Precise Probabilities for Hash Collision Paths

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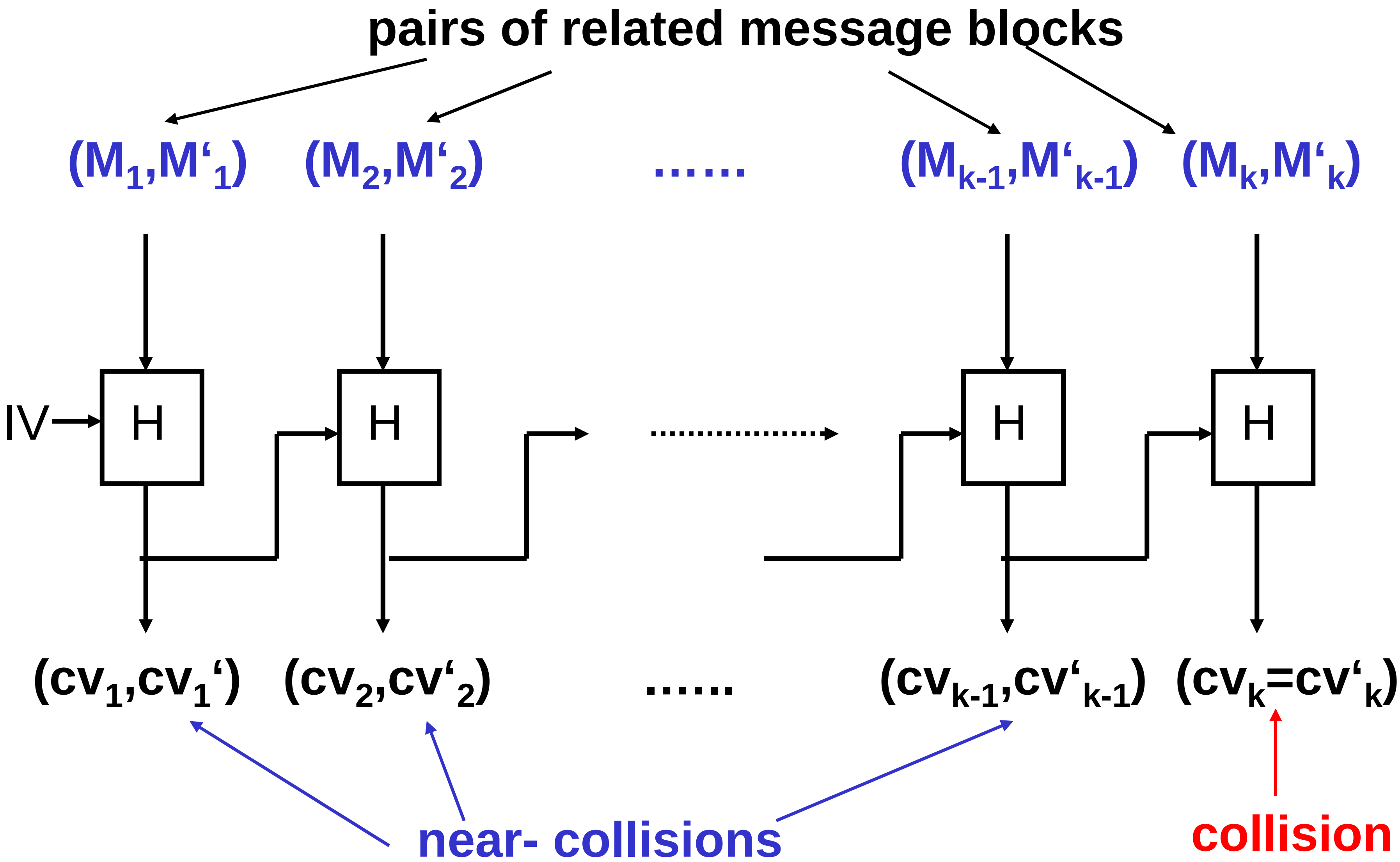
(Multiblock) Hash Collision Attack



Workload for a collision

- The pair (cv_j,cv'_j) is called a near-collision if both components are „almost“ equal, fulfilling a set of specified conditions.

- Workload = Workload (Block1) + ... + Workload (Block k)
  Consequence: The blocks may be analysed independently.

- Set of sufficient (bit) conditions *SC*
  - characterizes a (near-)collision path
  - $\rightarrow$ (near-)collision

Workload and Success Probability

- *SC = SC1* $\cup$ *SC2*
  - *SC1*: conditions can be guaranteed by message modification
  - *SC2* (conditions after message modification): fulfilled with a particular probability

- Prob(near-collision path) = Prob(all SC2-conditions are fulfilled)
  - Prob((near-)collision) $\geq$ Prob((near-) collision path)
  - $\rightarrow$ workload

The set *SC2*

- Example

  $SC2 := \{(r_{27,5}, r'_{27,5}) = (0,1), r_{34,5} = r_{33,5}, (r_{45,25}, r'_{45,25}) = (0,0), \ldots\}$

  where $r_{i,j}$ = register bit j in Step i

- „Rule of thumb“ (usually applied):

  Prob(near-collision path) =

  Prob(all cond's. of *SC2* are fulfilled) $\approx 2^{-|SC2|}$

  number of bit conditions

Goal of this contribution

- This rule of thumb provides only a rough estimate of the true probabilities.
- Deviations may be caused by various interfering effects:
  - cyclical shifts
  - addition of 32-bit words ($\rightarrow$ carry bits)
  - bit conditions on the chaining values (post addition with fixed values; bit counting is very inaccurate)

  NOTE: Specific effects have been addressed in literature (qualitatively and / or quantitatively)
- Our contribution supplies universal tools that support the systematic calculation of probabilities of (near-)collision paths.

