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We study counterfeiting of currency in a search-theoretic model of monetary exchange. In contrast to Nosal and Wallace (2007), we establish that counterfeiting does not pose a threat to the existence of a monetary equilibrium; i.e., a monetary equilibrium exists irrespective of the cost of producing counterfeits, or the ease with which genuine money can be authenticated. However, the possibility to counterfeit at money can affect its value, velocity, output and welfare, even if no counterfeiting occurs in equilibrium. We provide two extensions of the model under which the threat of counterfeiting can materialize: counterfeits can circulate across periods, and sellers set terms of trades in some matches. Policies that make the currency more costly to counterfeit or easier to recognize raise the value of money and society's welfare, but the latter policy does not always decrease counterfeiting.

Key words: Money, counterfeiting, search, bargaining..

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1 Introduction

Recent advances in technology, like digital printers and higher quality scanners, have greatly reduced the cost to produce counterfeits, posing serious challenges to currencies all over the world.[1] Despite the extent of counterfeiting being small in the U.S. – the value of counterfeit currency in 2005 was about 1 dollar for every \$12,400 in circulation – the U.S. government have responded to this threat by redesigning notes, and by enhancing the public's ability to verify the authenticity of currency through educational programs.[2] To assess the effectiveness of various measures to deter counterfeiting, one needs to understand the exact nature of the threat that counterfeiting poses on the economic activity. Does the possibility of counterfeiting threaten the very existence of the fiat money system? Under which conditions does the threat of counterfeiting materialize? And, is it necessary to continue efforts regarding the deterrence and suppression of counterfeiting even when counterfeiting seems insignificant?

To answer these questions, one needs a model of monetary exchange, where the societal benefits of fiat money are explicitly spelled out, and where genuine money is not perfectly recognizable and can be counterfeited at some cost.[3] This is precisely the approach of Nosal and Wallace (2007) – NW hereafter – who study counterfeiting using the off-the-shelves monetary model of Shi (1995) and Trejos and Wright (1995). They derive two main implications: Counterfeiting of currency

[1]According to "The use and counterfeiting of United States currency abroad," Part 3, Section 6.6, the cost to produce reasonably deceptive counterfeits could be as low as \$300. The digital counterfeited notes as a fraction of all passed notes increased from 0.5% in 1995 to 52% in 2005.

[2]In 2005, out of the \$760 billion of U.S. banknotes in circulation \$61 million of counterfeit currency was passed on to the public. See "The use and counterfeiting of United States currency abroad," Part 3, page 47. Also, according to the estimate by Judson and Porter (2003) counterfeit U.S. currency that has been passed into circulation is about one note in ten thousands of currency in circulation. Two recent programs to improve the design of the U.S. currency are the New Currency Design introduced in 1996 and the Series 2004 New Color of Money introduced in 2003. Both programs were preceded by educational campaigns.

[3]Surprisingly, very little work has been devoted to this topic. Kultti (1996) and Green and Weber (1996) are the first papers to study counterfeiting of currency in a random-matching model with exogenous prices. Williamson (2002) investigates the counterfeiting of banknotes in a random-matching model with indivisible money but divisible output. Nosal and Wallace (2007) introduce lotteries as a proxy for divisible money and show that it allows buyers to signal the quality of their money holdings. Cavalcanti and Nosal (2007) and Monnet (2005) adopt a mechanism design approach and focus on pooling allocations. In addition, Monnet (2005) does not restrict money holdings to lie in $\{0, 1\}$. Quercioli and Smith (2007) study counterfeiting in a non-monetary model but they introduce multiple denominations and a costly decision to verify currency. Papers that consider competing assets of which one is subject to the counterfeiting problem and study the liquidity differentials or acceptability of assets include, for example, Lester, Postlewaite and Wright (2007), and Kim and Lee (2008).

does not occur in equilibrium; The possibility of counterfeiting constitutes a threat to the existence of a monetary equilibrium, but the allocation in a monetary equilibrium is independent of the technology to produce counterfeits or policies intended to improve the recognizability of genuine money.

The objective of this paper is twofold. First, despite using a similar equilibrium concept, we obtain insights that are in contrast to the main results in NW.[4] We establish that counterfeiting does not pose a threat to the existence of a monetary equilibrium. That is, the cost of producing counterfeits and the recognizability of genuine money do not determine whether fiat money has value. The intuition is simple: since counterfeiting involves a fixed cost, it should not threaten a currency which is almost valueless. However, in contrast with the view that the allocation is independent of the possibility of counterfeiting as long as it is not realized, we find that the cost of producing counterfeits and the recognizability of genuine money can affect the value of fiat money, its velocity, output and welfare, even though no counterfeiting occurs in equilibrium. Our findings also imply a different objective for anti-counterfeiting policies: it is not to prevent the monetary equilibrium from breaking down as suggested in NW; it is to raise output and welfare by mitigating the threat of counterfeiting on the value of money.

Second, the prediction of no counterfeiting in NW is not consistent with the observed counterfeiting on major currencies. In 2005 in the U.S., $61 million of counterfeit currency was passed on to the public, 3717 counterfeiters were arrested, and 611 counterfeiting plants were suppressed.[5] As documented by Mihm (2007), counterfeiting was a widespread phenomenon in the U.S. during the 19th century. Another objective of the paper is thus to reconcile the prediction of the model with the observed counterfeiting, significant or not, in the real world. To do so, we provide two extensions of the model under which the threat of counterfeiting can materialize. The first extension relaxes the assumption of full confiscation in NW. We assume that the government has limited power to withdraw counterfeits so that individuals who receive them have a chance to pass them on to someone else in the future. We provide a simple condition under which a counterfeiting

[4]In short, NW omit some equilibria by implicitly assuming that in any equilibrium with no counterfeiting all offers made by buyers, even out-of-equilibrium ones, should be attributed to genuine buyers. See Appendix D for a more detailed comparison between our approach and the one in NW.

[5]These numbers are taken from "The use and counterfeiting of United States currency abroad," Part 3, page 47.

equilibrium exists, and we show that this equilibrium is separating – in meetings where the quality of money is not recognized, genuine buyers and counterfeiters make different offers.[6] Counterfeiting is more likely to take place if the stock of genuine money is low, if the cost to produce counterfeits is small, and if the ability of the government to confiscate counterfeits is limited.

The second extension consists in assuming that sellers can set terms of trade in some matches. Counterfeiting is shown to prevail even under full confiscation. When the quality of money is not recognized, no trade takes place if buyers make the offer, while terms of trade are pooled if sellers make the offer. This implies that sellers accept counterfeits in the instances where they cannot verify the authenticity of currency. The results also demonstrate the distortions that counterfeiting generates on the economic activity: in some occasions, mutually beneficial trades cannot take place, while in others trades occur but terms of trade are distorted by the private information problem. By enhancing the public's ability in authenticating currency the government can increase the value of money and society's welfare, but it may not always decrease the extent of counterfeiting.

The rest of the paper is organized as follows. Section 2 presents the environment. In Sections 3 and 4 we analyze a simple game where sellers are always uninformed and counterfeits are confiscated at the end of each period. In Section 5 we consider the case of heterogeneously informed sellers. Counterfeits are allowed to circulate across periods in Section 6 and sellers are endowed with some bargaining power in Section 7.

2 The model

The environment is similar to the one in Shi (1995) and Trejos and Wright (1995). Time is indexed by $t \in \mathbb{N}$. There is a large number of perfectly divisible and perishable goods and a unit measure of agents who are specialized in the goods they produce and consume. Agents do not consume their own output. Each agent's preference at time 0 is

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left[u(q_t) - h_t \right] \right],$$

[6] This implies that sellers accept counterfeits knowingly. Mihm (2007, p.221) provides examples where store keepers in the 19th century U.S. would accept notes even when they would suspect those notes were counterfeits. These episodes seem consistent with a separating equilibrium described here.

where $q_t \in \mathbb{R}_+$ is consumption at time t, $h_t \in \mathbb{R}_+$ is the effort devoted to production, and $\beta = (1+r)^{-1} \in (0,1)$ is the discount factor. We assume that $u(q)$ is twice continuously differentiable, strictly increasing and strictly concave. Moreover, $u(0) = 0$, and there exists $q^* > 0$ such that $u'(q^*) = 1$. Producing q units of output incurs disutility q.

Agents trade in bilateral matches. The pattern of specialization rules out double-coincidence-of-wants matches but it allows single-coincidence matches where one agent wishes to consume the good produced by his partner, but not vice versa. Agents cannot commit to future actions and the trading histories are private information, which eliminates the possibility of credit arrangements. Thus, trade must involve a tangible medium of exchange.

There are two types of money, genuine fiat money and counterfeits. Both objects are indivisible, perfectly durable and storable. For tractability, we assume money holdings lie in the set $\{0,1\}$. The quantity of genuine money is $m \in (0,1)$. Agents who do not hold genuine money at the beginning of a period decide whether or not to produce a counterfeit at a utility cost $k > 0$. As a result of this decision, the measure of agents holding counterfeited notes is $n \in [0, 1-m]$. At the end of each period, the government is able to take out of circulation a fraction $\delta \in (0,1]$ of counterfeits.[7] We assume that, though the government confiscates counterfeits, it does not know the identity or trading histories of agents, so the anonymity of agents is preserved. The measure of agents without money is $1-m-n$. Notice the key differences between genuine and counterfeit monies: while the former is in fixed supply, the latter is privately produced and it is subject to confiscation by the government.

Only meetings between agents holding money (genuine or counterfeit) and agents without money occur. In particular, two money holders never meet.[8] We assume a simple matching technology where the probability for a buyer to be matched with a seller is equal to the measure of sellers in the market, $1-m-n$.[9] The probability of a single coincidence match for an agent without money

[7] We assume the absence of punishment for an individual caught with a counterfeit. This is consistent with the situation in the US in the 18th and 19th century where prosecutions were rare. For a more accurate description of counterfeiting nowadays, one would also need to take into account the usually severe punishments for those producing or attempting to pass counterfeited notes.

[8] This rules out the possibility that genuine money is exchanged for a counterfeit note plus some output. See, e.g., Aiygari, Wallace and Wright (1996) and Li (2002).

[9] We could adopt different matching technologies without affecting the main results (see, e.g., Li and Rocheteau 2008). We could also add explicitly a probability of single coincidence as in NW.

is $m + n$. Conditional on being matched, the producer meets a money holder with genuine money with probability $m/(n + m)$.

In a match, a producer is informed about the quality of the money of his partner with probability η, in which case there is complete information in the match. With the complementary probability $1 - \eta$ the producer is uninformed and there is one-sided incomplete information. After matches are terminated, all agents learn the quality of their money holdings.

Terms of trade in bilateral matches are determined by a bargaining protocol where the buyer is chosen to make a take-it-or-leave-it offer with probability θ, while with the complementary probability $1 - \theta$ the seller is the one to make the offer. We assume $\theta = 1$ throughout except Section 7 where we consider $\theta < 1$. If the other agent in the match accepts the offer, trade is implemented. Agents can offer lotteries to overcome the indivisibility of money, as in Berentsen, Molico and Wright (2002). The ability to make two-dimensional offers permits buyers with genuine money to potentially separate themselves from counterfeiters.[10]

3 A simple counterfeiting game

Our main insights on how the threat of counterfeiting affects the equilibrium can be presented in a simple environment where sellers are always uninformed ($\eta = 0$), counterfeits cannot circulate across periods ($\delta = 1$) and only buyers make offers ($\theta = 1$). The first assumption allows us to focus on the determination of terms of trade in uninformed matches, which is key for the incentives to produce counterfeits. The second assumption (made in NW) implies that a counterfeit has no value for someone who holds it after matches are dissolved. The third assumption (also used in NW) allows us to focus on a bargaining game that has been used extensively in the literature. All these assumptions will be relaxed in the subsequent sections.

We first analyze the one-period game starting at the beginning of a period, taking as given the discounted continuation values of a money holder, v_1, and an agent without money, v_0. Later

[10] In search monetary models with indivisible money and complete information, the use of lotteries acts as an imperfect proxy for divisibility of money: it allows agents to extract larger gains from trade, and it eliminates some trade inefficiencies (see, e.g., Berentsen and Rocheteau, 2002). In Appendix E we develop a model with divisible money along the line of Lagos and Wright (2005), but with an overlapping generation structure, and we show that our main insights are preserved. See also Rocheteau (2008).

we will consider infinitely-lived agents that play the game repeatedly in a stationary economy. Formally, the game is defined as follows.

Players A measure $m \in (0,1)$ of buyers holding one unit of genuine money and a measure $1-m$ of agents without money.

Game tree First, each agent without money chooses to produce a counterfeit ($\chi = c$) or to become a seller ($\chi = s$). Second, buyers and sellers are matched at random. Third, each matched buyer makes an offer $(q,p) \in \mathbb{R}_+ \times [0,1]$ where q is the output produced by the seller and p is the probability that the buyer hands over his unit of money. Fourth, matched sellers accept or reject the offers they receive.

Information structure The type of money held by an agent is private information. The offers and acceptance decisions in a trade are private information to a match.

Payoffs The payoff of an agent without money is represented by the following v.N-M utility function,

$$U_0(\chi, q^b, q^s, d) = -k\mathbb{I}_{\{\chi=c\}} + u(q^b) - q^s + \upsilon_0\mathbb{I}_{\{d=0\}} + \upsilon_1\mathbb{I}_{\{d=1\}},$$

where $\mathbb{I}_A$ is an indicator function equal to one if A holds, $\chi \in \{c,s\}$ is the agent's decision to produce a counterfeit, $q^b \in \mathbb{R}_+$ is his consumption, $q^s \in \mathbb{R}_+$ his production and $d \in \{0,1\}$ his money holdings at the end of the game. (Recall that due to the single-coincidence matches, $q^bq^s = 0$.) Similarly, the payoff of an agent holding one unit of genuine money is represented by

$$U_1(q^b, d) = u(q^b) + \upsilon_0\mathbb{I}_{\{d=0\}} + \upsilon_1\mathbb{I}_{\{d=1\}}.$$

A (pure) strategy for an agent without money is composed of the decision to produce a counterfeit ($\chi = c$) or become a seller ($\chi = s$), which offer (q,p) to make if $\chi = c$, and which offers to accept if $\chi = s$. A strategy for a genuine buyer is an offer (q,p). The seller's belief that he faces a genuine buyer following an offer (q,p) is denoted $\lambda(q,p)$. We will assume that all sellers share the same belief system.

An equilibrium of this game is composed of a strategy for each player and a belief system λ that satisfy the following requirements:

Sequential rationality Given their belief λ, sellers accept offers that yield a non-negative surplus, i.e., $-q + p\lambda(q,p)(\upsilon_1 - \upsilon_0) \geq 0$.[11] Offers made by genuine buyers and counterfeiters are best responses to sellers' acceptance rule. (Since the no-trade offer $(0,0)$ is always accepted, we can, with no loss in generality, require that buyers only make offers that are accepted.) The offer of a genuine buyer solves

$$(q_b^u, p_b^u) = \arg\max_{q,p\leq 1} [u(q) - p(\upsilon_1 - \upsilon_0)] \quad \text{s.t.} \quad -q + p\lambda(q,p)(\upsilon_1 - \upsilon_0) \geq 0, \tag{1}$$

while the offer of a counterfeiter solves

$$(q_c^u, p_c^u) = \arg\max_{q,p\leq 1} u(q) \quad \text{s.t.} \quad -q + p\lambda(q,p)(\upsilon_1 - \upsilon_0) \geq 0. \tag{2}$$

The decision of an agent without money to produce a counterfeit or not is optimal given the measure of other agents who choose to counterfeit, and the offers and acceptance rules in the bargaining game, i.e.,

$$\chi \begin{array}{l} = c \\ = s \\ \in \{c,s\} \end{array} \quad \text{if} \quad -k + (1-m-n)u(q_c^u) \begin{array}{c} > \\ < \\ = \end{array} m\left[-q_b^u + p_b^u(\upsilon_1 - \upsilon_0)\right] + n(-q_c^u). \tag{3}$$

Consistency of beliefs with strategies For any offer (q,p) made in equilibrium, $\lambda(q,p)$ is derived from Bayes' rule.

