [0,1]$ of his bonds to make a payment in a bilateral match during the day. If $g = 0$, bonds are fully illiquid, whereas if $g = 1$, they are perfectly liquid. One can interpret this illiquidity of bonds as stemming from the fact that bonds are not as easily transportable as money, are not perfectly divisible, or involve costs to be recognized. One can also view this constraint as an arbitrary form of a cash-in-advance requirement. Since terms of trade are determined by take-it-or-leave-it offers by buyers, the quantities traded in bilateral matches during the day satisfy

$$c(q) = \phi(m + gb),$$

which is the seller's participation constraint.

The value function for buyers, $V^b(m,b)$, which is similar to (68), can be expressed as

$$V^b(m,b) = \sigma\{u[q(m,b)] + W^b(m - d_m, b - d_b)\} + (1 - \sigma)W^b(m,b),$$

where

$$W^b(m,b) = \phi(m + b) + \max_{\hat{m},\hat{b}}\left\{-\phi\hat{m} - \omega\hat{b} + \beta V^b\left(\hat{m},\hat{b}\right)\right\},$$

and $d_m(d_g)$ represents the transfer of money (bonds) from the buyer to the seller in the morning. The buyer's portfolio problem is given by the solution to

$$\max_{\hat{m},\hat{b}}\left\{-\omega\hat{b}(r - r_b) - \phi\hat{m}r + \{\sigma u[q(m,b) - c[q(m,b)]\}\right\}.$$

The first-order conditions of this problem with respect to $\hat{b}$ and $\hat{m}$, respectively, are

(73)
$$\left(\frac{u'(q)}{c'(q)} - 1 \right) = \frac{\omega(r - r_b)}{\sigma g \phi}$$

and

(74)
$$\left(\frac{u'(q)}{c'(q)} - 1 \right) = \frac{r}{\sigma}.$$

Equating the right-hand sides of (73) and (74), so that buyers are indifferent between holding money and bonds, we obtain

$$r_b = \frac{r(1 - g)}{1 + gr}.$$

The one-period rate of return on government bonds depends on the degree of liquidity of bonds. If bonds are perfectly liquid, i.e., $g = 1$, then $r_b = 0$ and $\omega = \phi$. If bonds are illiquid, i.e., $g = 0$, then $r_b = r$.

An alternative way to formalize legal restrictions is to describe the government as a subset of sellers in the economy whose trading behaviors are specified exogenously. Government agents refuse to accept government bonds in payment and can also influence terms of trade by choosing the price at which it will sell its output. In general, the interest rate on government bonds would depend on the size of the government, i.e., the fraction of sellers who are government agents, as well as the government's trading policy, i.e., the price of its output. We can rationalize the coexistence of interest-bearing bonds and money if bonds are illiquid. But so far we have not explained why bonds should be illiquid: The presence of illiquid bonds does not generate better allocations.

In order for government bonds to pay interest, they must be illiquid. And, up to this point, we have arbitrarily imposed a form of illiquidity on bonds. But why should bonds be illiquid? An interesting way to justify the illiquidity of bonds is to show that their presence can, in fact, raise society's welfare. To make this point, the environment can be modified as follows: Suppose that each period is divided into three subperiods; the early morning, the day, and the night. The day and night are as before. During the early morning, however, buyers receive a preference shock: with some probability they want to consume, and with the complement probability they do not want to consume. The buyer's portfolio of money and bonds is chosen at night, before the buyer knows whether he will want to consume the next period, and buyers are allowed to readjust their portfolios in the early morning after they receive their preference shock. If bonds are perfectly liquid, there is no reason to trade money for bonds in the early morning, since they are perfect substitutes. If bonds are made illiquid, however, buyers will have an incentive to reallocate their portfolios in the early morning. Buyers with a positive $\hat{}$ preference shock will

sell bonds to be able to consume more during the day, while buyers with a negative preference shock will buy bonds that pay interest. An alternative interpretation of this result is as follows. Buyers with a positive preference shock would like to borrow from buyers with a negative shock. Since they cannot commit to repay their debts, the (private) loan market is inactive. In contrast to private agents, the government can commit to pay off in the future. Therefore, instead of selling their own debt, buyers with a positive preference shock sell the government's debt and buyers with a negative shock buy it.

Related Literature

Kiyotaki and Wright (1989) constructed an environment where all commodities can serve as means of payment and where agents can choose which one to use. Models of commodity monies include Sargent and Wallace (1983), Burdett and Wright (2001), Velde, Weber, and Wright (1999), and Li (2003). The existence of a monetary equilibrium when agents have access to a linear storage technology was studied by Wallace (1980) in the context of an overlapping-generations model. Lagos and Rocheteau (2004) studied how money and capital can compete as means of payment in a search environment. Shi (1999), Aruoaba and Wright (2003), and Aruoba, Waller, and Wright (2004) described search economies where agents can accumulate capital but capital is illiquid, in the sense that it cannot be used as a means of payment in bilateral matches.

The first search-theoretic environment with two currencies was proposed by Matsuyama, Kiyotaki, and Matsui (1993) and was extended by Zhou (1997) to allow for currency exchange. They considered a two-country economy where the two countries are imperfectly integrated and establish conditions on parameters under which one currency is used as an international currency. They also showed that a uniform currency dominates in terms of welfare. Head and Shi (2003) extended the previous analysis to propose a dual-currency economy where terms of trade are endogenous and monies are perfectly divisible. Legal restrictions were introduced by Li and Wright (1998). Trejos and Wright (1996) and Craig and Waller (2000) survey the search literature on dual-currency payment systems. The proposition of the indeterminacy of the exchange rate was established by Karakeen and Wallace (1981) in an overlapping-generations economy.

The coexistence of money and bonds has been discussed by Bryant and Wallace (1979), Aiyagari, Wallace, and Wright (1996), Kocherlakota (2001), Shi (2004a,b), and Wallace and Zhu (2004). According to Bryant and Wallace (1979), interest-bearing government bonds are socially inefficient because of intermediation costs to transform large-denomination bonds into perfectly divisible intermediary liabilities. Aiyagari, Wallace, and Wright (1996) introduced government agents to explain why government bonds are sold at a discount. The effects of the government's trading behavior on the equilibrium outcome have been studied more generally in Li and Wright (1998). Kocherlakota (2001) established the proposition according to which illiquid bonds can raise society's welfare when agents are subject to idiosyncratic shocks.

Banking

Payment systems involve financial intermediaries—in particular, banks—that not only supply liabilities that can circulate as mediums of exchange but also provide credit to finance productive investments. What kind of economic environments give rise to intermediaries that can issue debt that can be used as a mediums of exchange? What is the relationship between inside (bank) money and outside (fiat) money? Although the theory of banking is still in a very early stage of development, we present here two models that attempt to address these questions.

Banks and Safekeeping Services

Historically, banks have played a role in providing safekeeping services by storing gold and silver specie in their vaults. In exchange for their assets, agents receive bank notes that are much safer to hold. Since bank notes can serve as means of payment—and, as a result, circulate in the economy—banks are able to loan out some the assets that they hold for safekeeping.

We now describe a simple model that can account for a demand for safe bank notes. Buyers can hold two types of assets: commodity money and bank notes. Commodity money can be "minted" from general goods, according to a linear technology. In particular, each unit of night good can be transformed into one unit of commodity money, and this process is fully reversible (e.g., "coins" can be minted or melted at no cost, and melted coins can be consumed). Commodity money is exactly like the capital described in the section on Money and Capital, when $A = 1$.

At night, banks exchange bank notes for commodity money. Let ϕ represent the value of a bank note in terms of general goods. For convenience, we assume that each bank note is a claim to one unit of commodity money. Therefore, $\phi = 1$. Banks charge agents a per period fee equal to γ, measured in terms of the general good, for each unit of commodity money deposited in their vaults. The fee is payable to the bank at the end of the following period.