Stochastic Model

- Step functions (examples)
  - (MD5) $r_i = r_{i-1} + (\ \Phi_i(r_{i-1},r_{i-2},r_{i-3}) + r_{i-4} + m_i + const_i\ )^{<<<s} \pmod{2^{32}}$
  - (SHA-1) $r_i = r_{i-1}^{<<<5} + \Phi_i(r_{i-2},r_{i-3},r_{i-4}) + r_{i-5} + m_i + const_i \pmod{2^{32}}$

    $r_{i-2} = r_{i-2}^{<<<30}$

- Stochastic model

  We interpret the intermediate register values $(r_1,r'_1), (r_2,r'_2),\ldots$ and the message blocks $(m_1,m'_1), (m_2,m'_2),\ldots$ as values assumed by random variables $(R_1,R'_1), (R_2,R'_2),\ldots$ and $(M_1,M'_1), (M_2,M'_2), \ldots$, respectively.

  These random variables have specific properties which depend on the hash function and the near-collision path.

Relevant Types of Probabilities

- Notation:
  - The random variables X, X', Y, Y' assume values in $Z_{2^{32}}$
  - $S_1, S_2, S_3 \subseteq Z_{2^{32}} \times Z_{2^{32}}$ denote specific subsets ($\rightarrow$ bit conditions)
  - $T_i := pr_1(S_i) \subseteq Z_{2^{32}}$ (projection onto the 1st component)

- Relevant types of conditional probabilities:
  - $\mathrm{Prob}((X,X') + (Y,Y') \pmod{2^{32}} \in S_3 \mid (X,X') \in S_1, (Y,Y') \in S_2)$
  - $\mathrm{Prob}((X,X')^{<<<s} + (Y,Y') \pmod{2^{32}} \in S_3 \mid (X,X') \in S_1, (Y,Y') \in S_2)$
  - $\mathrm{Prob}((X,X')^{<<<s} + (Y,Y') \pmod{2^{32}} \in S_3 \mid (X-X') \pmod{2^{32}} = \Delta, (Y,Y') \in S_2)$

Main results

- Under suitable assumptions the conditional probabilities from the last slide can be simplified to
  - $\mathrm{Prob}(X+Y \pmod{2^{32}} \in T_3 \mid X \in T_1, Y \in T_2) * 1_{\{0\}}(A[S_1,S_2,S_3])$
  - $\mathrm{Prob}(X^{<<<s} + Y \pmod{2^{32}} \in T_3 \mid X \in T_1, Y \in T_2) * 1_{\{0\}}(B[s,S_1,S_2,S_3])$
  - $\mathrm{Prob}(X^{<<<s} + Y \pmod{2^{32}} \in T_3 \mid X \in V[s,S_1,S_2,S_3], Y \in T_2) * \mathrm{Prob}(X \in V[s,S_1,S_2,S_3])$

The paper provides characterisations for the conditions $A[S_1,S_2,S_3]$, $B[s,S_1,S_2,S_3]$ and for the set $V[s,S_1,S_2,S_3]$ that are appropriate for concrete calculations.

Example: MD5, Block 1 (1)

Stochastic model: $\rightarrow$ paper

Impact of bit conditions on the chaining values:

Post additions in Steps 61- 63: 6 bit conditions

- Wang Conditions (Eurocrypt 2005, PAPER):
  - Transition probability for standard IV $\approx$ 0.005

- Wang Conditions (Eurocrypt 2005, PUBLISHED EXAMPLE):
  - Transition probability for standard IV $\approx$ 0.095
  - Transition probability for IV = (0x 80000000, 0x 00000000, 0x 82000000, 0x 10325476) = 0.5
  - Transition probability for IV=(0x 00000000, 0x 82000000, 0x 80000000, 0x 10325476) = 0

Example: MD5, Block 1 (2)

- We analysed three different near-collision paths after message modification:
  - Path 1: Wang Conditions (PAPER, Eurocrypt 2005)
  - Path 2: Wang Conditions (PUBLISHED EXAMPLE)
  - Path 3: “Almost”-Wang conditions

	Path1	Path 2	Path3
# bit conditions	38	38	39
calculated probability	$2^{-41.64}$	$2^{-37.41}$	$2^{-36.61}$
empirical ($2^{41.87}$ samples)		$2^{-37.11}$	$2^{-36.25}$

Conclusion

- “Bit condition counting” yields only rough estimators for the probabilities of (near-)collision paths.
- Our contribution provides universally applicable theorems that support the precise computation of collision path probabilities.
- These theorems do not support the search for new (near-) collision paths.
- Our formulae were empirically confirmed by concrete MD5 near-collision paths.

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