Because there is little discipline on the out-of-equilibrium beliefs this game admits a large number (continuum) of sequential equilibria. In particular, any offer that is acceptable by sellers can be part of an equilibrium by using the threat that all other offers are attributed to counterfeiters. The following lemma identifies that some strategies for agents without money are strictly dominated.

Lemma 1 *The strategy of an agent without money that consists in producing a counterfeit ($\chi = c$) and offering (q,p) such that $-k + (1-m)u(q) < 0$ is strictly dominated by the strategy that consists in becoming a seller ($\chi = s$) and rejecting all offers.*

[11] Here we adopt the tie-breaking rule that a seller accepts any offer that makes him indifferent between accepting or rejecting. This enables us to find a solution to the buyer's problem.

A natural requirement is that a seller assigns no probability to his opponent playing a strictly dominated strategy. Together with Lemma 1 this leads to the following refinement on the belief system.[12]

Refinement 1a (Elimination of strictly dominated strategies) $\lambda(q,p) = 1$ *for all* (q,p) *such that* $-k + (1-m)u(q) < 0$.

Refinement 1a simply says that, if a seller receives an offer such that the expected utility of consumption adjusted by the matching probability in an economy without counterfeiters is less than the cost of producing a counterfeit then he should attribute it to a genuine buyer.

Lemma 2 *The infimum for the surplus of a genuine buyer in a bilateral match for all belief systems* λ *that satisfy Refinement 1a is*

$$\max_{q,p\leq 1} \left[u(q) - p(\upsilon_1 - \upsilon_0)\right] \tag{4}$$

$$s.t. \qquad -q + p(\upsilon_1 - \upsilon_0) \geq 0 \tag{5}$$

$$-k + (1-m)u(q) \leq 0. \tag{6}$$

Provided that $k > 0$ a genuine buyer can always secure a strictly positive surplus even though the seller cannot recognize the quality of his money. Intuitively, a genuine buyer can always ask for some low level of output which does not generate enough consumption value to cover the entry cost of a counterfeiter and, hence, the offer would be attributed with probability one to a buyer holding genuine money. Moreover, the higher the production cost of a counterfeiter, the larger the surplus that the genuine buyer can secure.[13] We will see later that this is the insight that explains the robustness of the monetary equilibrium.

In order to refine the beliefs further, we follow NW and adopt a forward-induction argument, in spirit of the Intuitive Criterion of Cho and Kreps (1987).[14]

[12] Our game has a similar structure as the one used in Fudenberg and Tirole (1991, p.438, Fig. 11.1) to motivate the notion of strategic stability.

[13] This finding is in sharp contrast with the outcome of the same bargaining game under pure adverse selection, i.e., when the distribution of types is exogenous. In that case, the no-trade outcome is obtained under the Intuitive Criterion.

[14] The game we are considering is not a signaling game with exogenous types as in Cho and Kreps (1987), i.e., the types that are relevant in the bargaining game are chosen endogenously. The refinement we are using, however, is

Refinement 1b (Forward induction) *Consider an equilibrium where* $\mathcal{U}_1$ *and* $\mathcal{U}_0$ *are the expected payoffs of a genuine buyer and an agent without money, respectively, and* n *is the measure of counterfeiters. The proposed equilibrium is disqualified if there exists an out-of-equilibrium offer* (q', p') *such that:*

$$-k + (1-n-m)u(q') + \upsilon_0 < \mathcal{U}_0 \tag{7}$$

$$(1-n-m)\left[u(q') - p'(\upsilon_1 - \upsilon_0)\right] + \upsilon_1 > \mathcal{U}_1 \tag{8}$$

$$-q' + p'(\upsilon_1 - \upsilon_0) \geq 0. \tag{9}$$

From (7) an agent without money would not choose to produce a counterfeit to make an offer (q', p') that reduces his payoff compared to the proposed equilibrium, irrespective of how a seller would interpret this offer (even under the most favorable beliefs of sellers). It should be clear that (7) is equivalent to $q' < q$ (where q is the output at the proposed equilibrium), i.e., a counterfeiter would not propose to consume less than what he can obtain in equilibrium. From (8) the offer (q', p') would benefit a genuine buyer if it were to be accepted; from (9) it is acceptable provided that $\lambda(q', p') = 1$.[15]

Proposition 1 *Under Refinement 1b, there is no equilibrium with* $n > 0$*. In any equilibrium with* $n = 0$ *the offer made by a genuine buyer solves (4)-(6).*

There cannot be a pooling offer with active trades under Refinement 1b (same as Proposition 1 in NW). If such a pooling offer were made in equilibrium then a buyer with genuine money could signal the quality of his currency by proposing an (out-of-equilibrium) offer where he transfers money with a smaller probability and he consumes less output.

Under the Cho-Kreps refinement the genuine buyer's payoff is exactly the solution to (4)-(6). This suggests that the refinement we use does not confer an additional advantage to fiat money

inspired by the same forward-induction logic as the one underlying the Intuitive Criterion. There are other refinements for signaling games (e.g., the undefeated equilibrium from Mailath, Okuno-Fujiwara and Postlewaite, 1993) but they are not directly applicable to games with endogenous types. For a method to analyze a class of signaling games with endogenous types, see In and Wright (2008). We apply this methodology in a different version of our model with divisible money and obtain similar insights. See Appendix E.

[15] Notice that Refinement 1b is consistent with Refinement 1a in the following sense. Consider a proposed equilibrium where the surplus of the genuine buyer is less than the lower bound defined in Lemma 2. This equilibrium would have to be sustained by a belief system that violates Refinement 1a, and it could be dismissed by Refinement 1b.

relative to Refinement 1a. Hence, we conjecture that if there is a monetary equilibrium under our refinement, there should also be a monetary equilibrium under an alternative specification for sellers' beliefs provided that they comply with Refinement 1a.

According to (4)-(6) genuine buyers choose the offer that maximizes their payoff and that deters counterfeiting. The terms of trade in bilateral matches solve

$$q_b^u = \min[q^*, \upsilon_1 - \upsilon_0] \quad \text{if} \quad u(\min[q^*, \upsilon_1 - \upsilon_0]) \leq \frac{k}{1-m}. \tag{10}$$

$$= u^{-1}\left(\frac{k}{1-m}\right) \quad \text{otherwise.} \tag{11}$$

Moreover,

$$p_b^u = \frac{q_b^u}{\upsilon_1 - \upsilon_0}. \tag{12}$$

Provided that k is sufficiently large, buyers make the same offer as the one they would make in an economy where counterfeiting is not even a possibility. However, if k is not too large so that (6) binds then buyers lower the output they ask for so that to deter the entry in counterfeiting. A belief system of sellers to sustain the offer (q_b^u, p_b^u) as an equilibrium offer is such that all offers that satisfy $-k+(1-m)u(q) \leq 0$ are attributed to genuine buyers while all other offers are attributed to an agent without money who chose to be a counterfeiter. As a consequence, all offers that violate (6) are rejected by sellers.

In the equilibrium of the one-period game, all agents without money choose to be sellers. Genuine buyers propose (q_b^u, p_b^u) solution to (10)-(12). Agents without money who would have chosen to produce counterfeits would offer the highest q consistent with $-q + p(\upsilon_1 - \upsilon_0) \geq 0$ and $-k+(1-m)u(q) \leq 0$. Sellers accept all offers such that $-q+p(\upsilon_1-\upsilon_0) \geq 0$ and $-k+(1-m)u(q) \leq 0$.

4 The threat of counterfeiting in equilibrium

So far we have studied the game that takes place within a period taking as given the continuation values υ_1 and υ_0. We now consider our model where the game repeats itself every period and the time horizon is infinite. An agent's strategy is restricted not to depend on his past private histories since such histories are payoff-irrelevant. Moreover, we focus on stationary equilibria where υ_0 and υ_1 are constant across periods.

From Proposition 1, $n = 0$ and $\upsilon_0 = 0$. The value of a genuine buyer, υ_1, solves the following flow Bellman equation,

$$r\upsilon_1 = (1-m)\left[u(q_b^u) - q_b^u\right], \tag{13}$$

where $r = \beta^{-1} - 1$ and q_b^u solves (4)-(6).[16] As is standard, (13) states that the flow return of holding genuine money is the expected surplus from a match in case trade occurs.

A monetary equilibrium in this economy consists of $\langle q_b^u, p_b^u, \upsilon_1 \rangle$ such that (q_b^u, p_b^u) solves (4)-(6) and $\upsilon_1 > 0$ solves (13).

Proposition 2 *There exists a unique monetary equilibrium (under Refinement 1b) iff*

$$r < (1-m)\left[u'(0) - 1\right]. \tag{14}$$

According to Proposition 2, the possibility of counterfeiting does not constitute a threat to the existence of a monetary equilibrium in the sense that the set of parameters that determine whether fiat money is valued does not include the cost to produce counterfeits.[17] In particular, if $u'(0) = +\infty$ then there always exist a monetary equilibrium even if fiat money can be counterfeited and sellers are unable to recognize the quality of the money held by buyers. This result relies on a key characteristic of counterfeiting activities – a fixed entry cost. From Lemma 2, it is this fixed cost that secures a positive surplus for genuine buyers, taking υ_1 as given. Put it differently, nobody would pay the cost to imitate an object which has almost no value, so that the existence of a valued fiat money is never threatened.

[16] Recall that υ_1 is the present value of an agent holding a unit of genuine money at the beginning of the next period. Let $V_1 = (1+r)\upsilon_1$. Then, V_1 satisfies the following Bellman equation:

$$V_1 = (1-m)\left[u(q_b^u) + (1-p_b^u)\beta V_1\right] + m\beta V_1.$$

With probability $1-m$ a genuine buyer meets a seller, then he consumes q_b^u and delivers money with probability p_b^u. Subtract βV_1 on both sides and use from (12) that $p_b^u \beta V_1 = q_b^u$ to get (13).

[17] Our Proposition is in contrast with Proposition 2 in NW where a necessary and sufficient condition for the existence of a monetary equilibrium is $k > (1-m)u(\bar{q})$ where $\bar{q}$ is the equilibrium quantity in an economy where counterfeiting is not a possibility. The difference between their result and ours can be traced back to their problem 1 (p.997) that determines terms of trade. They assume that the seller's belief is independent of the offer made by the buyer and is equal to the fraction of genuine buyers in equilibrium. This belief system is even more restrictive than the Cho-Kreps refinement, which explains why NW does not uncover all the equilibria. We provide a more detailed discussion in Appendix D.

The threat of counterfeiting, however, matters for the allocation and welfare. Let υ_1^* be the unique positive solution to $r\upsilon_1 = (1-m)\{u[q(\upsilon_1)] - q(\upsilon_1)\}$ where $q(\upsilon_1) = \min(q^*, \upsilon_1)$. Here υ_1^* is the value of genuine money in an economy where there is no possibility to counterfeit currency.

Proposition 3 *Suppose $r < (1-m)[u'(0) - 1]$. Then, there is $\bar{k} = (1-m)u[\min(q^*, \upsilon_1^*)]$ such that for all $k < \bar{k}$ then $\frac{d\upsilon_1}{dk} > 0$, $\frac{dq_b^u}{dk} > 0$ and $\frac{dp_b^u}{dk} > 0$. As $k \to 0$ then $q_b^u \to 0$ and $\upsilon_1 \to 0$.*

Even though no counterfeiting takes place in equilibrium, the mere possibility of counterfeiting affects the equilibrium provided that the cost of producing counterfeits is not too large. This is another example where an out-of-equilibrium threat affects the equilibrium outcome. Intuitively, buyers never trade more than the quantity that would give incentives to agents without money to produce counterfeits; i.e., the existence of a non-counterfeiting equilibrium can be a result of the market discipline. Even so, our model legitimates policies that consist in making a currency harder to counterfeit: by raising the cost to produce counterfeits the policy-makers can increase the velocity of money, output and welfare. Notice that the monetary equilibrium disappears at the limit when the cost to produce counterfeits is driven to zero. If the cost is above a certain threshold, the possibility of counterfeiting cannot affect the economic activity so that the threat of counterfeiting is inactive.

5 Heterogeneous information

We extend the model to allow sellers to receive informative signals about the authenticity of the money held by buyers, i.e., $\eta \in (0,1)$. The objectives are twofold. By introducing a proxy for the recognizability of money we will be able to assess the effects of advertising campaigns that make the public more able to verify the authenticity of currency. Secondly, we will show that under some conditions the terms of trade can vary across matches depending on whether the quality of money is recognized or not.

The signal received by the seller is common knowledge in the match. The definition of equilibrium must be adapted so that the strategy of an agent without money specifies his acceptance rule depending on the signal he receives. Similarly, the strategy of a buyer must specify an offer as a function of the signal received by the seller he is matched with. The terms of trade in informed matches solve

$$(q_b^i, p_b^i) = \arg\max_{q,p} [u(q) - p\upsilon_1] \quad \text{s.t.} \quad -q + p\upsilon_1 \geq 0, \tag{15}$$

if the buyer holds a genuine unit of money. If the buyer in the match holds a counterfeit then $q_c^i = 0$ and $p_c^i \in [0, 1]$.

Regarding the terms of trade in uninformed matches, we adopt a forward-induction argument similar to Refinement 1b. Following the same logic as in Proposition 1, the equilibrium of the counterfeiting game involves no counterfeiting, $n = 0$, and the terms of trade in uninformed matches solve

$$(q_b^u, p_b^u) = \arg\max_{q,p} [u(q) - p\upsilon_1] \tag{16}$$

$$\text{s.t.} \quad -q + p\upsilon_1 \geq 0 \tag{17}$$

$$-k + (1-m)(1-\eta)u(q) \leq 0. \tag{18}$$

A belief system that sustains (q_b^u, p_b^u) as an equilibrium offer is $\lambda(q,p) = 1$ if (18) holds, and $\lambda(q,p) = 0$ otherwise. The novelty in (18) comes from the fact that an agent who chooses to produce a counterfeit can obtain some output only when the quality of his money is not recognized, with probability $1 - \eta$.

The value of genuine money in a non-counterfeiting equilibrium solves

$$r\upsilon_1 = (1-m)\eta \left[u(q_b^i) - q_b^i\right] + (1-m)(1-\eta)\left[u(q_b^u) - q_b^u\right]. \tag{19}$$

A monetary equilibrium is a list $\left\langle (q_b^i, p_b^i), (q_b^u, p_b^u), \upsilon_1 \right\rangle$ such that (q_b^i, p_b^i) solves (15) and (q_b^u, p_b^u) solves (16)-(18), and $\upsilon_1 > 0$ solves (19). There exists a unique monetary equilibrium under condition (14).

We establish the effects of the recognizability of money on the economic activity in the following proposition .

Proposition 4 *Suppose* $r < (1-m)[u'(0) - 1]$ *holds. Then, there is* $\bar{k} = (1-m)(1-\eta)u[\min(q^*, \upsilon_1^*)]$ *such that:*

1. *For all* $k < \bar{k}$ *,* $\frac{dq_b^u}{d\eta} > 0$ *and* $\frac{d\upsilon_1}{d\eta} > 0$*. Moreover,* $q_b^u < q_b^i$ *and* $p_b^u < p_b^i$*.*

2. *For all* $k \geq \bar{k}$ *then* $(q_b^u, p_b^u) = (q_b^i, p_b^i)$*.*

In equilibrium, sellers know that they meet genuine buyers with probability one. However, provided that the cost to produce counterfeits is sufficiently low, agents trade lower quantities and spend their money with a lower probability in matches where the authenticity of money is not verified. The recognizability of money has real effects – it matters for the distribution of terms of trade, the value and velocity of money, output and welfare. Once again even though no counterfeiting occurs in equilibrium, the possibility of counterfeiting (an out-of-equilibrium threat) affects the economic activity. Also, in contrast to standard search equilibrium models, a distribution of terms of trade emerges even though agents on both sides of the market are homogenous.