Buyers and sellers are matched at the beginning of each period. A buyer is matched with a seller with probability one. Sellers are divided into two types: honest sellers and thieves. The fraction of sellers who are thieves is equal to p. Honest sellers can produce the search good during the day market (in exchange for money) and consume at night. Thieves cannot produce anything during the day, but have the ability to steal a buyer's commodity money. We assume that whenever a thief meets a buyer, he can steal a fraction λ of the buyer's commodity money: Thieves are unable to steal bank notes. Thieves, like honest sellers, consume at night. An agent can sell some or all of his commodity money to a bank, receiving one bank note for each unit of commodity money deposited.

Consider a buyer with a portfolio of z units of commodity money and b bank notes at the beginning of a period. The buyer's lifetime expected utility satisfies

$$(75) \quad V^b(z,b) = pW^b[(1-\lambda)z,b] + (1-p)\{u[q(z,b)] + W^b[z - d_z(z,b), b - d_b(z,b)]\}.$$

Equation (75) has the following interpretation: A buyer meets a thief with probability p. The thief steals a fraction λ of the buyer's commodity money, so that the buyer enters the night market with $(1-\lambda)z$ units of commodity money and b units of bank notes. With probability $1 - p$, the buyer meets an honest seller, in which case he trades d_z units of commodity money and d_b bank notes for q units of the search good. Terms of trade (q, d_z, d_b) depend on the buyer's portfolio (z,b).

The utility of the buyer at night satisfies

$$(76) \quad W^b(z,b) = z + b + \max_{\hat{z}, \hat{b}}\left\{-\hat{z} - \hat{b}(1 + \beta\gamma) + \beta V^b(\hat{z}, \hat{b})\right\}.$$

According to (76), at night the buyer chooses his portfolio $(\hat{z}, \hat{b})$ of commodity money and bank notes for the following period. Recall that one unit of commodity money can be melted into one unit of general goods at no cost, one unit of general goods can be minted into one unit of commodity money, and, for each bank note, the buyer must pay a fee of γ to the bank in the following period.

The terms of trade in bilateral matches are determined by take-it-or-leave-it offers by buyers. Therefore, $q = q^*$ if $z + b > c(q^*)$; otherwise $q = c^{-1}(z + b)$. Substituting $V^b(z,b)$ by its expression given by (75) into (76), the buyer's portfolio choice problem at night can be reformulated as

$$(77) \quad \max_{\hat{z}, \hat{b}}\left\{-\hat{z}(r + p\lambda) - \hat{b}(r + \gamma) + (1 - p)\left\{u\left[q(\hat{z}, \hat{b})\right] - c\left[q(\hat{z}, \hat{b})\right]\right\}\right\}.$$

The terms $r + p\lambda$ and $r + \gamma$ in (77) represent the costs of holding commodity money and bank notes, respectively. Since commodity money and bank notes are perfect substitutes in the day market, buyers choose to hold all their wealth in bank notes whenever $p\lambda > \gamma$ and they choose to hold all their wealth in commodity money whenever $p\lambda < \gamma$. In the knife-edge case where $p\lambda = \gamma$, buyers are indifferent between holding bank notes or money. These conditions are quite intuitive. The cost of depositing one unit of money at the bank is γ, and the benefit of holding a bank note is to avoid the loss of a fraction λ of one's monetary wealth when meeting with a thief, an event that occurs with probability p. So, for example, if $p\lambda < \gamma$, the cost associated with holding bank notes exceeds the benefit; therefore, agents will hold all of their wealth in commodity money. In the case where $p\lambda > \gamma$, only bank notes are used in the day to implement trades. Those bank notes are fully backed by commodity money in banks' vaults.

Private Money

We now consider an environment where banks provide two types of services: They help to finance productive investments and they issue notes that can serve as mediums of exchange. The benchmark model will be slightly modified in order to accommodate banking. Instead of having marketplaces that open sequentially within a period, the "day–night" structure, we will assume that within a period two *sectors* are open simultaneously. There is a search sector, which mirrors the search market in the benchmark model, where buyers and sellers are bilaterally matched and sellers can produce the search good. There is also a banking sector, where agents can trade with a bank. Trading with a bank entails either selling an investment project to the bank in exchange for a bank note, where the input for an investment project is the general good, or selling a bank note to the bank in exchange for the general good, which is obtained by liquidating an investment project from the bank's portfolio. The bank sector here mirrors the night subperiod in the benchmark model.

At the beginning of each period, agents are allocated randomly between the two sectors. With probability π, an agent visits the search sector; with probability $1 - \pi$, the agent visits the banking sector. If an agent enters the search sector, then with probability $\frac{1}{2}$ he is a buyer and with probability $\frac{1}{2}$ he is a seller. (An agent's preferences over search goods are the same as in the benchmark model and are given by $u(q) - c(q)$. In the search sector, buyers and sellers are matched with probability one. Whether or not a buyer and a seller can trade depends on the each agent's portfolio of assets at the time they are matched.

In the banking sector, an agent always has the opportunity to fund an investment project. The investment project is indivisible and costs y units of the general good to be initiated; that is, to initiate the project, the agent must produce y units of the general good at a utility cost of y. The project pays off Ay in terms of utility when it is liquidated, where $A > 1$. The investment project, which is not portable, is "deposited" at the bank, in exchange for an indivisible bank note. The investment project can be liquidated in any period after the project is initiated; an investment project will be liquidated (and consumed) if an agent presents the bank with a bank note. We assume that an agent cannot hold more than one bank note at a time; denote ρ as the proportion of agents holding a bank note at the end of a period.

We focus on steady-state equilibria where bank notes circulate in the search sector. Let V_i denote the value of an agent in the search sector holding $i \in \{0,1\}$ bank notes at the beginning of a period, and W_i the value of an agent in the banking sector. Consider first an agent in the banking sector. The value functions W_0 and W_1 satisfy the following Bellman equations:

(78) $\quad W_0 = \max\{-y + \pi\beta V_1 + (1-\pi)\beta W_1, \pi\beta V_0 + (1-\pi)\beta W_0\},$

(79) $\quad W_1 = Ay + \max\{-y + \pi\beta V_1 + (1-\pi)\beta W_1, \pi\beta V_0 + (1-\pi)\beta W_0\}.$

According to (78), an agent without money in the banking sector can choose either to fund an investment project or not; if a project is funded, then the agent receives a bank note. If the agent funds an investment project, he produces y units of the general good and starts the next period with one bank note. In the subsequent period, the agent goes either to the search sector with probability π or, with probability $1-\pi$, remains in the banking sector. If the agent chooses not to fund an investment project, then he starts the next period with no bank note. According to (79) an agent with a bank note in the banking sector redeems this bank note for Ay units of consumption goods. The agent can then decide whether or not to fund a new investment project. Equations (78) and (79) imply that

(80) $\quad W_1 = W_0 + Ay.$

We are interested in steady-state equilibria where bank notes circulate in the search sector. For such an equilibrium to occur, some agents who enter the banking sector must be willing to fund an investment project; otherwise, the stock of bank notes would fall to zero, as bank notes are redeemed over time. It must also be the case that not all agents who enter the banking sector want to fund an investment project; otherwise, all agents in the economy will eventually end up with a bank note and no trades would take place in the search sector. In equilibrium, agents must be indifferent between funding an investment and not funding an investment, which implies that

(81) $\quad -y + \pi\beta V_1 + (1-\pi)\beta W_1 = \pi\beta V_0 + (1-\pi)\beta W_0.$

Now let's turn to the value functions of an agent in the search sector. The value functions for an agent in the search market with and without a bank note, V_1 and V_0, respectively, satisfy the following two Bellman equations:

(82) $\quad V_0 = \frac{\rho}{2}[-c(q) + \pi\beta V_1 + (1-\pi)\beta W_1] + \left(1 - \frac{\rho}{2}\right)[\pi\beta V_0 + (1-\pi)\beta W_0],$

(83) $\quad V_1 = \frac{1-\rho}{2}[u(q) + \pi\beta V_0 + (1-\pi)\beta W_0] + \left(1 - \frac{1-\rho}{2}\right)[\pi\beta V_1 + (1-\pi)\beta W_1].$

According to (82), an agent with no bank note becomes a seller with probability $\frac{1}{2}$ and, with probability ρ, is matched with a buyer who has a bank note. In this case, the agent produces q units of the search good and starts the following period with one bank note, after which he will go either to the banking sector, with probability π or to the search

sector, with probability $1-\pi$. With probability $1-\rho/2$, an agent with no bank note does not trade in the search market and starts the next period as an agent with no bank notes. Equation (83) has a similar interpretation. An agent holding a bank note becomes a buyer with probability $\frac{1}{2}$ and, with probability $1-\rho$, is matched with a seller who does not have a bank note. In this case the agent gets q units of the search good and starts the subsequent period without a bank note. With probability $1-(1-\rho/2)$, an agent with a bank note does not trade in the search market and starts the next period once again as an agent with a bank note.

