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**STICKY PRICES, MONEY, AND BUSINESS FLUCTUATIONS**

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## Abstract

Can nominal contracts create monetary nonneutrality if they arise endogenously in general equilibrium? Yes, if (1) agents have complete information about the money stock and (2) shocks to the system are purely redistributive and private information, precluding conventional insurance markets. Without contracts, money is neutral toward aggregate quantities. However, risk-sharing between suppliers and demanders creates an incentive for both parties to use nominal contracts. In particular, if an increase in the money growth rate signals a rise in the dispersion of shocks to demanders' wealth, then prices adjust only partially to monetary shocks and money is positively associated with output.

## Introduction

Many macroeconomists believe that some form of price stickiness underlies the observed positive association of high money growth and high real activity at business-cycle frequencies. Often, this price stickiness is asserted to arise from explicit or implicit contracts. Model economies that do not include nominal contracts are consequently viewed as omitting the basic cause of monetary nonneutrality. For example, Lucas's (1972) pathbreaking general-equilibrium model of business fluctuations -- which employs imperfect aggregate information to generate monetary nonneutrality -- has been widely criticized for excluding nominal contracts, even though no economic forces would lead these to arise endogenously. Yet, in the past decade, few similarly explicit model economies have been produced that (1) derive a role for nominal contracts from underlying assumptions about the economic environment and (2) explain the implications of contract arrangements for money and business cycles.<sup>1</sup> Contract theory seemingly could not justify nominal contracts; today, the foundations of sticky prices rest more on the cost of price changes (Rotemberg [1982], Parkin [1986]), or on the multiplicity of rational-expectations equilibria (Azariadis and Cooper [1985b]).

This paper provides a simple rational-expectations general-equilibrium model in which endogenously generated contracts make a difference. That is, under some fiscal-monetary regimes, contracts simultaneously make prices sticky (so that they respond less than proportionately to changes in the quantity of money) and lead to a causal positive relationship between

contemporaneously observed money and production/effect. Further, our model economy is a variant of Lucas's (1972) setup. The difference is that we assume monetary changes are neutral toward real aggregates in the absence of contracts because economic agents accurately perceive these changes.<sup>2</sup>

These results derive from four underlying assumptions about the preferences, technology, and information structure of a stochastic consumption-loans model that is in most other ways identical to the full-information version employed by Lucas. First, risk-averse demanders of money are subject to idiosyncratic individual disturbances that are private information. That is, there is a demand for insurance against idiosyncratic disturbances, but the fact that these are private rules out the operation of conventional insurance markets, which make payments contingent upon verifiable losses. Second, the growth rate of money is positively associated with the dispersion of individual disturbances. This assumption, though not standard in formal modeling, has received attention from both monetary theorists and policymakers. Third, prior to the realization of money growth or individual shocks, suppliers of goods can compete by offering alternative contracts that specify a relationship between money growth and price adjustments. Fourth, the technology of exchange dictates that an individual visit only one supplier after realization of aggregate and individual disturbances.

In this environment, welfare can be improved by competitive contracts that embody a shifting of risk, with resources being transferred between suppliers and demanders in contingencies that involve high individual uncertainty.

Because money growth is an indicator of the extent of individual uncertainty, prices rise less than proportionately and production/effort expands when money growth is high. Conversely, prices fall less than proportionately and production/effort contracts when money growth is low.<sup>3</sup>

In our model economy, a Phillips curve emerges under two conditions: (1) an interaction of individual and aggregate uncertainty and (2) an incompleteness of markets, which is due to private information. We conjecture that our analysis illustrates a more general idea; that is, our results depend more on the existence of market incompleteness than on the specific rationale.

This paper contributes to a growing area of the microfoundations literature that uses contract theory to model the real effects of monetary disturbances. Not all of this literature attempts to model sticky prices. For example, Farmer (1988) and Bernanke and Gertler (1986) pinpoint credit as the transmission mechanism. Naturally, such studies have a distinctive emphasis and use quite different techniques.

Much of the literature, however, does try to justify the sticky prices and wages so central to the policy-oriented models of Gray (1976), Fischer (1977), and Taylor (1980). In some cases, sticky prices emerge almost as an afterthought; in Rogerson and Wright (1988), positive money shocks create an inflation tax and reduce wealth, which in turn affect labor supply and thus unemployment. In contrast, the bubble (or self-fulfilling-prophecy) literature attempts to explain sticky prices, output fluctuations, and other business-cycle phenomena as market-based occurrences that depend on expectations, not contracts (Azariadis and Guesnerie [1986]).

In the broadest class, and the one in which our paper best fits, contracts insure against risk.<sup>4</sup> As in Azariadis and Cooper (1985a) and Cooper (1988), contracts produce sticky prices to provide insurance against social risk, ensuring that risk is shared optimally across different groups.

Several salient features distinguish this work from that of Azariadis and Cooper, both in terms of modeling techniques and results. On the technical level, our model uses distributional rather than aggregate risk: Each individual's position is uncertain, but the total wealth of society is not. One advantage of this approach is that it produces sticky prices by using only monetary shocks. Slowly adjusting prices help to insure consumers against random monetary injections by shifting some of the risk to producers. The risk-sharing arrangement in this study also differs from that of Azariadis and Cooper. Here, contracts spread the risk among all agents in the economy; this broad distribution makes sense because all parties are then risk-averse. In Azariadis and Cooper, risk is shifted to the risk-neutral producer class (perhaps imperfectly, because of inefficiencies that result).