6 The materializing threat: Circulating counterfeits

The model with full confiscation that we have considered so far predicts no counterfeiting in any equilibrium (as in NW). One key assumption that prevents counterfeiting from emerging is that counterfeit money cannot circulate across periods. To account for the extent of counterfeiting, in this section we assume that the government has limited ability to take the counterfeits out of circulation: only a fraction $\delta \in (0,1)$ of the counterfeits are confiscated at the end of each period. We show that counterfeiting can occur in a monetary equilibrium, and study the effectiveness of the policies against counterfeiting.[18]

As a consequence of the assumption $\delta \in (0,1)$, agents at the beginning of each period can be divided into three types: agents holding genuine money, agents holding counterfeits inherited from the previous trades, and agents without money. Let υ_c denote the present value of an agent holding a counterfeit at the beginning of the next period. We denote $\omega_1 \equiv \upsilon_1 - \upsilon_0$ the value of genuine money and $\omega_c \equiv (1-\delta)(\upsilon_c - \upsilon_0)$ the value of a counterfeit.

As in the previous sections, an equilibrium consists of a profile of strategies (i.e., offers, acceptance rules and decision to produce counterfeits) and a belief system for sellers that satisfy sequential rationality and consistency of beliefs with strategies. The determination of the terms of trade in informed matches is given by (15) with υ_1 replaced by ω_1, if the buyer holds a unit of

[18] In Appendix B we demonstrate that a non-counterfeiting monetary equilibrium can arise provided k or δ is high enough.

genuine money, and

$$(q_c^i, p_c^i) = \arg\max_{q,p} [u(q) - p\omega_c] \quad \text{s.t.} \quad -q + p\omega_c \geq 0, \tag{20}$$

if the buyer holds a counterfeit (irrespective of whether it has been produced or acquired in past trades). In uninformed matches an agent holding a counterfeit makes an offer that solves

$$(q_c^u, p_c^u) = \arg\max_{q,p\leq 1} [u(q) - p\omega_c] \quad \text{s.t.} \quad -q + p\left\{\lambda(q,p)\omega_1 + [1-\lambda(q,p)]\,\omega_c\right\} \geq 0. \tag{21}$$

In this section we look for counterfeiting equilibria with $n > 0$ and $\omega_1 > \omega_c$, i.e., genuine money is more valuable than counterfeits. We adopt the following forward-induction argument to refine sellers' beliefs in uninformed matches.

Refinement 2 *Consider a proposed equilibrium with $n > 0$ where the trade surpluses of a genuine buyer and a counterfeiter in uninformed matches are $\mathcal{S}_1^u$ and $\mathcal{S}_c^u$, respectively. This equilibrium is disqualified if there is an out-of-equilibrium offer (q', p') such that the following is true:*

$$u(q') - p'\omega_1 \quad > \quad \mathcal{S}_1^u, \tag{22}$$

$$u(q') - p'\omega_c \quad < \quad \mathcal{S}_c^u, \tag{23}$$

$$-q' + p'\omega_1 \quad \geq \quad 0. \tag{24}$$

According to (22), the out-of-equilibrium offer (q', p') would make a buyer with genuine money strictly better off if it were accepted. According to (23), the offer (q', p') would make a buyer with a counterfeit strictly worse off. That is, an agent without money would not choose to produce a counterfeit to make such an offer since in the proposed counterfeiting equilibrium he is just indifferent between being a seller or a counterfeiter. According to (24) the offer is acceptable provided that the seller believes it comes from a buyer with genuine money.

Lemma 3 *The offer made by a counterfeiter in an uninformed match solves*

$$(q_c^u, p_c^u) = \arg\max_{q,p\in[0,1]} [u(q) - p\omega_c] \quad s.t. \quad -q + p\omega_c \geq 0. \tag{25}$$

The offer made by a genuine buyer solves

$$(q_b^u, p_b^u) = \arg\max_{q,p\in[0,1]} [u(q) - p\omega_1] \quad s.t. \quad -q + p\omega_1 \geq 0, \tag{26}$$

$$u(q) - p\omega_c \leq u(q_c^u) - p_c^u\omega_c. \tag{27}$$

There cannot be a pooling offer with active trades under our forward-induction refinement. If such a pooling offer were made in equilibrium then a buyer with genuine money could signal the quality of his currency by proposing an (out-of-equilibrium) offer where he transfers money with a lower probability and he consumes less output. The logic is similar to the one used to dismiss pooling equilibria in Proposition 1. A belief system consistent with the offers in (25)-(27) is such that sellers attribute all offers that violate (27) to buyers with counterfeits, and all other out-of-equilibrium offers to buyers with genuine money.

The solution to the problem of the genuine buyer is characterized in the following lemma.

Lemma 4 *If $\omega_c > 0$ then there is a unique solution to (26)-(27) and it is such that*

$$p_b^u = \frac{q_b^u}{\omega_1}, \tag{28}$$

$$u(q_b^u) - \frac{\omega_c}{\omega_1} q_b^u = u(q_c^u) - q_c^u. \tag{29}$$

Moreover, $q_b^u < q_c^u$ and $p_b^u < p_c^u$.

If $\omega_c > 0$ then the offer is separating and the buyer with genuine money obtains less output than the buyer with a counterfeit, but spends his money with a lower probability.

From the characterization of the terms of trade above it is immediate that $\upsilon_0 = 0$. The flow Bellman equation for the value of an agent holding a counterfeit is[19]

$$r\upsilon_c = (1 - m - n)\left[u(q_c^i) - q_c^i\right] - \delta\upsilon_c. \tag{30}$$

From Lemma 3, counterfeiters make their complete information offer in all matches, i.e., $q_c^i = q_c^u = q_c = \min[q^*, \omega_c]$ and $p_c = q_c/\omega_c$. The last term on the right-hand side of (30) captures the fact that counterfeits are confiscated with probability δ. Using the definition $\omega_c \equiv (1-\delta)\upsilon_c$, it becomes

$$\left(\frac{r+\delta}{1-\delta}\right)\omega_c = (1 - m - n)\left[u(q_c^i) - q_c^i\right]. \tag{31}$$

[19] Let $V_c = (1+r)\upsilon_c$, then it solves the following Bellman equation

$$V_c = (1 - m - n)\left[u(q_c^i) + (1 - p_c^i)(1-\delta)\beta V_c\right] + (m + n)(1-\delta)\beta V_c.$$

Subtract $(1-\delta)\beta V_c$ from both sides and use, from (20), that $q_c^i = p_c^i(1-\delta)\beta V_c$ to get (30).

Counterfeiting occurs in equilibrium if $\omega_c = \beta(1-\delta)k$, which from (31) implies

$$1 - m - n = \frac{[1-(1-\delta)\beta]\,k}{u(q_c^i) - q_c^i}. \tag{32}$$

The value of genuine money, $\upsilon_1 = \omega_1$, obeys

$$r\upsilon_1 = (1-m-n)\left\{(1-\eta)\left[u(q_b^u) - q_b^u\right] + \eta\left[u(q_b^i) - q_b^i\right]\right\}. \tag{33}$$

Given that $\upsilon_0 = 0$, we define a monetary equilibrium as follows.

Definition 1 *A counterfeiting equilibrium is a list* $\left\langle (q_b^i, p_b^i), (q_b^u, p_b^u), (q_c^i, p_c^i), (q_c^u, p_c^u), \upsilon_1, \upsilon_c, n \right\rangle$ *that satisfies: (i) the equations for the determination of the terms of trades, (15), (20), (25), (26)-(27); (ii) the Bellman equations, (30) and (33); (iii) the entry condition in the counterfeiting sector, (32); and (iv) the conditions for a monetary equilibrium,* $\upsilon_1 > 0$*, and for genuine money to be more valuable than counterfeits,* $\upsilon_1 > (1-\delta)\upsilon_c$*.*

The counterfeiting equilibrium has a recursive structure. Equation (32) determines n while, given n, (33) generates υ_1.

Proposition 5 *If*

$$r' < (1-m)[u'(0) - 1], \tag{34}$$

where $r' = \frac{1-(1-\delta)\beta}{(1-\delta)\beta}$*, then there is* $\bar{k} > 0$ *such that for all* $k < \bar{k}$ *there exists a counterfeiting equilibrium.*

If $u'(0) = +\infty$ then there always exists a counterfeiting equilibrium provided that the cost of counterfeiting is small. Counterfeiting can prevail in equilibrium, even though counterfeiters and genuine buyers make different offers; i.e., people accept counterfeits knowingly. Just like genuine money, there is a chance that counterfeits can be passed on to someone else and traded for some output in the future. From Proposition 5, counterfeiting is more likely to take place if the stock of genuine money is low, if the cost to produce counterfeits is small, and if the ability of the government to confiscate counterfeits is limited.

We now study the effects of various policies on the extent of counterfeiting and the value of money. Let $n(m,k,\eta,\delta)$ and $\omega_1(m,k,\eta,\delta)$ denote the equilibrium measure of counterfeiters and the equilibrium value of genuine money as functions of exogenous variables.

Proposition 6 *Assume (45) holds. Then: (i)* $\frac{\partial n}{\partial m} = -1$ *and* $\frac{\partial \omega_1}{\partial m} = 0$*; (ii)* $\frac{\partial n}{\partial k} < 0$ *and* $\frac{\partial \omega_1}{\partial k} > 0$*; (iii)* $\frac{\partial n}{\partial \eta} = 0$ *and* $\frac{\partial \omega_1}{\partial \eta} > 0$*; (iv)* $\frac{\partial n}{\partial \delta} < 0$ *and* $\frac{\partial \omega_1}{\partial \delta} > 0$*.*

The fraction of agents holding a unit of money (genuine or fake) is uniquely pinned down by the free entry in the counterfeiting sector. Consequently, an increase in the supply of genuine money has a complete crowding-out effect on counterfeits, leaving the value of genuine money unchanged. By endowing a larger number of buyers with genuine money the monetary authority reduces the available space to counterfeiters.

In terms of anti-counterfeiting policies, higher confiscation rate of counterfeits reduces the extent of counterfeiting, and raises the value of genuine money. An increase in the recognizability of genuine money (η) raises the value of genuine money since the surplus enjoyed by genuine buyers is greater in informed matches than in uninformed matches. The improving recognizability, however, does not affect counterfeiting activity because in a separating equilibrium the offers made by counterfeiters depend on the production cost of counterfeits but not on the value of genuine money.

Society's welfare is measured by the sum of all agents' expected lifetime utility, i.e., $W = \beta^{-1}\left[(1-m-n)\upsilon_0 + m\upsilon_1 + n\upsilon_c\right] - \delta n k$, where the last term comes from the fact that a fraction δ of the counterfeits are confiscated and replaced in every period. Since $\upsilon_0 = 0$ and $\upsilon_c = \beta k$, we have

$$W = \beta^{-1}\left(m\omega_1 + n\omega_c\right). \tag{35}$$

The society's welfare is equal to the aggregate real balances in the economy, which also includes the real value of counterfeits.

Proposition 7 *Suppose (45) holds. Then,* $dW/dm > 0$ *and* $dW/d\eta > 0$*.*

An increase in the stock of genuine money raises welfare by substituting one-for-one counterfeits by more valuable genuine money. An increase in the fraction of informed matches raises welfare by making genuine money more valuable.

Whether a higher confiscation rate δ can improve welfare depends on the stock of genuine money in the economy. In Figure 1, we consider a numerical example where the stock of genuine money is substantially low, $m = 0.01$, a situation that can be interpreted as a currency shortage.[20] An

[20] The rest of the parametrization is $u(q) = 2\sqrt{q}$, $\eta = 0.5$, $r = 0.1$ and $k = 0.5$. Welfare is measured relative to the first best level, which is 1/4.

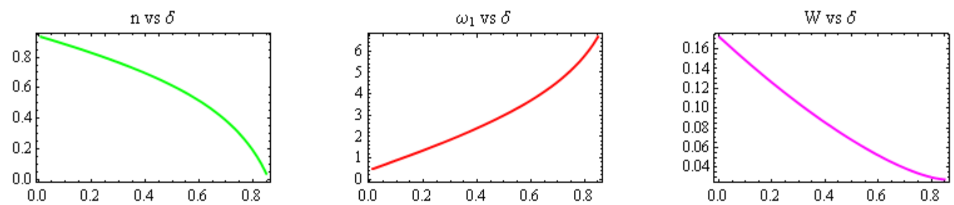

Figure 1: Confiscation of counterfeits under a currency shortage

increase in the confiscation rate deters counterfeiting and raises the value of genuine money, but it lowers welfare. Although it is believed that counterfeiting does harm to the economy and should be eliminated, our example suggests that the policy makers should let them circulate as long as the government cannot control the supply of genuine money and there is a severe currency shortage.[21]

To conclude this section, we investigate the limiting case where the government has no enforcement power and cannot confiscate counterfeits, i.e., we consider the limit of the equilibrium as $\delta \to 0$. The following proposition shows the existence of a counterfeiting equilibrium under an extremely lax anti-counterfeiting policy.

Proposition 8 *Consider the limit economy as $\delta \to 0$. If*

$$r < (1-m)[u'(0)-1],$$

then there is $\bar{k} > 0$ such that for all $k < \bar{k}$ there exists a counterfeiting equilibrium, and it is such that $\omega_1 = \omega_c = \beta k$.

Notice that the condition for the existence of a counterfeiting equilibrium here is identical to the condition for the existence of a monetary equilibrium in the previous sections (e.g., Proposition 2), with an additional requirement $k < \bar{k}$. When the confiscation rate tends to zero, counterfeits

[21] Mihm (2007) provides numerous quotes in support of the thesis that counterfeiting might have been a necessary evil to overcome the currency shortage in the U.S. before the civil war. For instance, Burroughs, the first famous counterfeiter in the U.S., in his memoirs (Cited in Mihm, 2007, p.41) quotes a friend saying: ‹‹An undue scarcity of cash now prevails [and] whoever contributes, really, to increase the quantity of cash, does not only himself, but likewise the community, an essentiel benefit.›› Daniel Mevis in his *Pioneer Recollections* (cited in Mihm, 2007, p. 159) wrote: ‹‹Counterfeiting and issuing worthless 'bank notes' ... was not looked upon as a felony as it would be today. Of course, it was taken for granted that it was a 'little crooked' but the scarcity of real money, together with the necessity of for a medium of exchange, made almost anything that looked like money answer the purpose.››

circulate as genuine money does. Furthermore, counterfeits and genuine money trade at the same value which, given $\omega_c = \beta k$, is determined by the entry cost in the counterfeiting sector.

7 The materializing threat: Pooling offers

The counterfeiting equilibria obtained in Section 6 have the feature that genuine buyers are able to separate themselves from counterfeiters by making an offer that the latter would not imitate. As a consequence, sellers accept counterfeits knowingly. In this section we consider the model as in Section 5 but assume $\theta < 1$. We will show that if sellers are informed in some matches ($\eta > 0$) a counterfeiting equilibrium can exist even if counterfeits cannot circulate across periods ($\delta = 1$).[22] In this equilibrium sellers accept counterfeits in the instances where they cannot recognize the authenticity of currency.

In informed matches buyers offer (q_b^i, p_b^i) solution to (15) while sellers offer (q_s^i, p_s^i) if the buyer holds a genuine note, with

$$(q_s^i, p_s^i) = \arg\max_{q,p\leq 1} \left[-q + p(\upsilon_1 - \upsilon_0)\right] \quad \text{s.t.} \quad u(q) - p(\upsilon_1 - \upsilon_0) \geq 0. \tag{36}$$

The solution to (36) is $q_s^i = \min\left[q^*, u^{-1}(\omega_1)\right]$ and $p_s^i = u(q_s^i)/\omega_1$. In uninformed matches, sellers may offer a menu of terms of trades as an attempt to screen buyers holding genuine money. We impose that sellers cannot commit to terms of trade that are not ex-post individually rational.

Lemma 5 *In uninformed matches, sellers offer a single contract solution to*

$$(q_s^u, p_s^u) = \arg\max \left[\frac{m}{n+m} p(\upsilon_1 - \upsilon_0) - q\right] \quad s.t. \quad u(q) - p(\upsilon_1 - \upsilon_0) \geq 0. \tag{37}$$

Given that counterfeits have no value, sellers cannot credibly offer to accept a counterfeit, and hence they must offer terms of trade that pool genuine buyers and counterfeiters. According to (37) sellers make an offer that maximizes the expected value of the currency they receive minus the production cost, taking into account the acceptance rule of genuine buyers only. The solution to (37) is $q_s^u = \min\left[u'^{-1}\left(1 + \frac{n}{m}\right), u^{-1}(\omega_1)\right]$ and $p_s^u = u(q_s^u)/\omega_1$.