The terms of trades in bilateral matches in the trading sector are determined by take-it-or-leave-it offers by buyers. A seller is indifferent between accepting a trade or rejecting it, if

$$(84) \qquad -c(q)+\pi\beta V_1+(1-\pi)\beta W_1 = \pi\beta V_0+(1-\pi)\beta W_0.$$

According to (84), the seller is indifferent between producing q units of the search good for the buyer, starting the next period with a bank note or producing nothing, starting the next period without a bank note.

From equations (78), (81), (82), and (84), we obtain that

$$W_0 = V_0 = \pi\beta V_0 + (1-\pi)\beta W_0.$$

It is easy to see that $V_0 = W_0 = 0$. Hence, from (81) and (84) we can deduce that $c(q) = y$. We will assume that $y \le c(q^*)$, so that in the search sector there is no need to introduce lotteries over money transfers. Hence, the purchasing power of a bank note in the search sector is determined by the cost of funding an investment project in the banking sector, i.e., one bank note buys $q = c^{-1}(y)$ units of the search good.

Finally, we determine the measure ρ of agents with bank notes in the steady state. Equation (81) represents the indifference between investing and not investing in the project. Since $V_0 = W_0 = 0$, and, from (80), $W_1 = Ay$, the value of holding a bank note in the search sector, as expressed by equation (81), can be rearranged to read

$$(85) \qquad V_1 = \left(\frac{1-(1-\pi)\beta A}{\pi\beta}\right)y.$$

Equation (85) implies that, as the return of an investment project increases, the value of holding a bank note in the trading sector must fall in order to keep agents indifferent between funding a project or not. The value of holding a bank note in the search sector, as expressed by (83), can be simplified to read

$$(86) \qquad V_1 = \frac{1-\rho}{2}u(q)+\left(1-\frac{1-\rho}{2}\right)y.$$

One can obtain an expression for ρ by equating the right-hand sides of equations (85) and (86); after some rearranging we obtain

$$(87) \qquad 1 - \rho = \left(\frac{1 - \beta A}{\pi \beta} + A - 1 \right) \frac{2y}{u(q) - y}.$$

So $\rho \in (0,1)$ if

$$(88) \qquad \frac{1 - \pi \beta \{1 + [u(q) - y]/2y\}}{1 - \pi} < \beta A < \frac{1 - \pi \beta}{1 - \pi}.$$

According to (88), the return of an investment cannot be too high or too low for bank notes to circulate. If the rate of return on an investment is too low, agents do not find it worthwhile to fund projects; if it is too large, all agents want to fund investment projects, so that no trades take place in the trading sector.

This model generates an equilibrium where banks play two roles. They "finance" investment projects by purchasing an illiquid asset with bank notes and, as a result, they provide the economy with a liquid asset: bank notes, which facilitate trades between buyers and sellers.

Related Literature

The model of banking based on crime was proposed by He, Huang, and Wright (2005). The model has been extended to endogenize the rate of crime (i.e., the number of thieves in the economy) and to allow for a money multiplier. The model of private money is similar to the one in Williamson (1999, 2002). In addition, Williamson shows that even if private monies can be subject to lemon problems and counterfeiting, the introduction of fiat money can decrease welfare. Cavalcanti and Wallace (1999a,b) and Wallace (2004) proposed a model of private money where a subset of agents, called banks, are monitored. Those agents can issue notes that can be used as a medium of exchange by nonbank agents. They show that the presence of inside money enlarges the set of allocations that are incentive-feasible. A model of private money with reserve management has been provided by Cavalcanti, Erosa, and Temzelides (1999).

Settlement

In actual economies, fiat money plays a dual role: It serves as a medium of exchange to facilitate trade and it is used to settle debt. In this section, we will consider economies where monetary exchange and credit coexist, and where debt must be settled with money. The fact that money is required to settle debt can generate liquidity problems in credit markets. These liquidity problems will affect the relative price of money, which in turn can distort the allocation of resources. Hence, liquidity problems in credit markets can spill over into product markets. This line of reasoning has been used to justify the need

for an elastic supply of currency, which is one of the founding principles of the establishment of the Federal Reserve System. In this section we examine a model where debt obligations must be settled with money (that is, a debt obligation cannot be settled by simply producing output). We introduce realistic frictions into the settlement process which, in turn, generate liquidity problems in credit markets. The specific nature of the settlement friction is a mismatch between the time a debtor can repay his debt and the time a creditor needs to be repaid.

The Environment

We consider an environment in which credit and money coexist and where money is used to settle debt obligations. In order to present the ideas in the most economical way, we make a slight departure from the benchmark model. We now divide a period into *four* subperiods: morning, day, night, and late night. As in the benchmark model, the day subperiod is characterized by bilateral matching with the production and consumption of the search good and the night subperiod is characterized by the production and consumption of the general good. In terms of the two new subperiods, the morning subperiod mirrors the night subperiod in that agents produce and consume the general good; the late-night subperiod is the time where agents settle their debts. In the late night, debtors and creditors ultimately go to a central meeting place in order to settle, with money, any outstanding debt that was issued during the previous subperiods.

In contrast to the benchmark model, where agents are infinitely lived, now agents live for only four subperiods. Buyers are born at the beginning of a period—in the morning—and die after the settlement phase in the late night of the same period. Sellers are born in the day subperiod and die at the end of the morning of the subsequent period. So in any particular morning, the economy will be populated with "young" buyers and "old" sellers; in all other subperiods, the economy is populated with buyers and sellers who are born in the same period. We assume that debts can only be issued in bilateral meetings and agents can commit to repay their debts.

During the day, buyers and sellers are matched, where buyers consume the search good while sellers produce it. In both the morning and night subperiods, the market for general goods is competitive. In contrast to the benchmark model, we assume that buyers can produce the general goods but obtain no utility from consuming them, while sellers consume market goods but cannot produce them. Buyers' preferences are described by the instantaneous utility function

$$U^b(q, y) = u(q) - y,$$

where y is the buyer's total production of the general good and q is the consumption of the search good. Similarly, the preferences for the seller are given by

$$U^s(q,x) = -c(q) + x,$$

where x is the seller's total consumption of the general good and q is the amount of the search good that is produced. Note that agents do not discount utility across subperiods.

In order to capture the coexistence of money and credit, we assume that buyers are heterogenous in terms of *when* they can produce. Half of the buyers can only produce in the morning market, and the other half can only produce in the night market. We call the former *early producers* and the latter *late producers*. Early-producer buyers can use money to trade in the day. In contrast, late-producer buyers are unable to obtain any money in the morning. But they are able to repay any debt that is issued in the day by producing for money at night. For simplicity, we eliminate any search-matching frictions by setting the matching probability σ to one.

To summarize, the timing of events and the pattern of trade will be as follows: At the beginning of a period, a measure one of buyers is born. Half of these buyers, the early producers, can produce in the morning. In a competitive market, buyers produce general goods in exchange for money and "old" sellers exchange money for the general good. Old sellers die at the end of the morning and are replaced by a measure one of new-born sellers at the beginning of the day. In the (day) search market, each buyer is matched with a seller. Half of the buyers—the early producers—trade with money and the other half—the late producers—trade with credit. In order to settle their debts at the end of the night period, buyers who traded with credit will produce general goods in exchange for money in a competitive market; sellers exchange money for the general good. In the late night, buyers and sellers arrive in a meeting place for the purpose of settling debts. Sellers who receive money in the late-night settlement subperiod will spend it in the morning of the following period before they die.