As might be expected, these different modeling techniques generate new and distinctive results. In our model, prices are sticky but not fixed; that is, they adjust -- although not proportionately -- to changes in the money supply. One important advantage of this approach over the fixed-price formulation is that it generates a Phillips curve (a positive relation between inflation and output). Azariadis and Cooper do not even permit the money supply to change. Our formulation, on the other hand, allows policy questions to be considered in a natural way: for example, how does increasing monetary

variability change the slope of the Phillips curve? When is monetary policy neutral?

The remainder of this paper is organized as follows: The basic structure of our model is outlined in section I. Competitive equilibrium without contracts is discussed in section II, and competitive equilibrium with contracts is developed in section III. Section IV summarizes and concludes.

### I. Structure of the Economy

In this section, we outline a stochastic consumption-loans model that draws heavily on Lucas (1972). In each period,  $N$  identical individuals are born, each of whom lives for two periods. In the initial period of the life cycle, effort is supplied in amount  $n$  and goods are consumed in amount  $c$ . In the latter period, goods are consumed in amount  $c'$  (a prime denotes an updated variable). Each individual's preferences for consumption and leisure are given by the utility function:

$$U(c, 1 - n) + V(c') \tag{1}$$

Following Lucas, we assume that: (1)  $U$  is increasing in consumption and leisure, strictly concave, and twice continuously differentiable; (2)  $V$  is increasing, strictly concave, and twice differentiable; (3)  $V$  is restricted so that current consumption and leisure are not inferior goods; and (4) agents' preferences are the expected value of equation (1) under situations of uncertainty. In addition to Lucas's preference assumptions, we require that old-age utility exhibit decreasing relative risk aversion.

Production takes place according to the simple scheme used by Lucas: One unit of effort yields one unit of output within the period, but goods are not storable.

There are a large number of islands (indexed by  $k = 1, 2, \dots, K$ ) in which productive activity occurs. At each date (indexed by  $t = 0, 1, \dots$ ), it is physically possible to transact (produce or consume) in only one of these marketplaces. In each period,  $J = N/K$  agents of each generation are presumed to transact in each market (in equilibrium). In contrast to Lucas, there are no exogenous shifts in demand across markets (caused by a random distribution of traders), and agents are fully cognizant of the terms of trade in other markets (although this information has no value in our setup). The importance of market structure is explained in more detail below.

Random money supply is the basic source of uncertainty in our model. Not only is the aggregate level of money uncertain, as in Lucas, but a source of individual uncertainty is added as well. On the aggregate level, we assume that money changes through time according to

$$m' = mx. \quad (2)$$

Here,  $m'$  is the next period's money supply,  $m$  is this period's money supply, and  $x$  is the growth factor; hence, the growth rate is  $x - 1$ . We assume that  $x$  is serially independent with mean  $\bar{x}$ . Thus, over a single period, the money supply grows by a random factor  $x$ , which is distributed as proportionate transfers to the holders of money (the elder generation), who therefore spend  $m'$ . Those currently young will take  $m'$  into the next period, where they will



spend  $m'x'$ . But in contrast to Lucas, we presume that, during the period, all agents know the values of  $x$  and  $m$ , which are the aggregate-state variables.<sup>5</sup>

This individual uncertainty, introduced through monetary transfers, is the key characteristic that distinguishes our study from that of Lucas. Each old agent receives a transfer,  $T$ , that has a nominal value of  $T = \eta xm$ , where  $\eta$  is the random shock that determines the amount of an individual's transfer. Transfers take this complicated form in order to prevent the nonneutralities that arise from a standard inflation tax. With a different specification, sticky prices would still exist, but the other effects would complicate the analysis. Within each island (and, a fortiori, in the aggregate), we require that transfers in each period aggregate to zero,  $\sum_{j=0}^J T_j = 0$ . This expected value of zero makes the transfers purely redistributive, and therefore the uncertainty about the transfers is not aggregate but purely individual. We further assume that  $\eta$  realizations are private information, so that conventional insurance arrangements are precluded. In addition, the distribution of the "shock,"  $\eta$ , may depend upon money growth,  $x$ , so that the conditional density functions of  $\eta$  can be written as  $g(\eta; x)$ .

This specification captures some of the uneven distribution of monetary injections (Friedman [1969, section III], Von Mises [1953, chapter VI]) and suggests that such dispersion increases with the size of the injection.<sup>6</sup> One realistic way that this could happen is if the various financial intermediaries react differently to monetary policies. Reserve and deposit growth would then be differentially distributed across firms and their

constituencies. It is likely that Gurley and Shaw (1960) originated this argument; by the 1970s, however, even the Federal Reserve recognized its validity (Burns [1978, p. 95]).

This connection between individual uncertainty and aggregate quantity is not standard: it destroys the simplicity of the representative-agent model. Nevertheless, Grossman, Hart, and Maskin (1983) use this kind of relationship to great effect. The specific interactions that we employ have often been considered, but they have never before been formally incorporated into a model.