We focus on a monetary equilibrium with counterfeiting, i.e., $n > 0$. The following lemma shows that no trade takes place in uninformed matches when buyers make the offer.

[22] In Appendix C we show that counterfeiting cannot occur in any equilibrium if $\eta = 0$.

Lemma 6 *In any counterfeiting equilibrium ($n > 0$) buyers in uninformed matches offer $(q_b^u, p_b^u) = (0, 0)$.*

Under the Intuitive Criterion (Refinement 1b) a genuine buyer can break out of a pooling equilibrium by offering to consume less output and to transfer money with a lower probability. The no-trade outcome is sustained by the sellers' belief that any offer with positive output comes from a counterfeiter, i.e., $\lambda(q, p) = 0$ whenever $q > 0$. Even though counterfeiters get zero consumption when they make offers, agents may be willing to produce counterfeits because they can obtain some surplus from the pooling offers made by sellers.

The value functions for agents in different states obey the following flow Bellman equations

$$\begin{aligned} rv_1 &= (1-m-n)\theta\eta\left[u(q_b^i) - q_b^i\right] && (38)\\ rv_0 &= (1-\theta)\left\{\eta m\left[u(q_s^i) - q_s^i\right] + (1-\eta)(n+m)\left[-q_s^u + \frac{m}{n+m}p_s^u(v_1 - v_0)\right]\right\} && (39)\\ rv_c &= (1-m-n)(1-\theta)(1-\eta)u(q_s^u) - (v_c - v_0). && (40)\end{aligned}$$

According to (38), a genuine buyer can only obtain a positive surplus in an informed match when he makes the offer. According to (39), a seller enjoys a positive expected surplus whenever he makes the offer. From (40), a counterfeiter enjoys some consumption if the seller makes the offer in an uninformed match.[23] Finally, assuming an interior solution, the measure of counterfeiters solves

$$v_c - v_0 = \beta k. \tag{41}$$

Definition 2 *A counterfeiting equilibrium is a list $\langle (q_b^i, p_b^i), (q_s^i, p_s^i), (q_s^u, p_s^u), v_0, v_1, v_c, n\rangle$ that satisfies: (i) the equations for the terms of trade, (15), (36), (37); (ii) the flow Bellman equations (38)-(40); (iii) the entry condition in the counterfeiting sector, (41); and (iv) the condition for a monetary equilibrium, $v_1 > 0$.*

Society's welfare, $\mathcal{W}$, is measured as the sum of the surpluses across all matches minus the cost

[23] Let $V_c = (1+r)v_c$, then it solves the following Bellman equation

$$V_c = (1-m-n)(1-\theta)(1-\eta)\left[u(q_s^u) + \beta V_0\right] + \left[1 - (1-m-n)(1-\theta)(1-\eta)\right]\beta V_0.$$

Since counterfeits are confiscated at the end of a period, the continuation value is βV_0 no matter the counterfeiter trades or not from a match. Subtract βV_c from both sides to get (40).

to produce counterfeits, i.e.,

$$\mathcal{W} = m(1-m-n)\left\{\theta\eta\left[u(q_b^i)-q_b^i\right]+(1-\theta)\eta\left[u(q_s^i)-q_s^i\right]\right\} \\ +(m+n)(1-m-n)(1-\theta)(1-\eta)\left[u(q_s^u)-q_s^u\right]-nk. \quad (42)$$

Counterfeiting exerts various adverse effects on society's welfare. No trade takes place in uninformed matches when buyers make offers even though some buyers hold genuine money. The trade surplus is lower in uninformed matches than in the informed matches since sellers make pooling offers. There is also a direct cost because resources are used to produce counterfeits.

We study the equilibrium through numerical examples. Our benchmark parametrization is $u(q)=2\sqrt{q}$, $r=0.1$, $m=0.3$, $\theta=\eta=0.5$ and $k=0.02$. In Figure 2 we report how changes in policy variables (k, η and m) and market structure (θ) affect the measure of counterfeiters, the value of genuine money, and welfare. In the last column of Figure 2, we express $\mathcal{W}$ as a fraction of the welfare at the first best allocation.[24]

Policies that raise the cost of producing counterfeits and make genuine money more recognizable have a positive effect on the value of genuine money and social welfare (see the first two rows in Figure 2). Moreover, a higher k reduces n while the relationship between n and η is non-monotonic. If it is very difficult to distinguish a unit of genuine money from a counterfeit then an improvement in the recognizability of money can in fact promote counterfeiting. The reason is that genuine money becomes more valuable which in turn makes the return of counterfeiting higher.

The third row in Figure 2 illustrates the effects of a change in buyers' market power. Since buyers holding genuine money get no surplus when sellers make offers, genuine money becomes more valuable as buyers get more opportunities to determine the terms of trade. An increase in θ has two opposite effects on agents' incentives to produce counterfeits. Genuine money becomes more valuable, and hence, agents have higher incentives to enter the counterfeiting sector. But a higher θ also implies that counterfeits can be traded less frequently because no trade takes place in uninformed matches when buyers make offers. The former effect dominates when θ is low.

Finally, the last row in Figure 2 reports the effects of a change in the supply of genuine money. If there is a shortage of genuine money then agents have an incentive to produce counterfeits since

[24] The first best is achieved when the number of matches is maximum ($m=1/2$) and agents trade q^* in all matches.

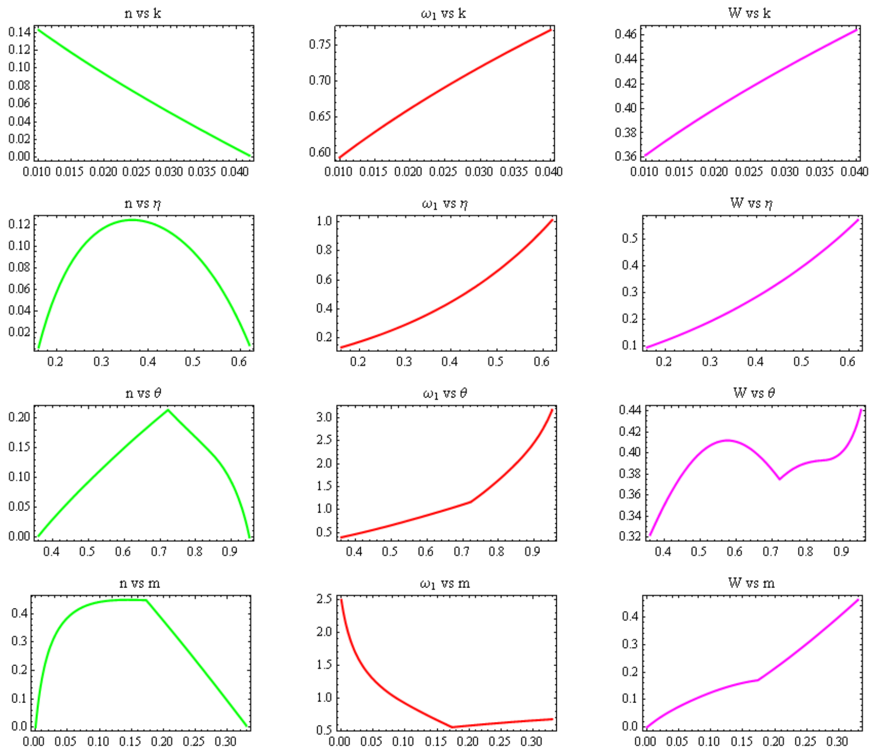

Figure 2: Counterfeiting equilibria

it is hard to trade as a seller. As the quantity of genuine money becomes sufficiently abundant the opposite occurs and an increase in m discourages counterfeiting. Society's welfare increases with the quantity of money (over the range where there is counterfeiting in equilibrium).

8 Conclusion

We have studied and extended the model of counterfeiting from Nosal and Wallace (2007). Our results challenge a key proposition in NW, that the threat of counterfeiting makes the existence of a monetary equilibrium less likely. We showed that the existence of a monetary equilibrium is not threatened by the possibility of counterfeiting. However, in contrast with what was previously thought, the value of genuine money and society's welfare can be directly affected by the cost of producing counterfeits and the recognizability of money, even if counterfeiting does not occur in equilibrium. An important policy implication is that, even though counterfeiting seems insignificant, government efforts such as introducing new design of currency or law enforcement that makes counterfeiting more costly should be necessary to prevent the threat to have adverse effects on output and welfare.[25]

We also showed that the model can account for the counterfeiting activity, even modest, that is observed in the U.S. and most countries. We have offered two extensions of the model under which the threat of counterfeiting is realized: if counterfeits can circulate across periods, or if sellers set terms of trade in some matches. Under both versions of the model, policies that improve the recognizability of the currency raise the value of genuine money and society's welfare. In some cases, however, those policies are ineffective in terms of reducing the extent of counterfeiting, or they may even have perverse effects.

Several other extensions would be worth exploring, such as introducing various punishments for people caught with counterfeits, or considering a costly technology to detect counterfeits. For tractability, we have assumed an agent's money holding lies in the set $\{0, 1\}$. This assumption can capture the phenomenon of currency shortage and the role of counterfeiting in promoting trades. Nonetheless, to study the effects of monetary policy such as inflation on counterfeiting, one may like to consider a version of the model with no restrictions on individuals' money holdings. This can be done by using a quasi-linear model with divisible money (e.g. Lagos and Wright 2005) in which some agents can choose to produce any quantity of counterfeits at a fixed cost.[26]

[25] "We have to stay ahead of technology, which is developing and progressing at an ever-increasing rate. Items like digital printers and higher quality scanners are becoming more readily available at cheaper prices. So we have to make our currency notes safer, smarter, and more secure in order to stay ahead of the would-be counterfeiters," by Tom Ferguson, Director of the Treasury's Bureau of Engraving and Printing, which produces U.S. currency. The quote is taken from the *Federal Reserve Bulletin*, Summer 2004.

[26] See Appendix E and Rocheteau (2008) for such a model.

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Appendix A.

A.1. Proof of Lemma 1.

The strategy of being a producer yields at least υ_0 since a seller always has the option to reject an offer that is proposed. The strategy of producing counterfeits and offering (q,p) yields an expected payoff no greater than $-k+(1-m)u(q)+\upsilon_0$ since the matching probability of the buyer is bounded above by $1-m$ and the buyer enjoys the utility of consuming q if the offer is accepted. Then, $-k+(1-m)u(q)+\upsilon_0<\upsilon_0$, irrespective of the choices of other agents, such as the decision of agents without money to produce counterfeits or the decision of sellers to accept or reject the offer (q,p).■

A.2. Proof of Lemma 2.

According to Refinement 1a, any offer (q,p) satisfying $-k+(1-m)u(q)<0$ is attributed to a genuine buyer ($\lambda(q,p)=1$), and the expected surplus of the seller is $-q+p(\upsilon_1-\upsilon_0)$. Moreover, if (5) holds then the offer is accepted. In order to find the greatest lower bound for the buyer's surplus, we consider offers in the closure of the set

$$\{(q,p)\in\mathbb{R}_+\times[0,1]:-q+p(\upsilon_1-\upsilon_0)\geq 0 \text{ and } -k+(1-m)u(q)<0\},$$

which gives (5) and (6).■

A.3. Proof of Proposition 1

(i) There is no equilibrium with $n>0$. The proof is by contradiction. Suppose $n>0$. Then, from (3) $\chi=c$ requires $q_c^u>0$. For the offer (q_c^u,p_c^u) to be acceptable,

$$-q_c^u+\lambda(q_c^u,p_c^u)p_c^u(\upsilon_1-\upsilon_0)\geq 0.$$

Consider the out-of-equilibrium offer (q',p') such that $p'=p_c^u-\varepsilon$, where $\varepsilon\in\left(0,p_c^u-\frac{q_c^u}{\upsilon_1-\upsilon_0}\right)$, and $q'<q_c^u$ satisfies (7)-(8) or, equivalently,

$$0<\frac{u(q_c^u)-u(q')}{\varepsilon}<\upsilon_1-\upsilon_0. \tag{43}$$

We first establish that the set of offers (q',p') is not empty. Using the fact that $\lambda(q_c^u,p_c^u)<1$, the seller's participation constraint at the proposed equilibrium implies $q_c^u<p_c^u(\upsilon_1-\upsilon_0)$ and hence

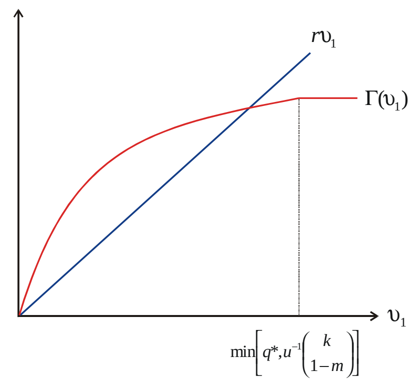
$\left(0, p_c^u - \frac{q_c^u}{\upsilon_1 - \upsilon_0}\right)$ is not empty. Moreover, for any $\varepsilon \in \left(0, p_c^u - \frac{q_c^u}{\upsilon_1 - \upsilon_0}\right)$ there is a $q' \geq 0$ that satisfies (43). Second, we show that any offer (q', p') disqualifies the proposed equilibrium according to Refinement 1b. By construction, from (43), (q', p') satisfies (7)-(8). Moreover, $\varepsilon < p_c^u - q_c^u/(\upsilon_1 - \upsilon_0)$ implies $q_c^u < (p_c^u - \varepsilon)(\upsilon_1 - \upsilon_0)$. From (43), $u(q_c^u) - u(q') > 0$ and therefore $q_c^u - q' > 0$. So (9) is satisfied as well.

(ii) In any equilibrium with $n = 0$ the offer made by a genuine buyer solves (4)-(6). Consider an equilibrium with $n = 0$ where the expected surplus of the genuine buyer is greater than that implied by (4)-(6). Then, (6) must be violated. But then, from (3), $n = 0$ implies $-q_b^u + p_b^u(\upsilon_1 - \upsilon_0) > 0$. With a reasoning similar as in (i), one can find (q', p') that solves (7)-(9) to disqualify the proposed equilibrium, violating Refinement 1b. To see this, take $q' = q_b^u - \varepsilon$ where $\varepsilon > 0$, and $p' = (q_b^u - \varepsilon)/(\upsilon_1 - \upsilon_0)$. Since $q' < q_b^u$, the offer would not be made by an agent without money who produces a counterfeit. As $\varepsilon \to 0$, the buyer's payoff tends to $u(q_b^u) - q_b^u$ which is the total surplus of the match. Since $-q_b^u + p_b^u(\upsilon_1 - \upsilon_0) > 0$ then $u(q_b^u) - p_b^u(\upsilon_1 - \upsilon_0) < u(q_b^u) - q_b^u$. Then, provided that ε is small enough, $u(q') - p'(\upsilon_1 - \upsilon_0) > u(q_b^u) - p_b^u(\upsilon_1 - \upsilon_0)$. So the offer is attributed to a genuine buyer and it is accepted.

A belief system for sellers to sustain the offer given by (4)-(6) as an equilibrium offer is such that all offers that satisfy $-k + (1 - m)u(q) \leq 0$ are attributed to genuine buyers while all other offers are attributed to an agent without money who chose to be a counterfeiter. The buyer's offer is optimal given that sellers believe it is made by the genuine buyers and accept it, and all offers that violate (6) are attributed to a counterfeiter and hence are rejected.■

A.4. Proof of Proposition 2

Define $\Gamma(\upsilon_1) = (1 - m)\{u[q_b^u(\upsilon_1)] - q_b^u(\upsilon_1)\}$. From (10)-(11), $\Gamma(\upsilon_1) = (1 - m)[u(\upsilon_1) - \upsilon_1]$ is strictly concave if both $\upsilon_1 < q^*$ and $u(\upsilon_1) < \frac{k}{1-m}$. Moreover, if $u(q^*) \leq k/(1 - m)$ and $\upsilon_1 \geq q^*$ then $\Gamma(\upsilon_1) = (1 - m)[u(q^*) - q^*]$; if $u(q^*) > k/(1 - m)$ and $\upsilon_1 > u^{-1}(k/1 - m)$ then $\Gamma(\upsilon_1) = (1 - m)\left[\frac{k}{1-m} - u^{-1}\left(\frac{k}{1-m}\right)\right]$. Hence, the existence of a monetary equilibrium requires $r < \Gamma'(0) = (1 - m)[u'(0) - 1]$. See Figure below.