We focus on stationary equilibria. Money is traded for general goods in competitive markets but in the two different subperiods, so we distinguish two prices for money. Let ϕ^{am} be the price of money in terms of general goods in the morning and ϕ^{pm} the price of money at night.

Frictionless Settlement

We consider first the case where there are no frictions in the late-night settlement phase: All debtors and creditors arrive simultaneously at a central meeting place and all debts are settled instantaneously.

Consider first a buyer who is an early producer. This buyer produces general goods in the morning to get m units of money, which he spends in a bilateral match in the day for q^m units of the search good. The quantity q^m is determined by a take-it-or-leave-it offer by the buyer. The seller's participation constraint is $-c(q^m) + \max(\phi^{\mathrm{am}}, \phi^{\mathrm{pm}})m \geq 0,$ since a seller can spend the money he receives either at night or in the following morning. If

$\phi^{\text{pm}} < \phi^{\text{am}}$, sellers will spend their money in the following morning; but this outcome is inconsistent with the clearing of the general goods market at night. Therefore, $\max(\phi^{\text{am}}, \phi^{\text{pm}}) = \phi^{\text{pm}}$. (Note that the buyer has no incentive to use debt because debt would have to be repaid with money at the end of the period. But the value of money received in the settlement phase is $\phi^{\text{am}} \leq \phi^{\text{pm}}$.) The early-producer buyer's choice of money balances solves the problem

(89) $\quad \max_{m, q^m}[-\phi^{\text{am}} m + u(q^m)],$

(90) $\quad$ s.t. $c(q^m) = \phi^{\text{pm}} m.$

Substituting m from (90) into (89) and taking the first-order condition for q^m, we obtain

(91) $\quad \dfrac{u'(q^m)}{c'(q^m)} = \dfrac{\phi^{\text{am}}}{\phi^{\text{pm}}}.$

From (91) $q^m = q^*$ if and only if $\phi^{\text{am}} = \phi^{\text{pm}}$; if $\phi^{\text{pm}} > \phi^{\text{am}}$, then $q^m > q^*$.

At the end of the morning, all of the money in the economy is held by half of the buyers, i.e., the early-producer buyers. Hence, equilibrium in the money market (or in the general goods market) in the morning implies $M = m/2$ and, from the seller's participation constraint (90), q^m satisfies

(92) $\quad c(q^m) = 2M\phi^{\text{pm}}.$

Now let's turn to the problem of a late-producer buyer. In his bilateral match during the day, the late-producer buyer must issue an IOU to pay for the search good, which will be repaid in the late-night settlement subperiod. The buyer repays the debt by producing output for money at night. The terms of trade in the day match are determined by a take-it-or-leave-it offer (q^d, d) by the buyer, where q^d is the amount of search good produced by the seller and d is the amount of dollars that the buyer commits to repay in the late-night settlement subperiod. (It might be convenient to think of the "m" in offer (q^m, m) as referring to a buyer who uses money to purchase search goods and the "d" in offer (q^d, d) as referring to a buyer who uses debt.) The offer (q^d, d) is given by the solution to the buyer's problem

(93) $\quad \max_{q^d, d}[u(q^d) - \phi^{\text{pm}} d]$

(94) $\quad$ s.t. $-c(q^d) + \phi^{\text{am}} d = 0.$

The seller's participation constraint has the price ϕ^{am}, since the seller spends the money obtained in the late-night settlement subperiod the next morning. The solution to the buyer's problem (93)–(94) is

(95) $$\frac{u'(q^d)}{c'(q^d)} = \frac{\phi^{\text{pm}}}{\phi^{\text{am}}}.$$

From (95), $q^d = q^*$ if and only if $\phi^{\text{am}} = \phi^{\text{pm}}$. If $\phi^{\text{am}} < \phi^{\text{pm}}$, then $q^d < q^*$.

If $\phi^{\text{pm}} > \phi^{\text{am}}$, then sellers holding money at the beginning of the night will spend all of it so that at the end of the night all of the money is held by the late-producer buyers, i.e., $d/2 = M$. If $\phi^{\text{pm}} = \phi^{\text{am}}$, then sellers holding money are indifferent between spending it at night or in the following morning. In this case, $d/2 \leq M$. From the seller's participation constraint (94),

(96) $$c(q^d)\begin{Bmatrix} = \\ \leq \end{Bmatrix} 2M\phi^{\text{am}} \qquad \text{if} \quad \phi^{\text{pm}} \begin{Bmatrix} > \\ = \end{Bmatrix} \phi^{\text{am}}.$$

A steady-state equilibrium is a list $(q^m, q^d, \phi^{\text{am}}, \phi^{\text{pm}})$ that satisfies (91), (92), (95), and (96). It is easy to check that $q^m = q^d = q^*$ and $\phi^{\text{am}} = \phi^{\text{pm}} = c(q^*)/2M$ is an equilibrium. (And one can show that this is the unique equilibrium for some specifications for u and c, e.g., $c(q) = q$ and $u(q) = 2\sqrt{q}$). In this equilibrium, the price of money is the same in the morning and night markets, and the efficient quantity of the search good q^* is traded in all matches. Note that early-producer buyers, who use money, are as well off as late-producer buyers, who use credit. So, if buyers could choose using money or debt in bilateral matches, they would be indifferent between the two means of payment.

Settlement and Liquidity

We now introduce settlement frictions. Settlement frictions are captured by assuming that debtors and creditors arrive and leave the late-night settlement period at different times. So a liquidity problem may arise if a creditor leaves the late-night settlement period before his debtor arrives. To be more specific, the timing during the late-night settlement period is as follows: All of the creditors and a fraction α of debtors arrive at the beginning of the late-night settlement period. Then a fraction δ of the creditors depart, after which the remaining $(1-\alpha)$ debtors arrive. Finally, the remaining $(1-\delta)$ creditors and all of the debtors leave the late-night settlement period; debtors, who are buyers, all die, and creditors move into the morning of the next period. We will sometimes refer to creditors (debtors) as being early-leaving (-arriving) and late-leaving (-arriving), where the meaning is obvious. These arrival and departure frictions will create a need for a resale market for debt during the late-night settlement period. We denote ρ as the value of one-dollar of debt in terms of money in this market.

There are agents in the economy who are neither creditors nor debtors; for example, sellers who produce search goods for money during the day. These sellers may have an incentive to forgo (some) consumption in the night market and instead provide liquidity in the late-night settlement period. More specifically, sellers who have money balances at the beginning of the night may, in the late-night settlement period, want to buy the IOUs

of early-leaving creditors that will be repaid by late-arriving debtors. For simplicity, and with no loss in generality, we assume that sellers with money who choose not to spend all of it at night always arrive at the beginning of the late-night settlement period and always stay until the end.

The problem of an early-producer buyer must now take into account the possibility that a seller who receives money for producing search goods during the day may want to use some of it to purchase debt in the late-night settlement period. A seller who receives one unit of money in a bilateral match during the day can spend it at night for ϕ^{pm} units of the general good, or he can buy $1/\rho$ IOUs in the late-night settlement period and then purchase ϕ^{am}/ρ units of the general good the following morning. Since $\phi^{\text{pm}} \geq \phi^{\text{am}}/\rho$ is required for the night money market to clear, the seller's participation constraint is still given by $c(q^m) = \phi^{\text{pm}}m$. Hence, the early-producer buyer's day-bargaining problem is (still) given by (89)–(90). As well, the solution to this problem is characterized by (91), and the quantity produced in this day match, q^m, satisfies equation (92).