Activities within each period adhere to the following sequence, illustrated in table 1. At the beginning of a period, prior to realization of shocks, old agents make locational decisions. In the contractual version of our model, this is the interval in which young agents in a specific market offer contracts in order to attract demanders. Subsequently, realization of  $x$  and  $\eta$  takes place, followed by production and consumption.

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Table 1		
Sequence of Activities within a Time Period		
(1)	(2)	(3)
location decisions; contracts offered	realization of shocks ( $x, \eta$ )	production; consumption

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## II. Competitive Equilibrium without Contracts

Because our analysis of the nature of competitive equilibrium without contracts is close to that of Lucas (1972), our treatment of this subject will be brief, developing material that will be useful in subsequent discussion.

Supply and demand for goods versus money determines the price level in our economy. The market-clearing value of this price (in any of the  $K$  identical islands) may be written as a function of the state of the economy  $(x,m)$ :

$$p = \psi(x,m). \quad (3)$$

Our analysis of the nature of this equilibrium price function follows Lucas.<sup>7</sup> Only young agents face a nontrivial decision problem: The old simply spend their accumulated cash balances, while the young must pick levels of consumption ( $c$ ), effort ( $n$ ), and money demand/saving ( $\lambda$ ). Recall that money serves as the intergenerational store of value in the overlapping-generations model; thus, money held ( $\lambda$ ) is also savings. The young choose savings and effort to maximize expected utility:

$$\begin{aligned} \max_{c,n,\lambda} [U(c, 1 - n) + EV(c')|x,m] & \quad (4) \\ \text{s.t. } p(n - c) - \lambda \geq 0 & \\ \lambda x' + \eta'x'm' - c'p' \geq 0, & \end{aligned}$$

where  $E(\cdot)|x,m$  denotes an expectation of a variable conditional on  $x$  and  $m$ ,  $\lambda$  is nominal money demand,  $p'$  is the future price level, and so on. The first constraint arises because money is the only store of value, so money-holding reflects the difference between current production and

consumption. The second constraint limits the next period's consumption. When old, the agent's money balance constrains consumption. The agent has savings (augmented by the proportional growth of money in the next period)  $\lambda x'$  and the random transfer  $\eta' x' m'$ .

It is useful to solve this maximization problem in two stages. First, consider picking efficient quantities of leisure and current consumption so as to maximize utility given a specific pattern of saving behavior. The results of this maximization process are an indirect utility function and a conditional demand for goods and leisure (or, equivalently, a supply of effort).

$$W\left(\frac{\lambda}{p}\right) = \max_{c, n} \{U(c, 1 - n)\} \text{ s.t. } n - c - \frac{\lambda}{p} \geq 0 \quad (5)$$

$$c = \phi_c\left(\frac{\lambda}{p}\right) \text{ and } n = \phi_n\left(\frac{\lambda}{p}\right) \quad (6)$$

Previous assumptions imply that  $W$  is twice continuously differentiable and that  $\phi_c$  and  $\phi_n$  are continuously differentiable. The assumption that consumption and leisure are normal goods implies that  $\phi'_c < 0$  and that  $\phi'_n > 0$ .<sup>8</sup> Second, consider selecting an efficient savings plan  $(\lambda/p)$  so as to maximize

$$W\left(\frac{\lambda}{p}\right) + EV\left(\frac{\lambda x'}{p'} + \frac{\eta' x' m'}{p'}\right) | x, m, \text{ or}$$

$$W\left(\frac{\lambda}{p}\right) + EV\left[\frac{\lambda}{p} \cdot R' + \frac{\eta' m'}{p} \cdot R'\right] | x, m,$$

where  $R' = px'/p'$  is the real return on money. The intertemporal efficiency condition for this plan is simply

$$W' \left( \frac{\lambda}{p} \right) + E[V'(\cdot) \cdot R'] | x, m = 0, \quad (7)$$

which states the standard first-order condition for a risky asset: equality between current utility forgone with a unit of saving  $(\lambda, p)$  and expected future utility received.

Individual income uncertainty  $(\eta'x'm')$  may raise the demand for saving as a "hedge," under conditions on  $V$  discussed by Sandmo (1970). This precautionary demand for saving is ensured if old-age marginal utility is convex ( $V'' > 0$ ), which is implied by diminishing absolute risk aversion. That is, savings will rise with greater second-period income uncertainty as long as the premium an individual must be paid to accept a fixed actuarially fair bet declines with the level of future consumption ( $c'$ ). Thus, in comparison to Lucas's setup -- which involves no idiosyncratic income shocks -- there will be more desired saving  $(\lambda p)$  at any rate of return  $R' = (px'/p')$ . In competitive equilibrium, money supply ( $xm$ ) must equal money demand  $(\lambda)$ . Requiring equation (7) to hold with  $\lambda = xm$ , it is direct that the price level is proportional to the money stock in competitive equilibrium; that is,  $p = \psi xm$ .

$$W'(\psi^{-1}) = -E[V'(1 + \eta')\psi^{-1}] | x, m \quad (8)$$

As in Lucas (1972, theorem 2), equilibrium is unique within the class considered here, because the left side of equation (8) is decreasing in  $\psi$  and the right side is increasing in  $\psi$  (see footnote 6).