■

A.5. Proof of Proposition 3

From (10) evaluated at υ_1^* and (6) held at equality, one can solve for $\bar{k}$. For all $k < \bar{k}$ the constraint (6) binds. Hence, $q_b^u = u^{-1}\left[k/(1-m)\right]$ and $\frac{dq_b^u}{dk} > 0$. Differentiating (13),

$$\frac{d\upsilon_1}{dk} = r^{-1}\left[1 - \frac{1}{u'(q_b^u)}\right].$$

Since (6) binds, $q_b^u < \min(\upsilon_1, q^*) \leq q^*$. Hence, $u'(q_b^u) > 1$ and $\frac{d\upsilon_1}{dk} > 0$. From (12), $p_b^u = \frac{q_b^u}{\upsilon_1} < 1$. Differentiating (12),

$$\frac{dp_b^u}{dk} = \frac{r\upsilon_1 - (1-m)\left[q_b^u u'(q_b^u) - q_b^u\right]}{(1-m)ru'(q_b^u)(\upsilon_1)^2}.$$

Using the strict concavity of $u(q_b^u)$, $r\upsilon_1 > (1-m)\left[q_b^u u'(q_b^u) - q_b^u\right]$ and hence $\frac{dp_b^u}{dk} > 0$.

Since $q_b^u = u^{-1}\left[k/(1-m)\right]$, as $k \to 0$ then $q_b^u \to 0$ and $\upsilon_1 \to 0$ from (13).■

A.6. Proof of Proposition 4

From (16)-(18), the lowest value of k above which (18) does not bind is $\bar{k} = (1-m)(1-\eta)u(q)$ with $q = \min(q^*, \upsilon_1^*)$. If $k < \bar{k}$ then $q_b^u = u^{-1}\left(\frac{k}{(1-m)(1-\eta)}\right)$ and hence $\frac{\partial q_b^u}{\partial \eta} > 0$. Moreover, if (18) binds then $q_b^u < q_b^i$ and hence $p_b^u = \frac{q_b^u}{\upsilon_1} < p_b^i = \frac{q_b^i}{\upsilon_1}$. Differentiating (19),

$$\frac{d\upsilon_1}{d\eta} = \frac{(1-m)\eta\left\{\left[u(q_b^i) - q_b^i\right] - \left[u(q_b^u) - q_b^u\right]\right\} + (1-m)(1-\eta)\left[u'(q_b^u) - 1\right]\frac{\partial q_b^u}{\partial \eta}}{r - (1-m)\eta\left[u'(q_b^i) - 1\right]\frac{\partial q_b^i}{\partial \upsilon_1}} > 0,$$

since at the monetary equilibrium the right-hand side of (19) intersects the left-hand side of (19) from above so that the denominator of the expression is positive, $u(q_b^i) - q_b^i > u(q_b^u) - q_b^u$ (because $q_b^u < q_b^i \leq q^*$) and $\frac{dq_b^u}{d\eta} > 0$.■

A.7. Proof of Lemma 3

We first show that there cannot be a pooling offer that is accepted such that $q > 0$. Suppose there is an equilibrium where both types of buyers offer $(\bar{q}, \bar{p})$ with $\bar{q} > 0$, i.e., $\lambda(\bar{q}, \bar{p}) \in (0, 1)$. The equilibrium payoffs are $\mathcal{S}_1^u = u(\bar{q}) - \bar{p}\omega_1$ and $\mathcal{S}_c^u = u(\bar{q}) - \bar{p}\omega_c$. Furthermore, the offer $(\bar{q}, \bar{p})$ is accepted if $-\bar{q} + \bar{p}\{\lambda(\bar{q}, \bar{p})\omega_1 + [1 - \lambda(\bar{q}, \bar{p})]\,\omega_c\} \geq 0$. Consider the out-of-equilibrium offer (q', p') such that $p' = \bar{p} - \varepsilon$, where $\varepsilon \in \left(0, \bar{p} - \frac{\bar{q}}{\omega_1}\right)$, and $q' < \bar{q}$ satisfies (22)-(23) or, equivalently,

$$\omega_c < \frac{u(\bar{q}) - u(q')}{\varepsilon} < \omega_1. \tag{44}$$

We first establish that the set of offers (q', p') is not empty. Using the fact that $\lambda(\bar{q}, \bar{p}) < 1$, the seller's participation constraint at the proposed equilibrium implies $\bar{q} < \bar{p}\omega_1$ and hence $\left(0, \bar{p} - \frac{\bar{q}}{\omega_1}\right)$ is not empty. Moreover, for any $\varepsilon \in \left(0, \bar{p} - \frac{\bar{q}}{\omega_1}\right)$ there is a $q' \geq 0$ that satisfies (44). To see this, rewrite (44) as

$$u(\bar{q}) - \omega_1\varepsilon < u(q') < u(\bar{q}) - \omega_c\varepsilon.$$

Since $\varepsilon < \bar{p} - \frac{\bar{q}}{\omega_1}$ we have $\varepsilon\omega_1 < \bar{p}\omega_1 - \bar{q}$ and hence $u(\bar{q}) - \omega_1\varepsilon > \mathcal{S}_1^u + \bar{q} > 0$. Second, we show that any offer (q', p') disqualifies the proposed equilibrium according to Refinement 2. From (44), (q', p') satisfies (22)-(23). Moreover, $\varepsilon < \bar{p} - \bar{q}/\omega_1$ implies $\bar{q} < (\bar{p} - \varepsilon)\,\omega_1$. From (44), $u(\bar{q}) - u(q') > 0$ and therefore $\bar{q} - q' > 0$. So (24) is satisfied as well.

We next ask whether a pooling offer of $q = 0$ exists. If $q = 0$ then $\mathcal{S}_1^u = \mathcal{S}_c^u = 0$ (since the equilibrium payoff of a buyer is always non-negative). But $\mathcal{S}_c^u \geq \max_{q,p\leq 1}[u(q) - p\omega_c]$ s.t. $-q + p\omega_c \geq 0$, which implies that this case is only relevant when $\omega_c = 0$.

Unless $q = 0$ the equilibrium of the bargaining game is separating. So the buyer with counterfeit money cannot do better than proposing his complete information offer. Hence, his offer (q_c^u, p_c^u) solves (25). The buyer with genuine money makes the separating offer that maximizes his payoff; any other separating offer violates Refinement 2.[27] Finally, a belief system consistent with the offers

[27] Suppose there is a separating equilibrium where the expected payoff of the buyer with a counterfeit is $\mathcal{S}_c^u$ and the

in (25)-(27) is such that sellers attribute all offers that violate (27) to buyers with counterfeits, and all other out-of-equilibrium offers to buyers with genuine money.■

A.8. Proof of Lemma 4

From Lemma 3, (q_b^u, p_b^u) is the solution to (26)-(27). Suppose first that (27) is not binding. Then, (q_b^u, p_b^u) is the complete information offer, i.e., $q_b^u = \min[q^*, \omega_1] \geq q_c^u$ and $p_b^u = q_b^u/\omega_1 > 0$. The payoff of a buyer with a counterfeit who offers (q_b^u, p_b^u) is then

$$\begin{aligned} u(q_b^u) - p_b^u \omega_c &= u(q_b^u) - q_b^u + p_b^u (\omega_1 - \omega_c) \\ &> u(q_c^u) - q_c^u, \end{aligned}$$

since $p_b^u > 0$ and $\omega_1 - \omega_c > 0$. But then (27) is violated. A contradiction.

Suppose next that the seller's participation constraint in (26) is not binding. Substitute $u(q_b^u) = p_b^u \omega_c + u(q_c^u) - p_c^u \omega_c$, given by (27) at equality, into the buyer's payoff to get

$$\max_{p \in [0,1]} \{ p [\omega_c - \omega_1] + u(q_c^u) - p_c^u \omega_c \},$$

which gives $p_b^u = 0$ and $u(q_b^u) = u(q_c^u) - q_c^u > 0$ (since $\omega_c > 0$). But then the seller's participation constraint, $-q_b^u \geq 0$, is violated. A contradiction.

So, (27) and the seller's participation constraint are binding. From the seller's participation constraint we obtain (28). Substitute p_b^u from (28) into (27) at equality to get (29).

In order to establish the existence of a unique solution to (26) and (27), notice that the left-hand side of (29) is first increasing and then decreasing in q_b^u, and it reaches a maximum greater than $u(q^*) - q^* \geq u(q_c^u) - q_c^u$ for some $q_b^u > q^*$ solution to $u'(q_b^u) = \frac{\omega_c}{\omega_1}$. So, there might be multiple solutions (at most two) to (29). However, only the lowest value for q_b^u maximizes the payoff of the buyer with genuine money. To see this, notice that

$$u(q_b^u) - q_b^u = u(q_b^u) - \frac{\omega_c}{\omega_1} q_b^u - \left(1 - \frac{\omega_c}{\omega_1}\right) q_b^u.$$

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expected payoff of the genuine buyer is $\mathcal{S}_1 \in [0, \mathcal{S}_1^g)$ where $\mathcal{S}_1^g \equiv u(q_b^u) - p_b^u \omega_1$ (this is the payoff of the genuine buyer at the Pareto-efficient separating equilibrium). Replace $u(q_c^u) - p_c^u \omega_c$ in (27) by $\mathcal{S}_c^u - \varepsilon$ with $\varepsilon > 0$, and denote $\mathcal{S}_\varepsilon^1$ the associated payoff for the genuine buyer. The set of acceptable and feasible offers is compact. From the Theorem of the Maximum, $\mathcal{S}_\varepsilon^1$ is continuous in ε and $\lim_{\varepsilon \to 0} \mathcal{S}_\varepsilon^1 = \mathcal{S}_1^g$. Hence, there is an $\varepsilon > 0$ such that $\mathcal{S}_\varepsilon^1 > \mathcal{S}_1$. The associated offer satisfies (22)-(24) so that the proposed equilibrium violates Refinement 2.

From (29),

$$u(q_b^u) - q_b^u = u(q_c^u) - q_c^u - \left(1 - \frac{\omega_c}{\omega_1}\right) q_b^u,$$

The right-hand side of the equation above is decreasing in q_b^u; hence, $u(q_b^u) - q_b^u$ is maximized at the lowest value of q_b^u that solves (29).

Next, we establish that $q_b^u < q_c^u$ and $p_c^u > p_b^u$. The left-hand side of (29) is increasing in q_b^u over $[0, q_c^u]$ from $0 < u(q_c^u) - q_c^u$ to $u(q_c^u) - \frac{\omega_c}{\omega_1} q_c^u > u(q_c^u) - q_c^u$. So, there is a unique $q_b^u \in (0, q_c^u)$. From (27), $(p_c^u - p_b^u)\omega_c = u(q_c^u) - u(q_b^u) > 0$, and hence $p_c^u > p_b^u$.■

A.9. Proof of Proposition 5

The proof proceeds in three parts by following the recursive structure of the equilibrium. First, we show that a unique solution $n \in (0, 1-m)$ to (32) exists provided that

$$1 - m > \frac{[1 - (1-\delta)\beta]\, k}{u(q_c^i) - q_c^i} \tag{45}$$

holds, where $q_c^i = \min[q^*, \beta(1-\delta)k]$. We then derive condition (34) from (45). Second, given condition (45), we show that there is a unique $\omega_1 > \omega_c$ that solves (33). Third, we establish that there is no equilibrium with $\omega_1 \leq \omega_c$.

(i) The right-hand side of (32) is independent of n while the left-hand side is strictly decreasing for all $n \in [0, 1-m]$. Hence, if a solution exists, it is unique. At $n = 1-m$, the left-hand side is 0. So a solution to (32) exists, and it is such that $n > 0$, if the left-hand side of (32) evaluated at $n = 0$ is greater than the right-hand side, which gives (45).

Let $\bar{k}$ denote the value of k that solves $1 - m = \frac{[1-(1-\delta)\beta]k}{u(q_c^i) - q_c^i}$. If $q^* \leq \beta(1-\delta)k$ then $\bar{k} = \frac{(1-m)[u(q^*)-q^*]}{1-(1-\delta)\beta} > 0$. For all $k < \bar{k}$, (45) holds, and there exists a counterfeiting equilibrium. Consider next the case $q^* > \beta(1-\delta)k$. Differentiate the following expression with respect to k,

$$\begin{aligned}
\frac{d}{dk}\left(\frac{u(q_c^i) - q_c^i}{k}\right) &= \frac{\left[u'(q_c^i) - 1\right]\beta(1-\delta)k - \left[u(q_c^i) - q_c^i\right]}{k^2} \\
&= k^{-2}\left[u(q_c^i) - q_c^i\right]\left\{\frac{\left[u'(q_c^i) - 1\right] q_c^i}{u(q_c^i) - q_c^i} - 1\right\},
\end{aligned}$$

where we have used that $q_c^i = \min[q^*, \beta(1-\delta)k] = \beta(1-\delta)k$. Using the strict concavity of $u(q_c^i) - q_c^i$ over $(0, q^*)$ and the fact that $u(0) = 0$, we have $\frac{[u'(q_c^i)-1]q_c^i}{u(q_c^i)-q_c^i} < 1$, and so $\frac{d}{dk}\left(\frac{u(q_c^i)-q_c^i}{k}\right) < 0$. Hence, the right-hand side of (45) is increasing in k. As $k \to 0$, $\frac{[1-(1-\delta)\beta]k}{u(q_c^i)-q_c^i} \to \frac{[1-(1-\delta)\beta]}{[u'(0)-1](1-\delta)\beta}$ (from L'Hopital's rule). Then, (45) becomes $(1-m)[u'(0)-1] > \frac{[1-(1-\delta)\beta]}{(1-\delta)\beta} = r'$, which gives (34).

(ii) Take as given the solution $n \in (0, 1-m)$ to (32). We start by establishing that a solution $\omega_1 > \omega_c$ to (33) exists. From the comparison of (15) and (26)-(27),

$$u(q_b^u) - q_b^u \leq u(q_b^i) - q_b^i,$$

with a strict inequality when $\omega_1 > \omega_c$. (To see this, recall from Lemma 4 that the incentive-compatibility condition (27) is binding.) At $\omega_1 = \omega_c$, $q_b^u = q_c^u = q_c^i$ (from (29)) and the right-hand side of (33) is

$$(1-m-n)\left[u(q_c^i) - q_c^i\right] = [1-(1-\delta)\beta]\, k > (1-\beta)(1-\delta)k = r\omega_c,$$

where we have used (32) to obtain the first equality, the assumption $\delta > 0$ to get the inequality, and $\omega_c = \beta(1-\delta)k$ to obtain the last equality. Consider next $\omega_1 = \tilde{\omega}_1 > \omega_c$ where $\tilde{\omega}_1$ is the unique solution to $r\tilde{\omega}_1 = (1-m-n)\left[u(q_b^i) - q_b^i\right]$ with $q_b^i = \min[q^*, \tilde{\omega}_1]$. In order to establish that $\tilde{\omega}_1 > \omega_c$, notice from (31) that ω_c solves $\left(\frac{r+\delta}{1-\delta}\right)\omega_c = (1-m-n)\left[u(q_c) - q_c\right]$ where $q_c = \min[q^*, \omega_c]$. Since $\delta > 0$ the result is immediate. Then,

$$\begin{aligned} & (1-m-n)\left\{(1-\eta)\left[u(q_b^u) - q_b^u\right] + \eta\left[u(q_b^i) - q_b^i\right]\right\} \\ < \quad & (1-m-n)\left[u(q_b^i) - q_b^i\right] = r\tilde{\omega}_1. \end{aligned}$$

By the Intermediate Value Theorem, there is a $\omega_1 \in (\omega_c, \tilde{\omega}_1)$ that solves (33).