The late-producer buyer's day-bargaining problem must also take into account the frictions that affect settlement in the late-night period. More specifically, since creditor sellers may have to sell their IOUs at a discount if they need to leave the settlement phase before their debtors arrive, the participation constraint of a seller who trades output for debt will be affected. Let θ denote the expected value to the seller of a one-dollar IOU expressed in dollars. The buyer's bargaining problem can be represented by

$$(97) \qquad \max_{q^d,d}[u(q^d) - \phi^{\text{pm}}d]$$

$$(98) \qquad \text{s.t. } c(q^d) - \theta\phi^{\text{am}}d = 0,$$

where θ satisfies

$$(99) \qquad \theta = \delta\alpha + \delta(1-\alpha)\rho + (1-\delta)\frac{\alpha}{\rho} + (1-\delta)(1-\alpha).$$

Equation (99) has the following interpretation. With probability δ a seller holding a one-dollar IOU must leave the late-night settlement place early. If his debtor has already arrived, an event which occurs with probability α, the IOU is redeemed for one dollar. Otherwise, the IOU is sold at the price ρ. With probability $1-\delta$, the seller with a one-dollar IOU does not need to leave early. Therefore, the IOU is redeemed for one dollar, irrespective of the arrival time of his debtor. However, if the debtor arrives early, an event which occurs with probability α, the creditor can use the dollar he receives to buy $1/\rho$ IOUs that will be redeemed for $1/\rho$ dollars at the end of the settlement phase. The solution to the late-producer buyer's bargaining problem (97)–(98) is given by

(100) $\qquad \dfrac{u'(q^d)}{c'(q^d)} = \dfrac{\phi^{pm}}{\theta \phi^{am}}.$

If $\phi^{pm} > \phi^{am}/\rho,$ then sellers who hold money at the beginning of the night prefer spending it at night rather than the following morning. As a consequence, the equilibrium of the money market implies $d/2 = M$. If, however, $\phi^{pm} = \phi^{am}/\rho,$ then sellers are indifferent between spending money at night or in the morning, so that $d/2 \le M$. Since $c(q^d) = \theta \phi^{am} d,$ we have

(101) $\qquad c(q^d) \begin{Bmatrix} = \\ \le \end{Bmatrix} 2M\theta\phi^{am} \qquad \text{if } \phi^{pm} \begin{Bmatrix} > \\ = \end{Bmatrix} \dfrac{\phi^{am}}{\rho}.$

Let us turn to the equilibrium of the second-hand debt market in the late-night settlement period. Denote $\Delta = M - d/2$ as the funds that sellers with money retain at night so that they can purchase second-hand IOUs in the late-night settlement period. Note that ρ, the price of IOUs in the late-night settlement period, cannot be greater than one; otherwise, no one would buy second-hand IOUs. Therefore, $\rho \le 1$. The supply of "funds," i.e., money, by creditors who are repaid early and leave late is $(1-\delta)\alpha d/2$. (Recall that half of the sellers in the day are paid with IOUs.) The supply of funds of sellers who transacted with money in the day is Δ. The demand of funds from creditors who leave early is $\delta(1-\alpha)d\rho/2$. From this, the market-clearing price ρ satisfies

(102) $\qquad \rho = \min\left[\dfrac{(1-\delta)\alpha d + 2\Delta}{\delta(1-\alpha)d}, 1 \right].$

A steady-state equilibrium is a list $(\phi^{am}, \phi^{pm}, \rho, q^m, q^d)$ that satisfies (91)–(92) and (100)–(102). We distinguish between two types of equilibria: one where $\rho = 1$ and one where $\rho < 1$. If $\rho = 1$, then there is no liquidity shortage in the late-night settlement period: Second-hand IOUs are sold at par, $\rho = 1$ and $\theta = 1$. The equilibrium conditions are then identical to those of the economy without any settlement frictions in the late-night settlement period, so that $q^m = q^d = q^*$ and $\Delta = 0$. From (102), $\rho = 1$ requires that $(1-\delta)\alpha/\delta(1-\alpha) \ge 1$, or equivalently, $\alpha \ge \delta$. Intuitively, there is no liquidity shortage if the measure of debtors who arrive early in the settlement place, α, is larger than the measure of creditors who leave early, δ. Creditors who are repaid by early-arriving debtors can use this money to purchase the IOUs of creditors who need to sell them, the earlier-leaving creditors.

Let us now turn to equilibria where second-hand debt is sold at a discount in the late-night settlement period, i.e., where $\rho < 1$. The equilibrium is liquidity-constrained in the sense that the amount of money available at the late-night settlement period just prior to the departure of the early-leaving creditors is insufficient to clear debts at their par value. One can first show that if $\rho < 1$, then $\Delta > 0$, which implies that sellers with money pro-

vide additional liquidity in the late-night settlement period by only spending a fraction of their money balances at night. To see this, suppose to the contrary that $\Delta = 0$. Then, from (102), $\rho = (1-\delta)\alpha / \delta(1-\alpha)$ and, from (99), $\theta = 1$. But then, the equations determining $(q^m, q^d, \phi^{am}, \phi^{pm})$ are exactly the same as those derived in the model with no settlement frictions implying that $\phi^{pm} = \phi^{am}$, which contradicts the no-arbitrage condition $\phi^{pm} \geq \phi^{am} / \rho$. Therefore, $\Delta > 0$, when $\rho < 1$.

When $\rho < 1$ and $\Delta > 0$, condition (101) implies that $\phi^{pm} = \phi^{am} / \rho$, which means that sellers with money are indifferent between spending money at night or the following morning. Since $\phi^{pm} > \phi^{am}$ and $\theta < 1$, equations (95) and (100) imply that the quantities traded in the day's bilateral matches must satisfy $q^d < q^* < q^m$: Buyers who trade with money in the day receive more output than those who trade with credit.

The liquidity shortage during the late-night settlement period affects the allocation of resources by making money more valuable at night than in the morning. Indeed, since unsettled debts are sold at a discount during the late-night settlement period, there is an additional demand for liquidity at night. The fact that money is more valuable at night allows early-producer buyers to consume more, whereas the consumption of late-producer buyers falls.

Settlement and Default Risk

We now introduce an idiosyncratic risk of late-producing buyers defaulting on their debt. We formalize the default risk by assuming that the debtor can produce at night with probability γ and, with probability $(1-\gamma)$, he is unable to produce and, hence, defaults on his debt. A debtor does not know whether or not he will default before night. Therefore, in the day, buyers and sellers have symmetric information in their bilateral matches. We assume that debtors who are unable to produce, and hence default on their debt, do not show up at the late-night settlement period.

The problem of an early-producer buyer is still given by (89)–(90) since transactions are conducted with money. The problem of a late-producer buyer, however, is now given by

(103) $\quad \max_{q^d, d}[u(q^d) - \gamma\phi^{pm}d]$

(104) $\quad$ s.t. $-c(q^d) + \theta'\phi^{am}d = 0.$

According to (103), the buyer receives q^d from the seller and repays his debt with probability γ. According to (104), the seller who receives a promise of d dollars can expect to have $\theta'd$ dollars, at the end of the period, that can be spent the following morning, where θ', the expected value of a one-dollar IOU, now reflects not only any settlement frictions but also the possibility of default. The solution to (103)–(104) implies that

(105) $\quad \dfrac{u'(q^d)}{c'(q^d)} = \dfrac{\gamma \phi^{\mathrm{pm}}}{\theta' \phi^{\mathrm{am}}}.$

In the absence of any settlement frictions, it will be the case that $\theta' = \gamma$ and, therefore, $q^d = q^*$. The default risk is reflected in the amount of money that the buyer commits to repay, but the quantity of output traded in bilateral matches remains efficient.

Consider a seller who has money at night and who contemplates buying a second-hand IOU in the late-night settlement period. The probability that a second-hand IOU will be repaid conditional on the fact that the debtor did not arrive early is $\gamma(1-\alpha)/(1-\gamma\alpha)$. (There are three possible events for an IOU: It is not repaid, which occurs with probability $1-\gamma$; it is repaid early, which occurs with probability $\gamma\alpha$; it is repaid late, which occurs with probability $\gamma(1-\alpha)$.) Therefore, the price ρ of a second-hand IOU cannot be greater than $\gamma(1-\alpha)/(1-\gamma\alpha)$, in order for debt to clear in the resale market. If liquidity in the central clearing market is not constrained, then $\rho = \rho^*$, where

(106) $\quad \rho^* = \dfrac{\gamma(1-\alpha)}{1-\gamma\alpha}.$

From (106), in the absence of a liquidity constraint, the price of IOUs simply reflects the probability of default (conditional on the fact that they have not been redeemed early).