Competitive equilibrium without contracts involves a neutrality of money, again following Lucas's 1972 study, because agents have accurate information on the money stock. Prices adjust proportionately to money shocks, and a high  $x$  is accurately reflected in prices,  $p = \psi xm$ .<sup>9</sup> The micro-level uncertainty leads to greater saving than Lucas found, however, so the price level is lower. This reflects a greater demand for money as a hedge against future income uncertainty. Nevertheless, realizations of these micro disturbances have no effect on the price level, although they do reallocate consumption across members of the elder generation.

### III. Competitive Equilibrium with Contracts

At the beginning of each period, prior to the realization of aggregate and individual shocks, we now permit the representative young agent in each market to offer a contingent contract (it is best to view each island's suppliers as clustered together into one firm so that no idiosyncratic demand risk is present). Specifically, we consider contracts that permit a demander to buy any quantity at the price

$$p = \pi(x)m, \tag{9}$$

where the "price contract" (that is, the function  $\pi[x]$ ) is chosen by suppliers so as to maximize their lifetime expected utility subject to

competition. Competition among islands implies that, in choosing an island, demanders must achieve a level of expected utility at least equal to that achievable elsewhere ( $\bar{V}$ ). Since a demander decides on a market prior to realization of  $x$  and  $\eta$ , the relevant constraint is thus  $E\left\{V\left(\frac{m'}{p} \frac{\eta m x}{p}\right)\right\} \geq \bar{V}$ .

The most general possible contract would allow an arbitrary exchange of goods for dollars and thus specify both prices and quantities; old agents might be unable to obtain all that they demand at the given price. In addition, it would allow contrived uncertainty through mixed strategies and lotteries. Thus, we compute the optimal contract over a limited -- though broad -- class of contracts. One reason for this is that more complicated contracting strategies are often unsustainable (Haubrich and King [1983]).

We restrict contracts mainly for tractability. A fully optimal analysis in an already incomplete market model (OLG) would be difficult, as would the approach of specifying the costs and information structure that would make our contracts optimal in the broader class. Still, the nonlinearity that we allow means that our contract should closely approximate the optimal one. In addition, since a contract that replicates the "no contract" case of section II is feasible, the optimal sticky-price contract represents an improvement.

In competitive equilibrium without contracts, the presence of a large number of islands is inconsequential. Prices and quantities are identical in each market. Here, suppliers in each island compete with those in other islands in offering contracts. The presence of a large number of markets permits us to reasonably treat  $\bar{V}$  as not influenced by the contract

offered by the market in question. We proceed to characterize all Pareto-efficient contracts, without determining the split of the gains from trade ( $\bar{V}$ ).

A young agent on an island competes with young agents on other islands by offering contracts. If a young agent tenders a contract that provides an expected utility of less than  $\bar{V}$ , he attracts no money and consequently has zero consumption in the next period. Conversely, if the contract gives expected utility of more than  $\bar{V}$ , everyone wants it. We do not allow subcontracting, so in the latter case the young agent would have to limit the number of contracts that he accepts (because meeting them all would leave him no leisure time), and therefore he could meet his demand for saving (money). If the contract gives expected utility exactly equal to  $\bar{V}$ , then we assume that the young agent obtains a proportional share (up to his demand) of the total money supply. (The exact rationing rule does not matter for the ultimate equilibrium.) In full equilibrium, each young agent maximizes expected utility given the contract choices of the others, and the supply of money equals the demand. We focus on the symmetric equilibrium, in which each young agent offers the same contract.

Given the setup of the model, the indirect utility function approach of the previous section remains helpful. But now, real saving ( $\lambda/\pi[x]m$ ) depends on the contract chosen. Additionally, even though  $\pi(x)$  is now an object of choice, we continue to view suppliers as treating the distribution of future prices as invariant to their current actions, that is, taking the form  $p' = \pi'(x')m'$ , where  $\pi'$  is not an object of choice. A currently



unborn generation chooses  $\pi'$ . An efficient contract may be found by maximizing expected young-agent utility (equation [11]) with respect to  $\pi(x)$ , subject to the demand constraint

$$\max E\left\{W\left[\frac{\lambda}{\pi(x)m}\right]\right\} \quad (10)$$

$$\text{s.t. } EV\left[\left(1 + \eta\right)\frac{x}{\pi(x)}\right] \geq \bar{v}.$$

It is possible to express this maximization problem as a control problem with an integral constraint as long as  $x$  is continuously distributed (see appendix for details).

The solution to equation (10) selects the efficient contract given the money demand,  $\lambda$ , and the reservation utility of the old,  $\bar{v}$ . From this set, the individual young agent chooses optimal money demand. The total of money demands must balance (the money market must clear). As in the case without contracts, the price level clears the market, adjusting to equate demand and supply. In the contract case, this involves shifting the level of  $\pi(x)$ , in turn changing  $\bar{v}$ . A high demand for real balances by the young means a low general price level; old agents get a lot for their money, giving them high expected utility,  $\bar{v}$ .