In order to establish uniqueness, rewrite (33) as

$$r\omega_1 - (1-m-n)\eta\left[u(q_b^i) - q_b^i)\right] = (1-m-n)(1-\eta)\left[u(q_b^u) - q_b^u\right].$$

The left-hand side (LHS) is convex in ω_1 (since $u(q_b^i) - q_b^i$ is concave), it is equal to 0 at $\omega_1 = 0$, it reaches a negative minimum at $\omega_1 = \bar{q}$ where $\bar{q}$ solves

$$u'(\bar{q}) = 1 + \frac{r}{(1-m-n)\eta},$$

and it is equal to $r\omega_1 - (1-m-n)\eta\left[u(q^*) - q^*\right]$ for all $\omega_1 \geq q^*$. Hence, LHS is increasing for all $\omega_1 > \bar{q}$ and it becomes positive for sufficiently large ω_1. The right-hand side is decreasing in ω_1.

To see this, differentiate (29) to get

$$\frac{\partial q_b^u}{\partial \omega_1} = \frac{-\omega_c q_b^u}{(\omega_1)^2 \left[u'(q_b^u) - \frac{\omega_c}{\omega_1}\right]} < 0$$

for all $\omega_1 > \omega_c$. (Recall from Lemma 4 that $q_b^u < q_c^u \leq q^*$ so that the denominator of the previous expression is positive.) Consequently, the solution $\omega_1 \in (\omega_c, \tilde{\omega}_1)$ to (33) is unique.

(iii) Suppose there is an equilibrium with $\omega_1 \leq \omega_c$. Following a similar reasoning as in the text, ω_1 and ω_c solve

$$r\omega_1 = (1-m-n)\left[u(q_b^i) - q_b^i\right] \tag{46}$$

$$\left(\frac{r+\delta}{1-\delta}\right)\omega_c = (1-m-n)\left\{(1-\eta)\left[u(q_c^u) - q_c^u\right] + \eta\left[u(q_c^i) - q_c^i\right]\right\}, \tag{47}$$

where, from Lemma 4, q_c^u is the smallest solution to

$$u(q_c^u) - \frac{\omega_1}{\omega_c} q_c^u = u(q_b^i) - q_b^i, \tag{48}$$

and $q_c^u \leq q_b^i \leq q_c^i$. Consequently, $u(q_c^u) - q_c^u \leq u(q_c^i) - q_c^i$, and hence, from (47), $\left(\frac{r+\delta}{1-\delta}\right)\omega_c \leq (1-m-n)\left[u(q_c^i) - q_c^i\right]$. From (46), $\omega_1 \leq \omega_c$ implies $r\omega_c \geq (1-m-n)\left[u(q_c^i) - q_c^i\right]$. Then,

$$\left(\frac{r+\delta}{1-\delta}\right)\omega_c \leq (1-m-n)\left[u(q_c^i) - q_c^i\right] \leq r\omega_c.$$

A contradiction whenever $\delta > 0$.■

A.10. Proof of Proposition 6

(i) Change in m. The right-hand side of (32) is independent of n and m. Hence, (32) determines a unique fraction of buyers, $n+m$. Consequently, $\partial(n+m)/\partial m = 0$ and $\partial n/\partial m = -1$. Since $\frac{\partial(n+m)}{\partial m} = 0$ then, from (33), $\frac{\partial \omega_1}{\partial m} = 0$.

(ii) Change in k. If $q^* \leq \beta(1-\delta)k$ then $q_c^i = q^*$ is independent of k and hence the right-hand side of (32) is increasing in k. Consider next the case $q^* > \beta(1-\delta)k$. Using the proof of *Proposition* 5, we know $\left(\frac{u(q_c^i)-q_c^i}{k}\right)' < 0$. The right-hand side of (32) is thus increasing in k. Hence, $\frac{\partial n}{\partial k} < 0$.

Let RHS denote the right-hand side of (33). Then, differentiating (33)

$$\frac{\partial \omega_1}{\partial k} = \frac{-r\omega_1}{\left(r - \frac{\partial RHS}{\partial \omega_1}\right)(1-m-n)} \frac{\partial n}{\partial k},$$

where $\frac{\partial RHS}{\partial \omega_1}$ is evaluated at the equilibrium. From the proof of Proposition 5, and the fact that there exists a unique $\omega_1 > 0$ solution to (33), $r - \frac{\partial RHS}{\partial \omega_1} > 0$ and $\frac{\partial \omega_1}{\partial k} > 0$.

(iii) Change in η. From (32), $\partial n/\partial \eta = 0$. From (33),

$$\frac{\partial \omega_1}{\partial \eta} = \frac{(1-m-n)\left\{\left[u(q_b^i) - q_b^i\right] - \left[u(q_b^u) - q_b^u)\right]\right\}}{r - \frac{\partial RHS}{\partial \omega_1}} > 0,$$

where RHS is the right-hand side of (33) and $\frac{\partial RHS}{\partial \omega_1}$ is evaluated at the equilibrium.

(vi) Change in δ. The numerator on the right-hand side of (32) is increasing in δ while the denominator is decreasing in δ, since $q_c^i = \beta(1-\delta)k$. Hence, $\frac{\partial n}{\partial \delta} < 0$. Following a similar reasoning as in (ii), $\frac{\partial \omega_1}{\partial \delta} > 0$.■

A.11. Proof of Proposition 7

(i) From Proposition 6 (i) and (35),

$$\frac{dW}{dm} = \beta^{-1}(\omega_1 - \omega_c) > 0.$$

(ii) From Proposition 6 (iii), $\frac{\partial n}{\partial \eta} = 0$ and $\frac{\partial \omega_1}{\partial \eta} > 0$. Hence, W increases with η.■

A.12. Proof of Proposition 8

The conditions are derived directly from (45) and the proof of Proposition 5 by taking $\delta \to 0$. Next we show $\omega_1 \to \omega_c$ as $\delta \to 0$. We have shown $\omega_1 \geq \omega_c$ when $\delta > 0$ in the proof of Proposition 5. Hence, it suffices to show that the inequality is not strict when $\delta \to 0$. Suppose not; $\omega_1 > \omega_c$ as $\delta \to 0$. Then ω_c and ω_1 solve

$$r\omega_c = (1-m-n)\left[u(q_c^i) - q_c^i\right] \quad (49)$$

$$r\omega_1 = (1-m-n)\left\{(1-\eta)\left[u(q_b^u) - q_b^u\right] + \eta\left[u(q_b^i) - q_b^i\right]\right\}, \quad (50)$$

respectively, as $\delta \to 0$. Then, $q_c^i = \min[q^*, \omega_c]$. From Lemma 4, $\omega_1 > \omega_c$ implies $q_b^u < q_c^i < q_b^i$. Consequently, $u(q_b^u) - q_b^u < u(q_b^i) - q_b^i$, and hence, from (50), $r\omega_1 < (1-m-n)\left[u(q_b^i) - q_b^i\right]$. From (49) $\omega_1 > \omega_c$ implies $r\omega_1 > (1-m-n)\left[u(q_b^i) - q_b^i\right]$. But then,

$$r\omega_1 < (1-m-n)\left[u(q_b^i) - q_b^i\right] < r\omega_1,$$

a contradiction. Hence, $\omega_1 \to \omega_c$ as $\delta \to 0$.■

A.13. Proof of Lemma 5

Consider the problem of the seller if we do not impose the contracts offered by the seller to be ex-post individually rational. Suppose that the seller chooses a menu of contracts $\{(q_g, p_g), (q_c, p_c)\}$, where (q_g, p_g) is the contract intended for genuine buyers and (q_c, p_c) is intended for counterfeiters, to maximize his expected surplus. The problem of the seller is thus

$$\max_{(q_g,p_g),(q_c,p_c)} \left[\frac{n}{n+m}(-q_c) + \frac{m}{n+m}(p_g\omega_1 - q_g) \right] \tag{51}$$

$$\text{s.t. } u(q_g) - p_g\omega_1 \geq 0, \tag{52}$$

$$u(q_g) - p_g\omega_1 \geq u(q_c) - p_c\omega_1, \tag{53}$$

$$q_c \geq q_g. \tag{54}$$

According to (51) the expected payoff of the seller is his production cost $-q_c$ if he trades with a counterfeiter, and the transfer of genuine money minus his production cost, $p_g\omega_1 - q_g$, if he trades with a genuine buyer. The constraint (52) indicates that the genuine buyer must obtain a positive surplus, while (53) requires that the genuine buyer prefers the contract that is intended to him. An incentive-compatibility constraint similar to (53) also holds for the counterfeiters, which reduces to (54), since counterfeits are not valued across dates. According to (54), a counterfeiter chooses the contract with the highest output. First, the constraint $q_c \geq q_g$ is binding. The proof is by contradiction. Suppose $q_c > q_g$. Then the seller could reduce q_c which would increase his payoff (see (51)) and would relax the incentive-compatibility condition (53) for genuine buyers. Second, given $q_c = q_g$ the seller can choose $p_c = p_g$, as p_c does not affect seller's objective function (51), to guarantee that the incentive-compatibility condition is satisfied. The seller's problem becomes

$$\max_{(q_g,p_g)} \left[\frac{m}{n+m} p_g\omega_1 - q_g \right] \tag{55}$$

$$\text{s.t. } u(q_g) - p_g\omega_1 \geq 0. \tag{56}$$

This pooling contract is ex-post individually rational since it does not reveal the type of the buyer. The menus $\{(q_g, p_g), (q_c, p_c)\}$ such that (q_g, p_g) solves (55)-(56), $q_c = q_g$ and $p_c > p_g$ would be payoff equivalent but would not be ex-post individually rational since a seller would not want to trade with a buyer choosing the contract (q_c, p_c).■

A.14. Proof of Lemma 6

The proof is by contradiction. Suppose there is an equilibrium such that $(q_b^u, p_b^u) \neq (0,0)$. By incentive-compatibility, a genuine buyer cannot make an active offer (with positive consumption) that is different from the one of a counterfeiter, i.e., $(q_b^u, p_b^u) = (q_c^u, p_c^u)$. By a reasoning similar to the one in the proof of Proposition 1, Part (i), or in Lemma 3, genuine buyers could make an alternative offer (q', p') such that $q' < q_c^u$, $p' < p_c^u$, $u(q') - p'\omega_1 > u(q_b^u) - p_b^u\omega_1$ and $p'\omega_1 - q' \geq 0$. Such an offer disqualifies the proposed pooling outcome according to our forward-induction argument. A belief system that sustains $(q_b^u, p_b^u) = (0,0)$ as a solution to (1) is $\lambda(q,p) = 0$ for all (q,p) such that $q > 0$.■

Appendix B. Non-counterfeiting monetary equilibrium under partial confiscation ($\delta < 1$)

In Section 6 we focused on equilibria with $n > 0$. We construct non-counterfeiting monetary equilibria when (45) does not hold. We adopt a belief system that is in the similar spirit as the one used in Sections 4 and 5. Taking ω_c as given,

$$\lambda(q,p) = \left\{ \begin{array}{c} 1 \\ 0 \end{array} \right. \text{ if } (1-m)\left\{\eta\left[u(q_c^i) - q_c^i\right] + (1-\eta)\left[u(q) - p\omega_c\right]\right\} + \omega_c \left\{ \begin{array}{c} \leq \\ > \end{array} \right. k \tag{57}$$

According to (57), an offer such that the sum of the expected trade surplus and the discounted value of a counterfeit is no greater than the cost of producing the counterfeit should not be attributed to a counterfeiter. All other offers are attributed to a buyer holding a counterfeit. We show below that such beliefs can be justified by a forward-induction argument when sellers get no surplus from trade.

Lemma 7 *Assume*

$$(1-m)\left[u(q_c^i) - q_c^i\right] + \omega_c \leq k. \tag{58}$$

Given the belief system specified in (57), the offer made by a genuine buyer in an uninformed match solves

$$(q_b^u, p_b^u) = \arg\max_{q,p\leq 1}\left[u(q) - p\omega_1\right] \tag{59}$$

$$s.t. \quad -q + p\omega_1 \geq 0 \tag{60}$$

$$(1-m)\left\{\eta\left[u(q_c^i) - q_c^i\right] + (1-\eta)\left[u(q) - p\omega_c\right]\right\} + \omega_c \leq k \tag{61}$$

Proof. Suppose an offer (q,p) is made that violates (61). From (58), $u(q) - p\omega_c > u(q_c^i) - q_c^i$ and hence, from (20), $-q + p\omega_c < 0$. From (57), $\lambda(q,p) = 0$ and hence (q,p) is rejected. If (q,p) satisfies (61) then $\lambda(q,p) = 1$. Hence, the seller's acceptance condition, $-q + p\left\{\lambda(q,p)\omega_1 + \left[1 - \lambda(q,p)\right]\omega_c\right\} \geq 0$, is equivalent to (60), which completes the proof. ■

Provided that $\omega_1 > \omega_c$ then (60) binds and $v_0 = 0$.[28] The Bellman equation for the value of holding a unit of genuine money is given by (33). The value function of a counterfeiter solves

$$(r+\delta)v_c = (1-m)\left\{\eta\left[u(q_c^i) - q_c^i\right] + (1-\eta)\max_{q,p\leq 1}\left[u(q) - p(1-\delta)v_c\right]\mathbb{I}_{\{(q,p)\in\mathcal{A}^u\}}\right\}, \tag{62}$$

[28] If (60) does not bind, then (61) binds; otherwise, $q_b^u = q_b^i$ and (60) binds. Then, the solution to (59)-(61) is also the solution to $\max_p p(\omega_c - \omega_1)$, which gives $p = 0$, and $q > 0$ solves (61) at equality. This solution violates (60).

where $\mathcal{A}^u$ is the set of offers accepted in uninformed matches.

Equilibria where (61) binds. From (61) and (62), $(r+\delta)\upsilon_c = k-(1-\delta)\upsilon_c$, and hence $\upsilon_c = \beta k$. From (33) at $n=0$, the value of genuine money solves

$$r\upsilon_1 - (1-m)(1-\eta)\left[u(q_b^u) - q_b^u\right] = (1-m)\eta\left[u(q_b^i) - q_b^i\right]. \quad (63)$$

We next show that there is a unique positive solution to (63). The right-hand side of (63) is strictly concave in υ_1 for all $\upsilon_1 < q^*$ and constant for all $\upsilon_1 \geq q^*$ (since $q_b^i = \min(q^*, \upsilon_1)$). Provided that (61) does not bind, the left-hand side of (63) is convex in υ_1 (since $q_b^u = q_b^i = \min(q^*, \upsilon_1)$). If (61) binds then $\upsilon_1 > \omega_c$.[29] Moreover, (q_b^u, p_b^u) is such that (60) and (61) hold at equality, i.e., q_b^u solves

$$(1-m)\left\{\eta\left[u(q_c^i) - q_c^i\right] + (1-\eta)\left[u(q_b^u) - q_b^u\frac{(1-\delta)\upsilon_c}{\upsilon_1}\right]\right\} = k\left(\frac{r+\delta}{1+r}\right), \quad (64)$$

where we take as given $\upsilon_c = \beta k$ and q_c^i. From (64), $\frac{\partial q_b^u}{\partial \upsilon_1} < 0$ (since the solution to (59)-(61) is such that $u'(q_b^u) - \frac{(1-\delta)\upsilon_c}{\upsilon_1} > 0$) and the left-hand side of (63) is increasing.[30] From the discussion above, provided that $r < (1-m)\left[u'(0) - 1\right]$, then there is a unique $\upsilon_1 > 0$ solution to (63). Denote υ_1^* the positive solution to $r\upsilon_1 = (1-m)\left[u(q_b^i) - q_b^i\right]$. The constraint (61) is binding if

$$(1-m)\left\{\eta\left[u(q_c^i) - q_c^i\right] + (1-\eta)\left[u(q_b^{i*}) - p_b^{i*}\left(\frac{1-\delta}{1+r}\right)k\right]\right\} > k\left(\frac{r+\delta}{1+r}\right), \quad (65)$$

where $q_b^{i*} = \min(q^*, \upsilon_1)$ and $p_b^{i*} = q_b^{i*}/\upsilon_1^*$. For an equilibrium, we also need to check that (58) holds, which requires

$$(1-m)\left[u(q_c^i) - q_c^i\right] \leq k\left(\frac{r+\delta}{1+r}\right). \quad (66)$$

This is the complement of (45).