The expected value of an IOU in the day satisfies

(107) $\quad \theta' = \gamma\left(\delta\alpha + (1-\delta)\alpha\,\dfrac{\rho^*}{\rho} + (1-\delta)(1-\alpha)\right) + \delta(1-\gamma\alpha)\rho.$

Equation (107) has the following interpretation: The debtor arrives early with probability $\gamma\alpha$. With probability δ the creditor leaves early, in which case he gets the par value of the IOU. With probability $1-\delta$, he can stay late and use his money to buy a second-hand IOU at the price ρ. The probability that the second-hand IOU is repaid is ρ^*. The debtor arrives late with probability $\gamma(1-\alpha)$. If the creditor can wait, with probability $1-\delta$, he receives one dollar at the end of the settlement phase. Finally, if the debtor does not arrive early (because he defaults or because he arrives late), an event that occurs with probability $1-\gamma\alpha$, and if the creditor leaves early, with probability δ, then the creditor sells his IOU at the price ρ. If liquidity is not scarce in the late-night settlement period, $\rho = \rho^*$ and $\theta' = \gamma$.

The equilibrium is not liquidity-constrained whenever $\gamma\alpha(1-\delta) \geq \delta(1-\alpha\gamma)\rho^*$ which, from (106), is equivalent to $\alpha \geq \delta$. This is precisely the condition we had in the absence of default risk. The fact that the rate of repayment γ does not influence the condition for a liquidity shortage can be explained as follows: Consider an increase in the repayment

rate. On the one hand, the number of creditors who are repaid early increases, so there is more liquidity in the late-night settlement period. On the other hand, the price of second-hand debt increases in the late-night settlement period, so the demand for liquidity is higher. The two effects just cancel each other.

Related Literature

The model of settlement presented in this section is closely related to the one by Freeman (1996a,b). Freeman considered an overlapping-generations economy with heterogenous agents. Some agents trade with debt, while others trade with money. As in our analysis, all debts are settled in a central clearing house. Freeman (1999) extends the model to allow for aggregate default risk. Green (1999) shows that the role of the central bank as a clearing house can be assumed by ordinary private agents. Zhou (2000) discusses the literature. Temzelides and Williamson (2001) consider two related models, a model with spatial separation and a random-matching model, and investigate different types of payment arrangements: monetary exchange, banking with settlement, and banking with interbank lending. They show that payment systems with net settlement generate efficiency gains, and interbank lending can support the Pareto-optimal allocation in the absence of idiosyncratic shocks.

Policy and Payments

Optimality of the Friedman Rule

In this section, we determine the optimal growth rate of the money supply in the economy with divisible money that was examined in the section on Divisible Money. Let M_t represent the stock of money at date t, and π the constant rate of growth of the money supply, i.e., $M_t = M_{t-1}(1+\pi)$. Money is injected or withdrawn in a lump-sum fashion in the centralized market: If $\pi > 0$, then injections of money take place at the beginning of the centralized market; if $\pi < 0$, then money is withdrawn at the end of the centralized market. Without loss of generality, we assume that money transfers go only to the buyers. We focus on steady-state equilibria where the real value of the money supply is constant over time, i.e., $\phi_t M_t = \phi_{t+1} M_{t+1}$. In equilibrium, the price of money in terms of general goods is falling at rate π.

To take into account that the price of money is not constant across time, we write the value functions V^b and W^b as functions of the buyer's real balances, $\phi_t m_t$. The Bellman equations for V^b and W^b are given by

$$V^b(\phi_t m_t) = \sigma \left\{ u(q(\phi_t m_t)) + W^b(\phi_t m_t - \phi_t d_t) \right\} + (1-\sigma)W^b(\phi_t m_t)$$
$$= \sigma \left\{ u(q(\phi_t m_t)) - \phi_t d_t \right\} + W^b(\phi_t m_t)$$

and

$$W^b(\phi_t m_t) = \max_{m_{t+1}}\{\phi_t m_t - \phi_t m_{t+1} + \beta V^b(\phi_{t+1}m_{t+1})\}.$$

We assume that prices are determined by take-it-or-leave-it offers by buyers. This implies that the quantity traded in a match satisfies $c(q_t) = \phi_t m_t$ whenever $\phi_t m_t \le c(q^*)$. The buyer's problem at time t can be generalized to read

$$(108) \quad \max_{\hat{m}}\left\{-\phi_t\hat{m} + \phi_{t+1}\hat{m} + \beta\left[\sigma\left(u\left(q(\phi_{t+1}\hat{m})\right) - c\left(q(\phi_{t+1}\hat{m})\right)\right)\right]\right\}.$$

According to (108), the buyer at night who wishes to hold $\phi_{t+1}\hat{m}$ real balances the following day must produce $\phi_t\hat{m}$ of night goods. In the following day, he trades with probability σ, in which case he extracts all the surplus of the match. Denote the buyer's choice of real balances as $z = \phi_{t+1}\hat{m}$ and the nominal interest rate as i, where $1+i = (1+r)(1+\pi)$. Through the seller's participation constraint, there is a one-to-one relationship between z and q for all $z \le c(q^*)$, i.e., $z = c(q)$. Hence, the buyer's problem (108) can be rewritten more compactly as a choice for q,

$$(109) \quad \max_{q\in[0,q^*]}\{-ic(q) + \sigma[u(q) - c(q)]\},$$

The first-order condition to the buyer's problem (109) is simply

$$(110) \quad \frac{u'(q)}{c'(q)} = 1 + \frac{i}{\sigma}.$$

This equation is similar to (53), except for the fact that the real interest rate, r, has been replaced by the nominal interest rate, i. The cost of holding real balances, i, generates a wedge that is proportional to the average length of time to complete a trade in the day market, $1/\sigma$.

   From (110), it is clear that the optimal monetary policy corresponds to the Friedman rule, which requires the nominal interest rate i to be set equal to zero, or equivalently, that the rate of growth of the money supply be negative and approximately equal to the rate of time preference. Intuitively, by reducing the cost of holding real balances to zero, the Friedman rule maximizes the demand for real balances and, therefore, the quantities traded in individual matches. The allocation of the monetary equilibrium under the Friedman rule coincides with the socially efficient allocation, i.e., $q = q^*$.

   The result that the Friedman rule generates the first-best allocation is sensitive to the choice of the bargaining solution. To see this, let's assume that the terms of trade, (q,d), are determined by the symmetric Nash solution, i.e., (q,d) is given by the solution to

(111) $\max_{q,d}[u(q) - \phi d][-c(q) + \phi d]$ s.t. $d \leq m$.

If the constraint $d \leq m$ binds (and it will in equilibrium), then the relationship between q and $z = \phi m$ is given by

(112) $z = \dfrac{c'(q)u(q) + u'(q)c(q)}{u'(q) + c'(q)}$.

Since there is a one-to-one relationship between z and q for all $q \in [0, q^*]$, the buyer's choice of real balances can be rewritten as a choice of q, i.e.,

(113) $\max_{q \in [0, q^*]} \{-iz(q) + \sigma[u(q) - z(q)]\}$.

At the Friedman rule, the buyer simply chooses q to maximize his surplus, $u(q) - z(q)$. Using (112), the buyer's surplus can be re-expressed as

(114) $u(q) - z(q) = \dfrac{u'(q)}{u'(q) + c'(q)}[u(q) - c(q)]$.

According to equation (114), the buyer receives a fraction $u'(q)/[u'(q) + c'(q)]$ of the match surplus. Since $u'(q)/[u'(q) + c'(q)]$ is decreasing in q, it is easy to show that the buyer's surplus is decreasing in q when q is close to q^*. Therefore, buyers choose an inefficiently low value for q, even when the cost of holding real balances is zero. This inefficiency is due to the nonmonotonicity of the Nash bargaining solution, according to which the buyer's surplus can fall even if the match surplus increases. However, despite the fact that real balances are "too low," the optimal monetary policy is still the Friedman rule.