Money market clearing imposes the equilibrium condition  $\lambda = xm$ . Substituting this into the equation (10) solution, we obtain the key necessary condition for optimal price policy, a variant of Borch's rule for risk-sharing. That is, it must be that

$$W'\left[\frac{x}{\pi(x)}\right] = \alpha E\left[\left(1 + \eta\right)V'\left[\left(1 + \eta\right)\frac{x}{\pi(x)}\right]\right] | x \quad (11)$$

at each point on the range of  $x$ , where  $\alpha$  is the value of the multiplier attached to the constraint in equation (10). This expression states equality (in each aggregate state  $x$ ) of the costs and benefits of transfers between the contracting parties.<sup>10</sup>

To examine how contract prices move in response to changes in  $x$  (that is, as one moves along the range of  $x$  realizations), we implicitly differentiate equation (11) and rearrange terms, yielding an elasticity

$$\frac{d \log \pi(x)}{d \log x} = \frac{d\pi(x)}{dx} \frac{x}{\pi} = (1 - a), \quad (12)$$

where

$$a = (\alpha\pi(x) \int V'(\cdot) \frac{\partial g}{\partial x}(\eta; x) d\eta) / (W'' - \alpha E[(1+\eta)^2 V''] | x) \geq 0 \text{ (also, } a < 1). \quad (13)$$

Roughly,  $a$  captures the shift in expected marginal utility induced by  $x$  because it shifts the distribution of  $\eta$ . Note first that if the conditional distribution  $\eta$  is independent of  $x$ , then the neutrality of money prevails in our contract equilibrium, because  $a = 0$ . That is, prices adjust proportionately to changes in money and, consequently, there are no real effects. We focus on the case where an increase in money growth ( $x$ ) induces a mean preserving spread on the distribution of individual shifts (see Rothschild and Stiglitz [1970] and Diamond and Stiglitz [1974]). When  $a > 0$ ,

prices respond less than proportionately to a change in money growth because old agents wish to purchase insurance against such aggregate states.<sup>11</sup> Sticky prices provide this insurance by giving the elderly more purchasing power in states of high uncertainty. The contract shifts some of the uncertainty's risk to the young.

Thus, the expected effect of changing  $x/\pi(x)$  on old-agent utility involves the interaction of the proportionate redistribution of money  $(1 + \eta)$  and its marginal utility value  $(V')$  in equation (13) above. Prices will be sticky if  $a > 0$ . The denominator is unambiguously positive from the definition of  $W$  and  $V$  in equations (4) and (5). With positive prices, the sign of  $a$  depends on  $(1 + \eta)V' \frac{\partial g}{\partial m} \int d\eta$ .

Sandmo's (1970) results on the theory of saving under uncertainty are pertinent to the interpretation of this condition; that is,  $E[(1 + \eta)V(c)]$  is exactly the expected utility reward for investing at the random gross return  $(1 + \eta)$ . He notes that, at a given level of saving, increases in the dispersion of  $\eta$  may either lower or raise the reward, even when  $V'$  is convex. This ambiguity reflects two offsetting economic elements. Individuals will want to save less to protect the income that they have, but they will also want to save more as protection against a "rainy day." That is, first, an increase in the level of interest-rate uncertainty leads an agent to feel less inclined to expose current resources to the possibility of loss; Sandmo identifies this effect formally with a negative substitution effect on saving. Second, greater uncertainty about interest rates leads to an increased potential for low consumption, which is highly valued. Thus, in a manner

formally identical to the effect of income uncertainty noted above, there is a positive impact on saving on this account. As a result, as  $x$  induces a mean preserving spread on  $\eta$ , it will raise  $E\{(1 + \eta)V'[(1 + \eta)\frac{x}{\pi(x)}]\}$

-- at a given  $\frac{x}{\pi(x)}$  -- as long as preferences are such that saving will rise with interest-rate uncertainty; that is, individuals are not too willing to substitute for old-age consumption.

Specifically, following Diamond and Stiglitz (1974), we require that  $\sigma(c) = -cV''(c)/[V'(c)] > 1$  and that  $\sigma(c)$  decreases with consumption, which ensures that savings will rise with an increase in interest-rate uncertainty.<sup>12</sup> In that case, there will be a contract specifying sticky prices and a positive relationship between money growth and output.

Figures 1a and 1b show the relationship between money growth, contract prices, and effort/production in our economy. Equation (13) also demonstrates that the model displays a variant of Lucas's (1973) hypothesis on the Phillips-curve slope, because greater variability in the growth component ( $x$ ) reduces the responsiveness of output to monetary shocks. Sticky prices probably do not uniquely support the insurance allocation. Other mechanisms can insure the elderly, such as a social security program with direct (incentive-compatible) payments that are linked to the monetary growth rate. Such schemes will not dominate the sticky-price contract, but merely support the same allocation in different ways.<sup>13</sup> We believe that the sticky-price contract is the best approximation of a real institution. Furthermore, the empirical predictions arising from our model -- connecting

monetary uncertainty with output and prices -- should in principle allow researchers to determine whether or not our mechanism provides the insurance.

Thus, if transfers are in fixed nominal terms, the fact that money growth induces an increase in the dispersion of individual shocks can lead to the Phillips-curve response illustrated in figure 1b, although it requires stronger restrictions on preferences than Lucas (1972) uses. In particular, we require that agents are relatively unwilling to substitute away from old-age consumption. With these preferences in place, increases in money growth are unmatched by proportional increases in the price level -- a result of the contract. Thus, an increase in money will provoke a positive output response.