[29] Suppose $\omega_1 \leq \omega_c$. Then, $u(q_b^i) - p_b^i\omega_c \leq u(q_b^i) - p_b^i\omega_1 \leq u(q_b^i) - q_b^i \leq u(q_c^i) - q_c^i$. From (58) it is in contradiction with the assumption that (61) binds.

[30] The left-hand side of (64) is strictly concave, hump-shaped function of q_b^u. Moreover, at $q_b^u = 0$ it is equal to $(1-m)\eta\left[u(q_c^i) - q_c^i\right]$ which, from (58), is smaller than the right-hand side of (64). So, in general, there is either 0 or two values of q_b^u that solve (64). If one substitutes $u(q)$ by its expression given by (61) into (59) one obtains

$$\frac{\left(\frac{r+\delta}{1+r}\right)k - (1-m)\eta\left[u(q_c^i) - q_c^i\right]}{(1-m)(1-\eta)} + p\left(\omega_c - \omega_1\right).$$

The payoff of the genuine buyer is maximized for the lowest value of p and hence the lowest value of q (from (60)). Since the left-hand side of (64) is maximum when $u'(q_b^u) - \frac{(1-\delta)\upsilon_c}{\upsilon_1} = 0$ then the solution must be such that $u'(q_b^u) - \frac{(1-\delta)\upsilon_c}{\upsilon_1} \geq 0$. To show that $q_b^u \leq q^*$ notice that otherwise the genuine buyer could lower q_b^u to q^* and p_b^u to q^*/υ_1 so as to increase his expected payoff while keeping the payoff of the seller equal to 0. The constraint (61) would still be satisfied since $u(q_b^u) - q_b^u\frac{(1-\delta)\upsilon_c}{\upsilon_1}$ is increasing in q_b^u when $u'(q_b^u) - \frac{(1-\delta)\upsilon_c}{\upsilon_1} \geq 0$ holds.

Equilibria where (61) does not bind. From (59)-(61), $(q_b^u, p_b^u) = (q_b^i, p_b^i)$ and $\upsilon_1 = \upsilon_1^*$. The constraint (61) does not bind if

$$(1-m)\left\{\eta\left[u(q_c^i) - q_c^i\right] + (1-\eta)\left[u(q_b^{i*}) - p_b^{i*}\omega_c\right]\right\} + \omega_c \leq k. \tag{67}$$

Since $u(q_b^i) - p_b^i\omega_c \geq u(q_b^i) - p_b^i\omega_1 = u(q_b^i) - q_b^i \geq u(q_c^i) - q_c^i$, it is immediate that (58) holds.

Appendix C. Sellers have some bargaining power ($\theta < 1$) and all matches are uninformed ($\eta = 0$)

We show that if sellers are uninformed in all matches then counterfeiting cannot be sustained in equilibrium.

Proposition 9 *Assume $\eta = 0$ and $\theta < 1$. Then, $n = 0$ in any equilibrium.*

Proof. The proof is by contradiction. Suppose $n > 0$. By the proof of Lemma 6, the only possible trade in matches where genuine buyers make the offer is $(q, p) = (0, 0)$. But then genuine money has no value, $\upsilon_1 = 0$, and $n = 0$. A contradiction. ■

The intuition for why counterfeiting cannot occur in equilibrium is as follows. If there were counterfeiting, then the only possible outcome in matches where buyers make offers that is consistent with our refinement is no trade, since genuine buyers would always deviate from a pooling offer. Furthermore, since genuine buyers get no surplus in matches where sellers make offers fiat money cannot be valued.

In an equilibrium with no counterfeiting, terms of trade when buyers make offers solve

$$(q_b^u, p_b^u) = \arg\max_{q,p} \left[u(q) - p(\upsilon_1 - \upsilon_0)\right] \tag{68}$$

$$\text{s.t.} \quad -q + p(\upsilon_1 - \upsilon_0) \geq 0 \tag{69}$$

$$-k + (1 - m)\left[\theta u(q) + (1 - \theta)u(q_s^u)\right] \leq 0. \tag{70}$$

The belief system is $\lambda(q, p) = 1$ if (70) holds and $\lambda(q, p) = 0$ otherwise. Offers in uninformed matches that do not generate an expected utility that is large enough to cover the entry cost into counterfeiting are attributed to genuine buyers. All other offers are assigned to counterfeiters.

The value functions for a genuine buyer and an agent without money solve the following flow Bellman equations:

$$r\upsilon_1 = (1 - m)\theta\left[u(q_b^u) - q_b^u\right], \tag{71}$$

$$r\upsilon_0 = m(1 - \theta)\left[u(q_s^u) - q_s^u\right]. \tag{72}$$

Proposition 10 *Assume $\eta = 0$ and $\theta < 1$. There exists a monetary equilibrium with $n = 0$ iff*

$$r < \left[u'(0) - 1\right]\left\{(1 - m)\theta - \frac{m(1 - \theta)}{u'(0)}\right\}. \tag{73}$$

Proof. Let $\omega_1 = v_1 - v_0$ and denote $\Gamma(\omega_1) \equiv (1-m)\theta\left[u(q_b^u) - q_b^u\right] - m(1-\theta)\left[u(q_s^u) - q_s^u\right]$. From (71) and (72) ω_1 solves $r\omega_1 = \Gamma(\omega_1)$. For all $\omega_1 \leq q^*$ and such that $-k+(1-m)\left[\theta u(\omega_1) + (1-\theta)\omega_1\right] \leq 0$ then $q_b^u = \omega_1$, $q_s^u = u^{-1}(\omega_1)$ and $\Gamma(\omega_1) \equiv (1-m)\theta\left[u(\omega_1) - \omega_1\right] - m(1-\theta)\left[\omega_1 - u^{-1}(\omega_1)\right]$. Thus, $\Gamma'(0) \equiv \left[u'(0) - 1\right]\left\{(1-m)\theta - \frac{m(1-\theta)}{u'(0)}\right\}$. Moreover, for all $\omega_1 \geq u(q^*)$, $u(q_s^u) - q_s^u = u(q^*) - q^*$ is constant and $\Gamma(\omega_1) > 0$ is constant as well. So, by continuity of $\Gamma(\omega_1)$, the existence of a monetary equilibrium requires $r < \Gamma'(0)$. ■

Appendix D. Accounting for the differences with NW

According to Problem 1 in NW sellers' beliefs are such that $\lambda(q,p) = \frac{m}{m+n}$ for all (q,p) in uninformed matches; i.e., an offer is attributed to a genuine buyer with a probability equal to the fraction of genuine buyers among all buyers. This belief is more restrictive than what is warranted by the intuitive criterion, and this is why NW cannot uncover all the monetary equilibria. In a non-counterfeiting equilibrium ($n = 0$), this belief system becomes $\lambda(q,p) = 1$ for all (q,p). Under this assumption, the genuine buyer offers

$$(q_b, p_b) = \arg \max_{q,p \in [0,1]} [u(q) - p\upsilon_1] \quad \text{s.t.} \quad -q + p\upsilon_1 = 0$$

in all matches.

NW assume that genuine buyers and counterfeiters make the same offer in uninformed matches (See definition of C in Eq. (3)). This assumption is not consistent with the assumed belief system. Instead, under the belief system $\lambda \equiv 1$ an agent who deviates and produces a counterfeit offers

$$(q_c, p_c) = \arg \max_{q,p \in [0,1]} u(q) \quad \text{s.t.} \quad -q + p\upsilon_1 = 0$$

in an uninformed match. Then, $(q_c, p_c) \neq (q_b, p_b)$ whenever $\upsilon_1 > q^*$ since in that case $(q_b, p_b) = (q^*, \frac{q^*}{\upsilon_1})$ and $(q_c, p_c) = (\upsilon_1, 1)$. The value of genuine money solves

$$r\upsilon_1 = (1-m)\left[u(q_b) - q_b\right],$$

where $q_b = \min[q^*, \upsilon_1]$. There is a $\upsilon_1 > 0$ solution to the above equation, and it is unique, if and only if $r < (1-m)\left[u'(0) - 1\right]$. The condition for $n = 0$ requires

$$-k + (1-m)u(q_c) \leq 0,$$

where $q_c = \upsilon_1$. This condition differs from the one in NW since $q_c \neq q_b$ when $\upsilon_1 > q^*$. So the condition for the existence of a monetary equilibrium derived in NW under the belief system $\lambda \equiv 1$ is incorrect, and the restriction on beliefs is not implied by the Intuitive Criterion.

Appendix E. A model of counterfeiting with divisible money

We develop a simple model of counterfeiting with divisible money based on a version of Lagos and Wright (2005). By using a very natural refinement for signaling games with endogenous types developed from In and Wright (2008), we obtain insights similar to the main results in the basic model of the paper. Moreover, since agents can accumulate money in the model with divisible money, it allows us to distinguish the fixed cost from producing counterfeits and the marginal cost. We set the marginal cost at zero for simplicity. Also, the model can be used to study how inflation affects the threat of counterfeiting.

Environment

Time is discrete, starts at $t = 0$, and continues forever. Each period has two subperiods, a morning (AM) followed by an afternoon (PM), where different activities take place. There is a continuum of agents divided into two types, called *buyers* and *sellers*, who differ in terms of when they produce and consume. The labels *buyers* and *sellers* indicate agents' roles in the PM market. There are two consumption goods, one produced in the AM and the other in the PM. Consumption goods are perishable.

Agents live for three subperiods. Buyers and sellers from generation t are born at the beginning of period t, and they die at the end of the AM in period $t + 1$. (See Figure 3.) Let $\mathcal{B}_t$ denote the set of buyers from generation t, $\mathcal{S}_t$ the set of sellers from generation t, and $\mathcal{J}_t = \mathcal{B}_t \cup \mathcal{S}_t$.[31] The measures of buyers and sellers are normalized to 1.

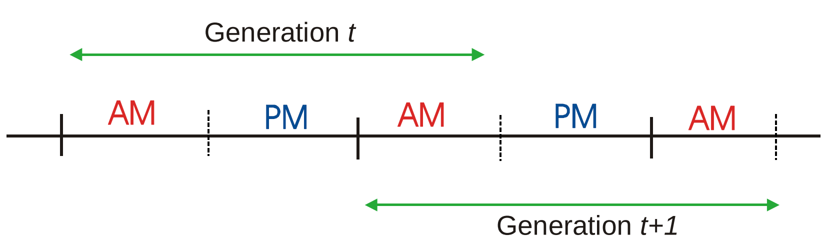

Figure 3: Overlapping generations structure

[31] This overlapping-generations structure facilitates the presentation of the model. For a related environment, see Zhu (2006) and Zhu and Wallace (2007).

Buyers produce in the first AM of their lives while sellers produce in the PM. This heterogeneity will generate a temporal double-coincidence problem (see Rocheteau and Wright 2005). The utility of a buyer born at date t is

$$U_t^b = -\ell_t + u(q_t) + \beta x_{t+1}, \tag{74}$$

where x_t is the AM consumption of period t, ℓ_t is the AM disutility of work, q_t is the PM consumption, and $\beta \in (0,1)$ is a discount factor. The utility function $u(q)$ is twice continuously differentiable, $u(0) = 0$, $u'(0) = \infty$, $u'(q) > 0$, and $u''(q) < 0$. Buyers can produce the AM-good according to a linear production technology, $y_t = \ell_t$. Buyers' endowment of labor is unlimited when young.

The utility of a seller born at date t is

$$U_t^s = -c(q_t) + \beta x_{t+1}, \tag{75}$$

where q_t is the PM production. The cost function $c(q)$ is twice continuously differentiable, $c(0) = c'(0) = 0$, $c'(q) > 0$, $c''(q) \geq 0$ and $c(q) = u(q)$ for some $q > 0$. Let q^* denote the solution to $u'(q^*) = c'(q^*)$.

Fiat money is durable, perfectly divisible, and it can be held in any nonnegative amount. The quantity of genuine money per buyer in the PM of period t is denoted M_t. It grows at a constant gross rate, $\gamma \equiv M_{t+1}/M_t$, where $\gamma > \beta$. New money is injected, or withdrawn if $\gamma < 1$, by lump-sum transfers $T_t = (\gamma - 1)M_{t-1}$, or taxes if $\gamma < 1$, to the young buyers.

The market structures are as follows. In the AM, there is a competitive market where agents can trade goods and fiat money. In the PM, each seller is matched bilaterally with a buyer drawn at random from the set of all buyers. All trades in the PM are *quid pro quo*, and matched agents can transfer any nonnegative quantity of PM-output and any quantity of their asset holdings. In order to guarantee that there is an essential role for a medium of exchange, we assume there is no public record of individuals' trading histories and agents cannot commit to future actions. Terms of trade in the PM are determined according to a simple bargaining game: The buyer makes an offer that the seller accepts or rejects. If the offer is accepted then the trade is implemented. At the end of the PM, agent pairs split apart.

We introduce counterfeiting of currency as follows. Instead of producing AM goods, buyers can choose to counterfeit fiat money. Production and counterfeiting of currency are two mutually exclusive activities. Given this setup and the overlapping generations structure, a buyer cannot accumulate both counterfeits and genuine money. There is a fixed cost k from engaging in counterfeiting, but the marginal cost from producing a counterfeit is 0. In the AM centralized market, counterfeits are recognized with probability one and they are automatically confiscated by the government. In the PM decentralized market a seller is not able to recognize the authenticity of money, and he does not observe the money holdings of the buyer.[32]

Equilibrium

In each period the following game is played (see Figure 4). At the beginning of the game, each buyer chooses to become a genuine buyer ($\chi = g$) or a counterfeiter ($\chi = c$). Buyers choose the amount of real balances (expressed in terms of AM good) to accumulate (z). Then, in the subsequent subperiod the buyer makes an offer (q, d) to the seller, where q is production/consumption of the PM good and $d \leq z$ is the transfer of real balances. The only action of the seller is to accept ($a = Y$) or reject ($a = N$) an offer. The payoff of the buyer and the seller are

$$\begin{aligned} U^b(\chi, z, q, d, a) &= -k\mathbb{I}_{\{\chi=c\}} - z\mathbb{I}_{\{\chi=g\}} + \left\{u(q) - \frac{\beta}{\gamma}d\mathbb{I}_{\{\chi=g\}}\right\}\mathbb{I}_{\{a=Y\}} + \frac{\beta}{\gamma}z\mathbb{I}_{\{\chi=g\}}, \\ U^s(\chi, q, d, a) &= \left\{-c(q) + \frac{\beta}{\gamma}d\mathbb{I}_{\{\chi=g\}}\right\}\mathbb{I}_{\{a=Y\}} \end{aligned}$$

where $\mathbb{I}_{\{\chi=c\}}$ is an indicator function that equals one if $\chi = c$ and where $a \in \{Y, N\}$ is the seller's response. The value of the money received in the PM is discounted at rate β, since it can only be spent in the next AM market, and it is scaled down by the gross inflation rate γ.

A strategy for a buyer is composed of the choice $\chi \in \{c, g\}$ of whether to counterfeit or not; the quantity of money z to accumulate as a function of the type χ; the offer (q, d) to make as a function of (χ, z). A strategy for the seller is a decision to accept or reject an offer, $a \in \{Y, N\}$, for each possible offer. In order to choose his action, the seller must form a belief about the buyer's type following every possible offer.

The following lemma establishes that a buyer who accumulates genuine money will never hold more than what he intends to spend in the bilateral match.

[32] As it will be clear later, the assumption that the seller does not observe the money holdings of the buyer is inessential for our results.