Trading Frictions and the Friedman Rule

There are two dimensions associated with trading in a search environment: the *quantities* traded in individual matches, sometimes called the *intensive margin*, and the number of matches, sometimes called the *extensive margin*. Monetary policy can affect both margins. As we have seen, monetary policy affects agents' choices of real balances and, therefore, the intensive margin. But it can also affect agents' costs of participating in the market and, therefore, the extensive margin. In the previous section, we saw that the Friedman rule takes care of the intensive margin because it maximizes buyers' real balances and, therefore, the quantities traded in bilateral matches. However, it is not at all clear whether the Friedman rule generates an efficient extensive margin and, more generally, whether it is the optimal monetary policy when the number of trades is endogenous.

In order to generate an extensive-margin effect, we have to slightly alter our benchmark model. Assume now that there is a unit measure of *ex ante* identical agents that can *choose* to be buyers or sellers in the day market. Suppose, for instance, that there are

two goods produced during the day: an intermediate good and a final consumption good, which requires the intermediate good as an input. The final consumption good can only be consumed by the agent who produces it, and agents have to specialize in one of the production technologies. Agents who produce intermediate goods are sellers, while those who produce final goods are buyers. Hence, the intermediate good will be produced and traded in bilateral matches between buyers and sellers during the day.

The fraction of buyers is denoted by n, whereas the fraction of sellers is $1 - n$. The matching technology between buyers and sellers is as follows: A buyer meets a seller with probability $1 - n$, the fraction of sellers in the population. A seller meets a buyer with probability n, the fraction of buyers in the population. As a consequence, the number of matches in the day market is $n(1 - n)$. The number of matches is maximized when $n = \frac{1}{2}$.

Clearly, the way in which the terms of trade are determined will affect an agent's choice of which side of the market to participate in, i.e., whether to be a buyer or a seller. Here we will assume that the terms of trade are determined by a simple proportional bargaining solution, according to which the buyer's surplus is a fraction $\theta \in (0,1)$ of the total match surplus, i.e.,

$$(115) \quad u(q) - \phi d = \theta[u(q) - c(q)],$$

where q is the level of intermediate goods produced in a match and d is the monetary transfer from the buyer to seller. Furthermore, the trade (q,d) is pairwise Pareto-efficient so that $q = q^*$ if $\phi m \geq (1-\theta)u(q^*) + \theta c(q^*)$ and $d = m$ otherwise. Assuming the constraint $d \leq m$ is binding, there is a simple relationship between q and $z = \phi m$,

$$(116) \quad z = (1-\theta)u(q) + \theta c(q).$$

The buyer's expected utility at night $W^b(\phi m)$ satisfies a Bellman equation similar to (113) except that σ is replaced by $1 - n$. Hence, the buyer's choice of real balances:

$$(117) \quad \max_z \left\{ -iz + (1-n)\theta\{u[q(z)] - c[q(z)]\} \right\}.$$

The problem in (117) takes into account the fact that the buyer obtains a fraction θ of the surplus of a match. The solution to the buyer's problem (117) is given by

$$(118) \quad \frac{i}{(1-n)\theta} = \frac{u'(q) - c'(q)}{(1-\theta)u'(q) + \theta c'(q)}.$$

An agent will be indifferent between a buyer and a seller if $W^b(\phi m) = W^s(\phi m)$. Since both value functions are linear, the choice of being a buyer or seller is independent of the

money balances of the agent when he enters the centralized market. After some calculation, the condition $W^b(0) = W^s(0)$ yields

$$(119) \quad -i[(1-\theta)u(q) + \theta c(q)] + (1-n)\theta[u(q) - c(q)] = n(1-\theta)[u(q) - c(q)].$$

Equation (119) can be explained as follows: The left-hand side is the buyer's payoff. It is the sum of two elements: the cost of carrying $z = (1-\theta)u(q) + \theta c(q)$ real balances and the expected surplus of a match. The right-hand side is the seller's payoff, which is simply the seller's expected surplus of a match.

We first ask whether the Friedman rule generates the first-best allocation. As the nominal interest rate, i, tends to zero, equation (118) implies that q approaches q^*, and equation (119) implies that n approaches θ, the buyer's bargaining power. As before, the Friedman rule generates the efficient intensive margin; this is true even though the buyer does not have all the bargaining power. However, if θ is different from one-half, the composition of buyers and sellers will be socially inefficient at the Friedman rule. The requirement that $\theta = \frac{1}{2}$ for the composition of buyers and sellers to be efficient is related to the Hosios (1990) condition for efficiency in search models, according to which efficiency is achieved when an agent's bargaining power coincides with his contribution to the matching process. A buyer's bargaining power is θ, whereas his contribution to the matching process is $1 - n$, the fraction of sellers in the economy. From equation (119), it is straightforward to see that the condition $\theta = n$ is satisfied at the Friedman rule. Therefore, if $i = 0$ and the buyer's bargaining power coincides with his contribution to the matching process, then $\theta = \frac{1}{2}$.

We now ask whether a deviation from the Friedman rule can be optimal. We measure social welfare by $\mathcal{W} = n(1-n)[u(q) - c(q)]$. The effect of a change in the inflation rate on the number of buyers is given by

$$(120) \quad \left.\frac{dn}{di}\right|_{i\downarrow 0} = -\frac{(1-\theta)u(q^*) + \theta c(q^*)}{u(q^*) - c(q^*)} < 0.$$

As the cost of holding real balances increases, the number of buyers decreases. This effect can be easily understood from equation (119). An increase in inflation has a direct negative effect on buyers, which is given by the right-hand side of (119). When a match occurs, this cost of holding the real balances is sunk and, hence, cannot be recovered by the buyer. The effect on welfare of a deviation from the Friedman rule is given by

$$(121) \quad \left.\frac{d\mathcal{W}}{di}\right|_{i\downarrow 0} = -2\left(\frac{1}{2} - \theta\right)[(1-\theta)u(q^*) + \theta c(q^*)].$$

So a deviation from the Friedman rule is optimal whenever $\theta > \frac{1}{2}$. In this case, there are

too many buyers and, therefore, too few trades. An increase in inflation reduces the number of buyers and increases the number of sellers, and, therefore, increases the number of trades. Inflation also reduces the quantities traded in individual matches, but since this has only a second-order effect close to the Friedman rule, overall welfare will increase.

Distributional Effects of Monetary Policy

Inflation can be beneficial when the number of trades is inefficient because it affects agents' decisions to participate in the market. Monetary policy can also have a positive effect on the extensive margin when the distribution of money balances is not degenerate. To capture this distributional effect of monetary policy, we modify the benchmark model as follows: Buyers and sellers live for only three subperiods—they are born at night and die at the end of the following period. Agents can, therefore, potentially trade three times: In the night when they are born, on the following day, and in the night just before they die. Assume that agents do not discount across periods. This implies that the Friedman rule corresponds to a zero inflation rate. In order to obtain a nondegenerate distribution of money balances across agents, we assume that only a fraction p of newly born buyers get access to the centralized general goods market, say, because they are productive. The remaining $1 - p$ (unproductive) buyers are excluded and, therefore, are unable to acquire money to be able to trade in the next day's market.

The problem of a newly born buyer who has access to the centralized night market, which is similar to problem (108), is

$$(122) \quad \max_{\hat{m}}\left\{-\phi_t\hat{m} + \phi_{t+1}\hat{m} + \sigma\{u[q(\phi_{t+1}\hat{m})] - c[q(\phi_{t+1}\hat{m})]\}\right\}.$$

Since the buyer has access to the centralized general goods market when he is born, he can produce to accumulate the $\hat{m}$ money balances he needs to trade in the next day's search market. If he doesn't meet a seller during the day, he spends his money balances in the night before he dies; if he does meet a seller, we assume that he captures the entire surplus from the match. Denote $z = \phi_{t+1}\hat{m}$ as the buyer's choice of real balances for the next day's search market. The buyer's problem (122) can be simplified to read

$$(123) \quad \max_{z}\left\{-\pi z + \sigma\{u[q(z)] - c[q(z)]\}\right\}.$$

The first-order condition for this problem is

$$\frac{u'(q)}{c'(q)} = 1 + \frac{\pi}{\sigma}.$$

Therefore, whenever the money supply is constant, i.e., when $\pi = 0$, newly born buyers who are not excluded from the centralized general goods market can trade $q = q^*$

units of the search good the following day. However, if the money supply is constant, then those (unproductive) buyers who are excluded from the night market when they are young receive no money transfers and, therefore, cannot consume during the day.