Intuitively, what the contract does is protect consumers (the old) from the uncertainty and risk associated with random money injections. Without contracts, producers (the young) bear none of that risk because they adjust prices proportionally. With contracts specifying sticky prices, however, producers do bear some of the risk. When the money supply is high, for example, they must produce more and work harder.

#### IV. Summary and Conclusions

This theoretical investigation was conducted under two guiding principles. First, the analysis of sticky prices must be conducted in a general-equilibrium setting in order to ensure consistent behavioral responses and to lay the groundwork for an examination of policy alternatives in accordance with the Lucas (1976) critique. Typical, sticky, nominal-price

stories such as Fischer (1977) postulate nominal contracts, exogenously imposing a pattern of arrangements on the labor market of an otherwise neoclassical model. No specific gains result from nominal contracting at the private or social level identified in Fischer's or Taylor's (1979, 1980) models.

These papers do demonstrate the important effects of nominal contracts, however. Without an explicit framework that generates contracts endogenously, it is possible that such sticky-price models are internally consistent, since factors motivating a demand for a specified wage contract may also restrict employment or consumption decisions. Moreover, these results are devoid of predictions about how contracts will change in the face of variations in the economic environment. Second, in our view, the analysis of sticky nominal prices requires explicit consideration of a monetary economy. There must be elements of real uncertainty associated with monetary movements if nominal price stickiness is to be explained as a result of contractual arrangements that arise for risk-allocating reasons. Other recent work also adheres to these principles. To the extent that menu-cost and multiple-equilibrium models lead to different testable implications, they indicate the necessity of ascertaining the true cause of price stickiness. Different sources will create different macroeconomic implications, reemphasizing the point made above about the inadequacy of models that impose contracts exogenously.

With these guiding principles, we opted to study a stochastic consumption-loans model that is a minor variation on Lucas (1972). In this

setup, monetary growth was assumed to be positively related to the dispersion of individual transfer payments. Although money was neutral toward real aggregate quantities when an exogenous restriction was placed on contracts, neutrality did not continue to prevail when the restriction was lifted: The stickiness partially insured consumers against random money injections by shifting some of that risk to producers. Rather, competitive contracts specified price stickiness -- in the sense of less-than-proportionate adjustment in prices -- and, consequently, a positive relationship between production and money growth. Thus, our model economy provides a counter-example to Barro's (1976) conjecture that efficient competitive contracts necessarily reduce the dependence of output on nominal money growth.

Finally, our model economy incorporates some of the features that McCallum (1982) identifies as central elements of business fluctuations. Suppliers set prices (contingency plans) in advance of the realization of demand. High money growth does lead to high output -- a result, one can argue, of prices that do not adjust enough. At the same time, our model is not obviously Keynesian; that is, important social costs of nominal contracting are not left unconsidered in private arrangements.

## APPENDIX

In this appendix, we obtain the optimal contract for our model economy by solving an integral-constraint control problem. To do this, we rely on methods provided by Takayama (1985, chapter 8, section C) in his discussion of Hestenes' theorem.

Recall that the island's objective is to maximize young-agent utility subject to the demand constraint that requires old-agent utility to at least equal that achievable elsewhere. The problem is to choose the price function (or, in particular,  $\pi[x]$ ) that maximizes  $E\{W[\frac{\lambda}{\pi(x)m}]\}$  subject to  $E\{V[(1 + \eta)\frac{x}{\pi(x)}]\} \geq \bar{V}$ .

If we let  $h(x)$  be the density function of  $x$  and  $g(\eta; x)$  be the conditional density function of  $\epsilon$ , the objective and constraint each take the form of an integral. Specifically, the constraint may be written as

$$\int_{\bar{x}}^{\bar{x}} \int_{\bar{\eta}}^{\bar{\eta}} [V[(1 + \eta)\frac{x}{\pi(x)}] - \bar{V}]g(\eta; x)d\eta)h(x)dx \geq 0. \quad (A1)$$

Forming the Hamiltonian according to Takayama's methods, we get

$$H[\pi(x), x] = \rho W[\frac{\lambda}{\pi(x)m}]h(x) - \alpha \int_{\bar{\eta}}^{\bar{\eta}} [V[(1 + \eta)\frac{x}{\pi(x)}] - \bar{V}]g(\eta; x) d\eta)h(x) \quad (A2)$$



where  $\rho$  and  $\alpha$  are multipliers.<sup>14</sup> Since our problem is a variable right-side endpoint problem (that is,  $\pi[\bar{x}]$  is not specified in advance), we can set  $\rho = 1$  in equation (A2) without loss of generality. Maximizing the Hamiltonian with respect to the control,  $\pi(x)$ , we obtain the necessary condition

$$\frac{\partial H}{\partial \pi} = W' \left[ \frac{\lambda}{\pi(x)^m} \right] h(x) \frac{\lambda/m}{\pi(x)^2} - \alpha \int_{\eta}^{\bar{\eta}} \left[ V' \left[ (1 + \eta) \frac{x}{\pi(x)} \right] \cdot (1 + \eta) \frac{x}{\pi(x)^2} \right] g(\eta; x) d\eta h(x).$$

This implies the key condition (Borch's rule), which, after imposing  $\lambda = mx$ , becomes

$$W' \left[ \frac{x}{\pi(x)} \right] = \alpha E \left[ (1 + \eta) V' \left[ (1 + \eta) \frac{x}{\pi(x)} \right] \right] | x, \quad (A3)$$

which is equation (11) in the main text.