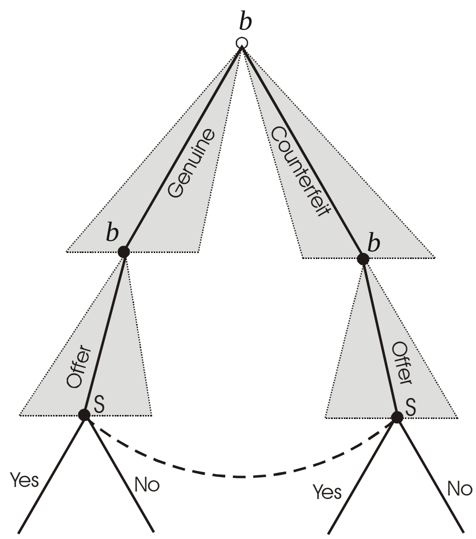

Figure 4: Original game

Lemma 8 *Any strategy such that $\chi = g$ and $d < z$ is strictly dominated.*

Proof. The buyer's payoff if he accumulates z units of real balances is

$$-z\left(1-\frac{\beta}{\gamma}\right)+\left\{u(q)-\frac{\beta}{\gamma}d\right\}\mathbb{I}_{\{a=Y\}}.$$

The seller's decision to accept or reject an offer is not conditional on the (unobserved) money holdings of the buyer. If $z > d$ then the buyer can increase his payoff irrespective of the seller's response by reducing his real balances z (since $\gamma > \beta$). ■

We use the previous lemma to restrict the set of strategies for genuine buyers to offers of the form (q, z). Since money balances are not observed, we adopt the same restriction for counterfeiters.

For an equilibrium, we need to specify the seller's belief following an offer (q, z). Sequential equilibrium imposes little discipline on those beliefs, which can lead to a plethora of equilibria. In order to obtain a tight characterization of the equilibrium, we follow the methodology in In and Wright (2008).[33] We consider the reverse-ordered game where the observed actions are chosen before the unobserved ones: the buyer makes first an offer (q, z) and then chooses whether to

[33] The methodology of In and Wright (2008) is based on the invariance condition of strategic stability from Kohlberg and Mertens (1986). It states that a solution of a game should also be the solution of any game with the same reduced normal form.

produce a counterfeit or to accumulate genuine money. (See Figure 5.) The order of the moves should not matter since it does not affect the payoffs and it does not convey any information. The original game and the reverse-ordered game have the same reduced strategic form. In this reverse-ordered game, a strategy for the buyer is an offer (q, z) and a decision $\chi : \mathbb{R}_{2+} \to \{c, g\}$ of whether to produce counterfeits or accumulate real balances as a function of the offer made at the beginning of the game. The belief of the seller will have to be consistent with the buyer's strategy in every subgame following an offer.

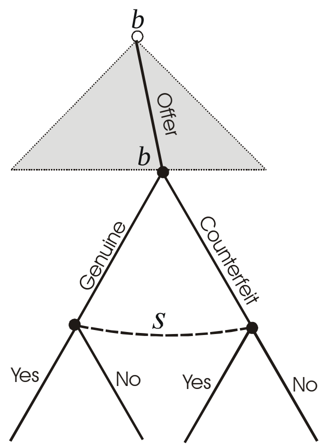

Figure 5: The reverse ordered game

Consider a subgame where the offer (q, z) has been made. Let $p \in [0, 1]$ denote the measure of sellers who accept the offer (q, z) and $\theta \in [0, 1]$ the measure of buyers who choose to accumulate genuine money. The seller's belief that he faces a genuine buyer following an offer (q, z) is denoted $\lambda(q, z)$. The consistency of sellers' beliefs with strategies implies $\lambda(q, z) = \theta$. Given θ, the decision of sellers to accept or reject an offer satisfies

$$-c(q) + \theta \frac{\beta}{\gamma} z \begin{array}{c} > 0 \\ < 0 \\ = 0 \end{array} \Longrightarrow p \begin{array}{c} = 1 \\ = 0 \\ \in [0, 1] \end{array} \tag{76}$$

Given p, the decision of a buyer to produce counterfeits or accumulate genuine money is given by

$$-z + \frac{\beta}{\gamma} z(1 - p) \begin{array}{c} > \\ < \\ = \end{array} -k \Longrightarrow \theta \begin{array}{c} = 1 \\ = 0 \\ \in [0, 1] \end{array} \tag{77}$$

A Nash equilibrium of the subgame following an offer (q, z) is a pair $(p, \theta) \in [0,1]^2$ that satisfies (76) and (77).

We first review all the Nash equilibria of the subgame following (q, z). See also Figure 6.

(i) $(p, \theta) = (1, 1)$**.** From (76) $p = 1$ requires $-c(q) + \frac{\beta}{\gamma} z \geq 0$. From (77) $\theta = 1$ requires $k \geq z$.

(ii) $(p, \theta) = (0, 1)$**.** From (76) $p = 0$ requires $-c(q) + \frac{\beta}{\gamma} z \leq 0$. From (77) $\theta = 1$ requires $z(1 - \frac{\beta}{\gamma}) \leq k$.

(iii) $(p, \theta) = (0, 0)$**.** From (76) $p = 0$ if $\theta = 0$. From (77) $\theta = 0$ if $z(1 - \frac{\beta}{\gamma}) \geq k$.

(iv) $(p, \theta) \in (0, 1)^2$**.** From (76) $p \in (0, 1)$ if

$$\theta = \frac{\gamma c(q)}{\beta z}.$$

The condition $\theta \in (0, 1)$ implies $c(q) < \frac{\beta}{\gamma} z$. From (77) $\theta \in (0, 1)$ if

$$p = \frac{k - z\left(1 - \frac{\beta}{\gamma}\right)}{\frac{\beta}{\gamma} z}.$$

The condition $p \in (0, 1)$ implies $z \in \left(k, \frac{\gamma}{\gamma - \beta} k\right)$.

(v) $(p, \theta) \in \{1\} \times (0, 1)$**.** From (77), $k = z$. From (76) $p = 1$ if $-c(q) + \frac{\beta}{\gamma} \theta z \geq 0$.

(vi) $(p, \theta) \in \{0\} \times (0, 1)$**.** From (77), $k = \frac{\gamma - \beta}{\gamma} z$. From (76) $p = 0$ if $-c(q) + \frac{\beta}{\gamma} \theta z \leq 0$.

(vii) $(p, \theta) \in (0, 1) \times \{1\}$**.** From (76), $\frac{\beta}{\gamma} z = c(q)$. From (77), $\theta = 1$ if $z\left[1 - \frac{\beta}{\gamma}(1 - p)\right] \leq k$.

(viii) $(p, \theta) \in (0, 1) \times \{0\}$**.** From (76), $q = 0$. From (77), $\theta = 0$ if $z\left[1 - \frac{\beta}{\gamma}(1 - p)\right] \geq k$.

In order to characterize the equilibrium for the whole game, we move backward in the game tree and we analyze the buyer's choice of which offer to make. The offer made by the buyer solves

$$(q, z) = \arg\max \left\{ -k\left[1 - \theta(q, z)\right] - z\left(1 - \frac{\beta}{\gamma}\right)\theta(q, z) + \left[u(q) - \frac{\beta}{\gamma} z\theta(q, z)\right] p(q, z) \right\}, \tag{78}$$

where $[\theta(q, z), p(q, z)]$ is an equilibrium of the subgame following the offer (q, z).

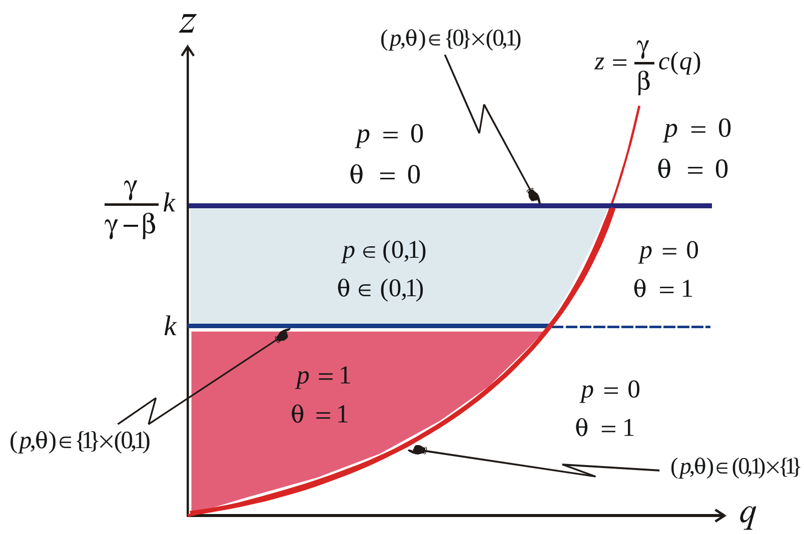

Figure 6: Equilibria of the subgame following (q, z)

Definition 3 *An equilibrium is a list* $\langle (q, z), \theta(q, z), p(q, z) \rangle$ *where, for given* (q, z), (θ, p) *solves (76)-(77) and* (q, z) *solves (78).*

Proposition 11 *There is a unique equilibrium and it is such that* $\theta = 1$ *and*

$$(q, z) = \arg\max \left\{ -\left(1 - \frac{\beta}{\gamma}\right) z + \left[u(q) - \frac{\beta}{\gamma} z \right] \right\} \tag{79}$$

$$s.t. \quad -c(q) + \frac{\beta}{\gamma} z \geq 0, \tag{80}$$

$$z \leq k. \tag{81}$$

Proof. Consider an offer such that $p(q, z) = 1$ and $\theta(q, z) = 1$, which requires $-c(q) + \frac{\beta}{\gamma} z \geq 0$ and $k \geq z$. If the inequalities are strict then the Nash equilibrium to the subgame following (q, z). The supremum to the buyer's payoff, U^*, is given by (79)-(81). It is achieved provided that the equilibrium of the subgame following the solution to (79)-(81) is $(p, \theta) = (1, 1)$.

The rest of the proof consists in eliminating other potential candidates for the equilibrium offer. From (78), any offer such that $p(q, z) = 0$ generates a payoff for the buyer equal to $-k[1 - \theta(q, z)] - z\left(1 - \frac{\beta}{\gamma}\right) \theta(q, z) \leq 0 < U^*$.

Consider next an offer such that $p(q,z) \in (0,1)$ and $\theta(q,z) \in (0,1)$. The Nash equilibrium of the subgame following (q,z) is unique and the buyer's payoff is $-k + pu(q)$ where $p = \gamma\left[k - z\left(1 - \frac{\beta}{\gamma}\right)\right]/\beta z$ is decreasing with z. The supremum for the buyer's payoff among such equilibria is

$$\begin{aligned}\bar{U} &= \sup\left\{-k + \frac{\gamma}{\beta}\left[\frac{k}{z} - \left(1 - \frac{\beta}{\gamma}\right)\right]u(q) : z \in \left(k, \frac{\gamma}{\gamma-\beta}k\right) \text{ and } c(q) = \frac{\beta}{\gamma}z\right\} \\ &= \sup\left\{-k + \left[k\frac{u(q)}{c(q)} - \left(\frac{\gamma}{\beta} - 1\right)u(q)\right] : c(q) \in \left(\frac{\beta}{\gamma}k, \frac{\beta}{\gamma-\beta}k\right)\right\}.\end{aligned}$$

Since $\frac{u(q)}{c(q)}$ is decreasing in q, $\bar{U} = -k + u(q)$ where $c(q) = \frac{\beta}{\gamma}k$. The supremum is not achieved since it requires $(p,\theta) = (1,1)$.

Consider an offer such that $p(q,z) = 1$ and $\theta(q,z) \in (0,1)$. Then, the buyer's payoff is $-k+u(q)$ with $c(q) < \frac{\beta}{\gamma}k$. The supremum of the buyer's payoff among such equilibria is also equal to $\bar{U}$, and it is not achieved.

Consider an offer such that $p(q,z) \in (0,1)$ and $\theta(q,z) = 1$. Then, the buyer's payoff is $-z\left(1 - \frac{\beta}{\gamma}\right) + p\left[u(q) - c(q)\right]$ with $z\left[1 - \frac{\beta}{\gamma}(1-p)\right] \leq k$ and $\frac{\beta}{\gamma}z = c(q)$. Consider first equilibria with $z \leq k$. Then, among all those equilibria, the supremum of the buyer's payoff is given by the solution to (79)-(81) but it is not achieved since $p < 1$. Consider next equilibria with $z \in \left[k, \frac{\gamma}{\gamma-\beta}k\right]$. Then, the supremum for the buyer's payoff is

$$\max\left\{\left[k\frac{u(q)}{c(q)} - u(q)\left(\frac{\gamma}{\beta} - 1\right)\right] - k : \frac{\gamma}{\beta}c(q) \in \left[k, \frac{\gamma}{\gamma-\beta}k\right]\right\} = \bar{U}.$$

The supremum is not achieved since $p < 1$. ■

As in Nosal and Wallace (2007), there is a unique equilibrium and it involves no counterfeiting. The refinement, however, does not rely on the Intuitive Criterion. Instead, it considers an equivalent game (with the same reduced normal form) that pins down the sellers' beliefs for all offers made by the buyer.

The program that determines the equilibrium outcome is similar to the one in a model with divisible money but no counterfeiting (e.g., Lagos and Wright, 2005) except that it incorporates an additional constraint, $z \leq k$. A buyer does not accumulate more money balances than the fixed cost to produce counterfeits. The solution to (79)-(81) is

$$\frac{u'(q)}{c'(q)} \leq \frac{\gamma}{\beta}, \quad \text{“ = ” if } q > 0 \tag{82}$$

$$z = \frac{\gamma}{\beta} c(q), \tag{83}$$

if $\frac{u'\left[c^{-1}\left(\frac{\beta}{\gamma}k\right)\right]}{c'\left[c^{-1}\left(\frac{\beta}{\gamma}k\right)\right]} \leq \frac{\gamma}{\beta}$. Otherwise,

$$q = c^{-1}\left(\frac{\beta}{\gamma}k\right), \tag{84}$$

$$z = k. \tag{85}$$

In Figure 7 the equilibrium value for q is determined at the intersection of the downward-sloping curve and the horizontal curve. The equilibrium is monetary if $z > 0$ (and $q > 0$).

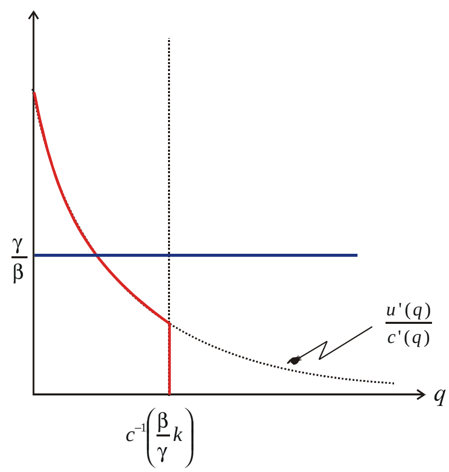

Figure 7: Determination of the equilibrium q

Proposition 12 *The equilibrium is monetary if and only if* $\frac{u'(0)}{c'(0)} > \frac{\gamma}{\beta}$.

Proof. From (82)-(85), $q = \min\left[\tilde{q}, c^{-1}\left(\frac{\beta}{\gamma}k\right)\right]$ where $\tilde{q}$ is the solution to (82). From (82), $\tilde{q} > 0$ if and only if $\frac{u'(0)}{c'(0)} > \frac{\gamma}{\beta}$. ■

The condition for a monetary equilibrium is independent of the fixed cost to produce a counterfeit, k. The threat of counterfeiting does not make the monetary equilibrium less likely to prevail. The equilibrium outcome, however, is affected by the threat of counterfeiting.

Proposition 13 *Assume* $\frac{u'(0)}{c'(0)} > \frac{\gamma}{\beta}$*. There is* $\bar{k} > 0$ *such that for all* $k < \bar{k}$*,* $\frac{\partial q}{\partial k} > 0$ *and* $\frac{\partial z}{\partial k} > 0$*.*

Proof. Direct from (84)-(85). The threshold $\bar{k}$ solves $\frac{u'\left[c^{-1}\left(\frac{\beta}{\gamma}k\right)\right]}{c'\left[c^{-1}\left(\frac{\beta}{\gamma}k\right)\right]} = \frac{\gamma}{\beta}$. ■

In order to see the effect of inflation on the threshold $\bar{k}$, assume $c(q) = q$. Then, $\bar{k}$ is decreasing with γ. So an increase in inflation makes the threat of counterfeiting less binding. However, the optimal monetary policy corresponds to the Friedman rule.

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