Assume now the there is a steady state inflation and that money is injected into the economy through lump-sum transfers to buyers. Let T_t denote a transfer at night in period t. We have $T_t = M_{t+1} - M_t = \pi M_t$. Let m_t represent the money balances of buyers in the morning of period t who had access to the centralized general goods market when they were young. Hence, equilibrium in the money market requires that

$$(124) \quad pm_t + (1-p)T_{t-1} = M_t.$$

Using the definition of T_t and (124), we obtain

$$(125) \quad m_t = M_t \left(\frac{1+p\pi}{p+p\pi} \right)$$

and

$$(126) \quad T_{t-1} = \frac{p\pi}{1+p\pi} m_t.$$

Note from (126), that $T_{t-1} < m_t$ so that unproductive buyers have less money balances than buyers who had access to the night market when young. Let $\tilde{q}$ denote the quantities traded by unproductive buyers. Since $c(q_t) = \phi_t m_t$ and $c(\tilde{q}_t) = \phi_t T_{t-1}$ from the buyer-takes-all assumption we have

$$(127) \quad c(\tilde{q}_t) = \frac{p\pi}{1+p\pi} c(q_t).$$

From (127) $\tilde{q}_t < q_t$ and $\lim_{\pi \to \infty} \tilde{q}_t = q_t$. So the planner faces a trade-off between smoothing consumption across buyers and preserving the purchasing power of real balances. Welfare is measured by $\mathcal{W} = \sigma p[u(q) - c(q)] + \sigma(1-p)[u(\tilde{q}) - c(\tilde{q})]$. It can be checked that a deviation from price stability has a beneficial effect on welfare, i.e.,

$$(128) \quad \left. \frac{dW}{d\pi} \right|_{\pi \downarrow 0} = \sigma(1-p) \left[\frac{u'(0)}{c'(0)} - 1 \right] pc(q^*) > 0.$$

An increase in inflation from $\pi = 0$ is optimal because it allows unproductive buyers to consume, while the negative effect on productive buyers' welfare is only second-order.

Settlements and Monetary Policy

Assume for the time being that there is no default risk and that there is a liquidity shortage in the late-night settlement period. There will be a liquidity shortage when the fraction of creditors who depart early, δ, is greater than the fraction of debtors who arrive

early, α. In this situation, the market clearing price for debt in the late-night settlement period, ρ, will be less than one. As well, sellers who produced search goods for money during the day will only spend a fraction of their balances in the competitive general goods market at night and will use the remainder to purchase IOUs in the late-night settlement period. This implies that the levels of search good production will be inefficient; in particular, $q^d < q^* < q^m$, where q^d is the amount of search goods produced for a buyer who purchases with an IOU and q^m is the amount of search goods produced for a buyer who purchases with money. Hence, the late-night liquidity problem results in inefficient levels of production of search goods during the day.

Suppose now that there exists a central bank that can provide "liquidity" to the late-night settlement period. More specifically, the central bank purchases Δ^{cb} amount of IOUs from early-leaving creditors in exchange for fiat money. When the late-arriving debtors come to the late-night settlement period, the central bank will exchange the IOUs for fiat money. Recall that the supply of funds by creditors who are paid early and stay late is $(1 - \delta)\alpha d / 2$ and that the face value of bonds of the creditors who leave early and whose issuers arrive late is $\delta(1 - \alpha)d / 2$. If $\Delta^{cb} \geq (\delta - \alpha)d / 2$, then the liquidity problem is solved and it will be the case that $\rho = 1$. This temporary supply of liquidity by the monetary authority resembles either a discount window policy or an open market operation. As an open market operation, the central bank purchases $(\delta - \alpha)d / 2$ units of bonds before the early-leaving creditors depart and "sells" the bonds back after the late arriving debtors arrive. As a discount window policy, the central bank stands ready to purchase second-hand IOUs at their par value, with the understanding that the IOUs have to be repurchased at their par value before the late-night settlement period ends. One can imagine one of the late-leaving creditors—call him the *clearinghouse*—gathers $(\delta - \alpha)d / 2$ units of IOUs from early-leaving creditors, exchanging them with the central bank for money, with the understanding that the IOUs will be repurchased before the late-night settlement period ends. When the late-arriving debtors arrive, the clearinghouse can obtain money from the debtors whose creditors have already left, repurchase the debt from the central bank, and return the IOUs to the original issuers. The increase in the money supply that results from the open market operation or discount window policy is not inflationary, since the IOUs purchased by the monetary authority are all redeemed within the period so that the stock of currency remains constant across periods. This policy is also in accordance with the real bills doctrine, which considers that the stock of money should be allowed to fluctuate to meet the needs of trade.

Note that a central bank is not needed in order to overcome the liquidity problem. Suppose that a late-leaving creditor, say a clearinghouse, purchases the debt of early-leaving creditors with his own IOUs, with the understanding that the IOU's of the clearinghouse can be exchanged for money at the beginning of the next period, before the early-morning general goods market opens. When the late-arriving debtors arrive, the clearinghouse

will exchange the debt that it holds for money. At the beginning of the next period, the clearinghouse can repurchase its debt with money before the general goods market opens. Hence, as long as the clearinghouse is able to repurchase the debt it has issued, the liquidity problem that arises due to the settlement frictions can be overcome by private agents: It is not necessary for a central bank to exist in order to deal with a liquidity problem that may arise due to settlement frictions.

Related Literature

The result according to which the optimal monetary policy requires setting the nominal interest rate to zero or, equivalently, deflating at the rate of time preference, comes from Friedman (1969). Different definitions and interpretations of the Friedman rule are discussed in Woodford (1990). The optimal monetary policy in a search model with divisible money was first studied by Shi (1997), who showed that the Friedman rule is optimal when agents' participation decisions are exogenous. The ability of the Friedman rule to generate the first-best allocation when the terms of trade are determined according to some bargaining solution are discussed in Rauch (2000), Lagos and Wright (2005), and Rocheteau and Waller (2004).

The importance of trading frictions and search externalities for the design of monetary policy was first emphasized by Li (1995, 1997), who established that an inflation tax could be welfare enhancing when agents' search intensities are endogenous. However, her results are subject to the caveat that prices are exogenous. Shi (1997) found a related result in a divisible-money model where prices are endogenous. In Shi's model, each household has a large number of members who can be divided between buyers and sellers. When the composition of buyers and sellers is inefficient, a deviation from the Friedman rule can be welfare improving. Rocheteau and Waller (2004) discuss Shi's finding under alternative bargaining solutions. Berentsen, Rocheteau, and Shi (2001) established the result according to which the efficient allocation is achieved when both the Hosios rule and the Friedman rule are satisfied. A necessary condition for a deviation from the Friedman rule to be optimal is that the Hosios condition is violated. Rocheteau and Wright (2004) study the optimal monetary policy in a model with free entry of sellers under alternative pricing mechanisms.

The welfare-improving role of a monetary expansion through distributional effects has been studied by Levine (1991), and in a search-theoretic environment by Molico (1997), Deviatov, and Wallace (2002), and Berentsen, Camera, and Waller (2004).

Freeman (1996a) and (1996b) provides a framework to study the settlement of debt with money in a model with no aggregate risk. Freeman (1999) adds aggregate risk to the analysis. Green (1999) provides a general overview of a settlements model and demonstrates that a central bank is not needed to provide liquidity to the economy if there is a "shortage"; agents in the model are able to provide the requisite liquidity.

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