## FOOTNOTES

1. In a modification of Lucas's (1972) setup that incorporates entrepreneurs and relatively risk-averse workers, Azariadis (1978) demonstrates that endogenous labor market risk-allocating arrangements -- which require an enforceable contingent contract -- may enhance the real effects of imperfectly perceived nominal disturbances. Efficient ex ante arrangements in Azariadis's model do not permit real quantities (hours worked or total compensation) to depend on contemporaneously perceived monetary disturbances.
2. It is useful to establish some terminology concerning monetary neutrality. The traditional view (Patinkin [1965, chapter IV]) is that a money change is neutral only if all real variables for all individuals are left unaltered in equilibrium. Our focus is on economies in which monetary events are interconnected with uninsurable redistributive events at the individual level, necessarily violating the Patinkin definition of neutrality. We employ a weaker neutrality concept -- invariance of aggregate real variables -- throughout our discussion.
3. Our model thus illustrates a general principle (discussed in more detail by Haubrich [1983]) concerning price movements in model economies that have (1) incomplete insurance due to private information and (2) contractual exchange contingent on aggregate variables. The principle is that aggregate disturbances may have different qualitative effects on near-representative-agent economies with and without contracts if these aggregate shocks alter the dispersion of individual circumstances.  
  
Grossman, Hart, and Maskin (1983) also discuss the role of aggregate shocks as signals of the unobservable individual disturbances upon which we focus here. However, they pinpoint economies in which asymmetric information between firms and workers is key, but do not explore the neutrality of money to any important degree.
4. Smith (1985) takes a different approach, using contracts to add uncertainty.
5. This notation, though standard in Lucas (1972), may be a bit confusing. Here,  $m$  is the inherited stock of money  $m_{t-1}$ , and  $m'$  is the period  $t$  stock of money (spent by the old in  $t$ ), while  $x$  is the period  $t$  money shock.

6. Von Mises (1953) assigns an important role to the distribution of monetary injections, even suggesting that, with such dispersion, prices adjust less than the quantity theory would predict. He states:  
 "This increase in the stock of money, as we have seen, starts with the original owners of the additional quantity of money and then transfers itself to those that deal with these persons, and so forth... at first only certain economic agents benefit and the additional quantity of money only spreads gradually through the whole community... There is no increase in the available stock of goods; only its distribution is altered... It is true that the prices paid for these commodities were higher than would have corresponded to the earlier purchasing power of money; nevertheless, they were not so high as to make full allowance for the changed circumstances. Europe had exported ships and rails, metal goods and textiles, furniture and machines, for gold which it little needed." (pp. 208-211)
7. We follow Lucas (1972) in restricting attention to the stationary price functions and considering only monetary equilibria. We now know that even this is a broad class, so we consider only "fundamental" equilibria, ruling out "sunspots" -- equilibria with stationary random prices unrelated to the environment's intrinsic uncertainty (see Azariadis and Cooper [1985b] and Azariadis and Guesnerie [1986]). Some of these can depend on the economy's entire history.
8. Following Lucas, we use the prime symbol to denote a derivative, although it also represents updated variables.
9. McCallum (1984) notes that this result derives from two facts: (1) money growth is permanent and (2) the proportionate distribution of new money effectively gives money a positive nominal return. This rules out nonneutrality due to inflationary finance.
10. The Borch-Arrow condition, equation (11), also equilibrates demand and supply, because it determines the price,  $\pi(x)$ , that clears this market. In most cases, since the young meet increased demand for goods by working more, the market clears. Rationing takes place only if a very low  $\pi(x)$ , and resulting high demand by the old, would require more than 24 hours of work ( $n > 1$ ). An Inada condition prevents this. If the marginal utility of the young approaches infinity when either consumption or leisure approaches 0, the left side of equation (1),  $W'$ , also approaches infinity. The right side must then increase, implying lower consumption by the old. Intuitively, a nonzero probability of rationing shifts too much risk to the young, and the price of output rises until the old demand less.

11. In Haubrich (1983), changes in an aggregate-state variable alter the level of efficient risk-pooling in the banking model developed by Haubrich and King (1983). Here, by contrast, the aggregate-state variable alters the extent of efficient risk-shifting. In both cases, it is central that the aggregate shock have implications for the dispersion of individual circumstances.
12. With convexity, this implies that both relative and absolute risk aversion must decrease with consumption. Lucas (1972), with a somewhat different problem, assumes  $\sigma(c) > 0$ .
13. Smith (1985) takes a similar position. In that paper, nominal contracts provide lotteries, which remove a nonconvexity. Other methods could provide the lotteries, but, as in our paper, Smith concentrates on explaining the observed contract.
14. If  $x$  had a discrete distribution,  $\rho$  and  $\alpha$  would be a series of Lagrange multipliers -- one pair for each point in the  $x$  distribution. However, a continuous  $x$  distribution permits us to discuss marginal changes more readily, although it requires the control problem.

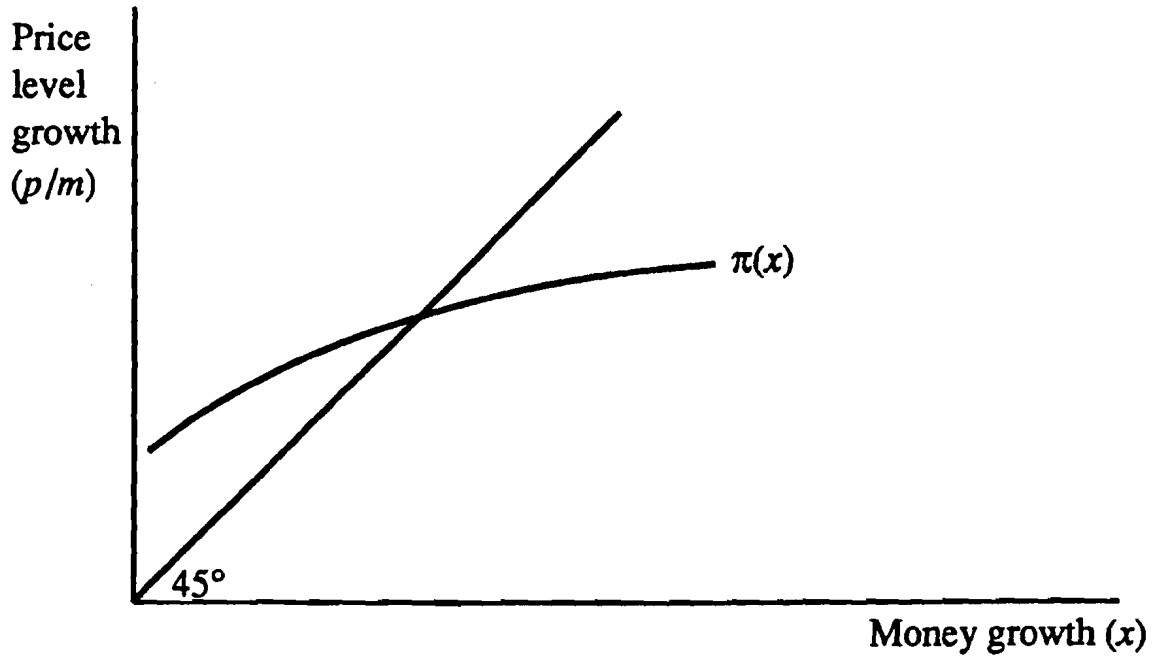
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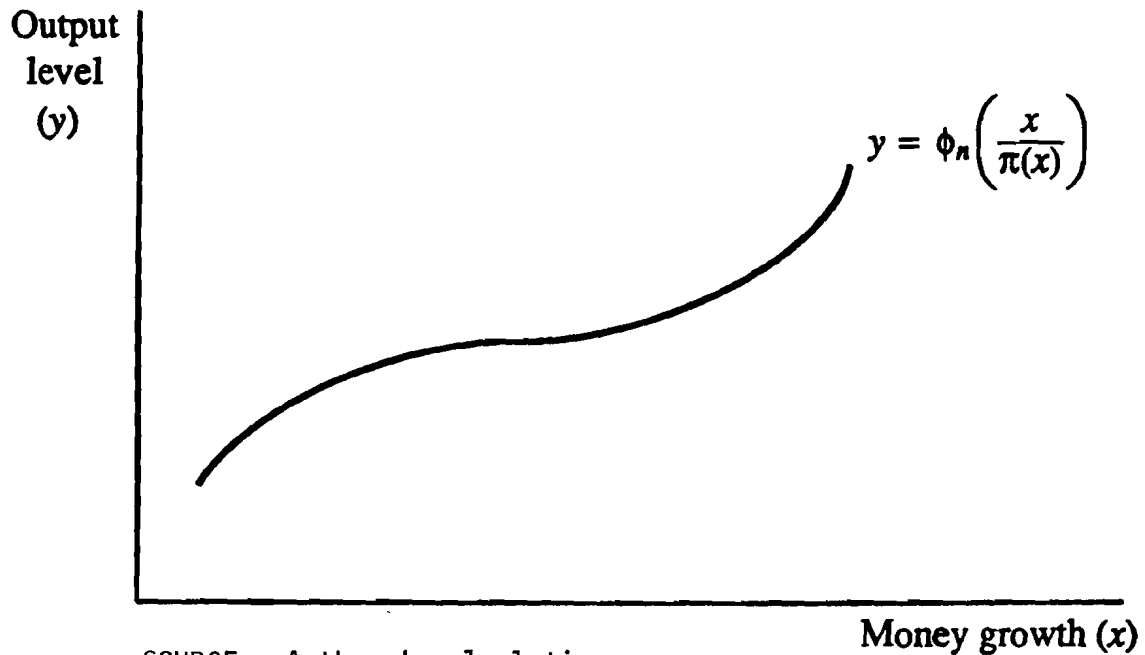
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**Figure 1a**  
**Money and Prices under Contracting**



SOURCE: Authors' calculations.

**Figure 1b**  
**Money and Output under Contracting**



SOURCE: Authors' calculations.