Title: Quadrilaterals Aren't Just Squares

Brief Overview:

This is a collection of the first three lessons in a series of seven lessons studying characteristics of quadrilaterals, including trapezoids, parallelograms, rhombi, rectangles, squares, kites and mid-point quadrilaterals. Students will investigate the characteristics of each group of quadrilaterals using compass and straight edge, Geometer's Sketchpad software, and other optional hand-held learning tools.

NCTM Content Standards/National Science Education Standards:

Geometry:

Students should analyze properties of and determine attributes of two-and three-dimensional geometric shapes

Students should explore and develop mathematical arguments about geometric relationships (including congruence) among classes of two-dimensional geometric objects, make conjectures about them, and solve problems involving them.

Students should establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others.

Students should draw and construct representations of two-and three- dimensional geometric objects using a variety of tools.

Students should specify locations and describe spatial relationships using coordinate geometry and other representational systems.

Algebra:

Represent and analyze mathematical situations and structures using algebraic symbols.

Grade/Level:

Grades 8-12 Geometry

Duration/Length:

3 class periods of 45-60 minutes in length

Student Outcomes:

Students will:

Recognize the unique properties of each subset of the category of four sided polygons, including trapezoids, parallelograms, rhombi, rectangles, squares, kites and mid-point quadrilaterals

Compare and contrast groups of quadrilaterals possessing special characteristics.

Construct each type of quadrilateral using compass and straightedge.

Create and manipulate drawings of quadrilaterals using dynamic Geometer's Sketchpad software to discover special relationships in specific groups of quadrilaterals.

Materials and Resources:

- Geometer's Sketchpad software
- Student worksheets (activity sheets, homework sheets and assessments)
- Graph paper
- Teacher notes and lesson plans for using GSP software demonstrations
- Textbook

Summative Assessments:

A short pre-assessment will be included in the plan for day one.

A partial chapter assessment will be included after lesson three, and after lesson seven (in Part II.)

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Lesson 1 – Defining and Constructing Quadrilaterals

Preassessment/Launch:

Distribute the **Quadrilateral Properties Chart** to the students and ask them to complete the chart simply based on any knowledge they may already have about quadrilaterals.

Teacher Facilitation/Student Application:

Display (perhaps on an overhead) "Quadrilateral Definitions" (Student Resource Sheet 12). Have the students break up into groups. Each group can construct one of the types of quadrilaterals (this quadrilateral can be chosen by the students, or assigned by the teacher). Have each group share its construction with the other groups in the class, allowing them to perform the construction as it is being shown to them.

Embedded Assessment/Reteaching/Extension:

Distribute a new copy of the **Quadrilateral Properties Chart** to the students and have them complete the chart as the class investigates each type of quadrilateral throughout the unit.

Quadrilateral Properties Chart

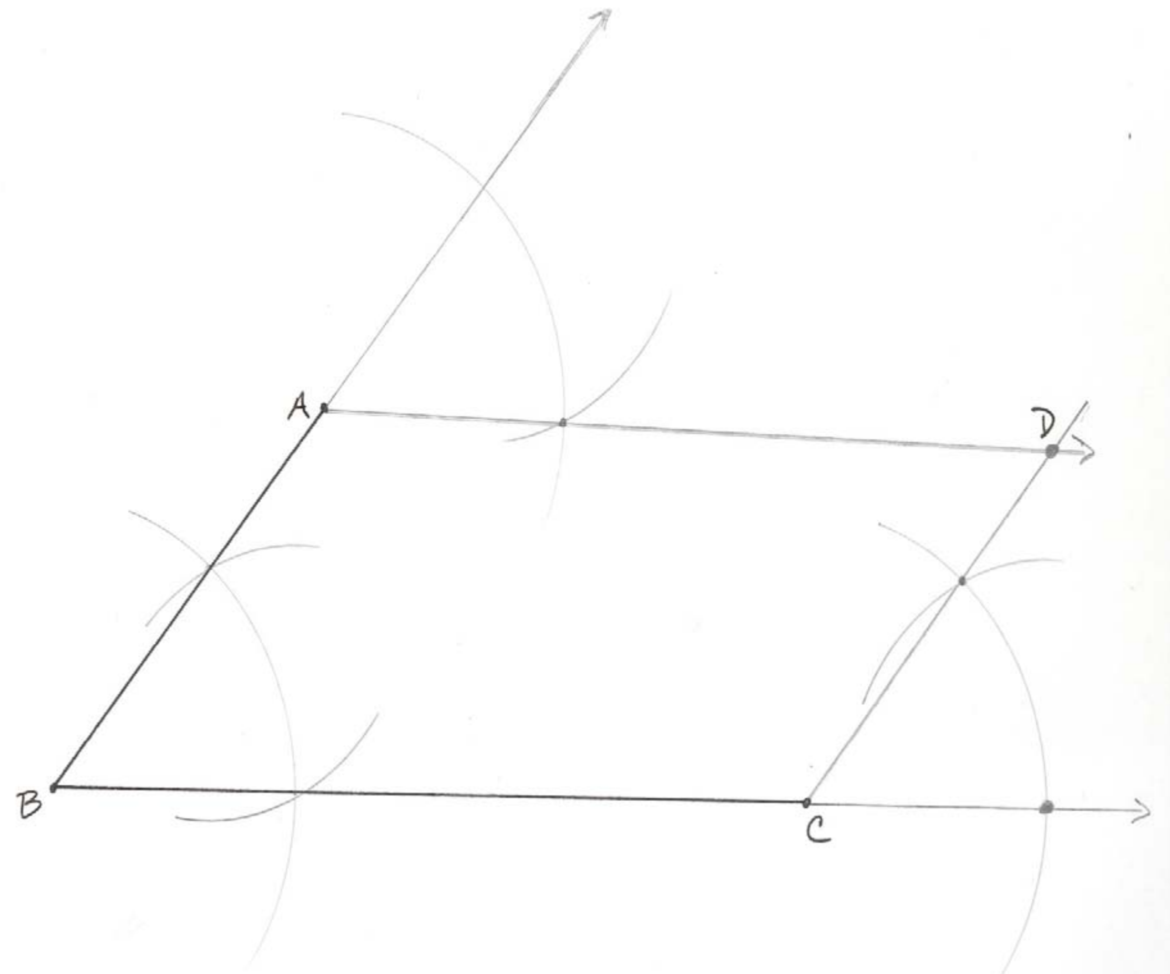
In the chart below, place a ☑ in the boxes if you think the quadrilateral listed along the top row has the properties listed in the left column.	Parallelogram	Rectangle	Rhombus	Square	Trapezoid	Isosceles Trapezoid	Kite
Opposite sides $\cong$							
Opposite $\angle$s $\cong$							
Consecutive $\angle$s sum = 180°							
Diagonals $\cong$							
Diagonals $\perp$							
Diagonals bisect each other							

	Check this box if you have no idea, or if you were just guessing.

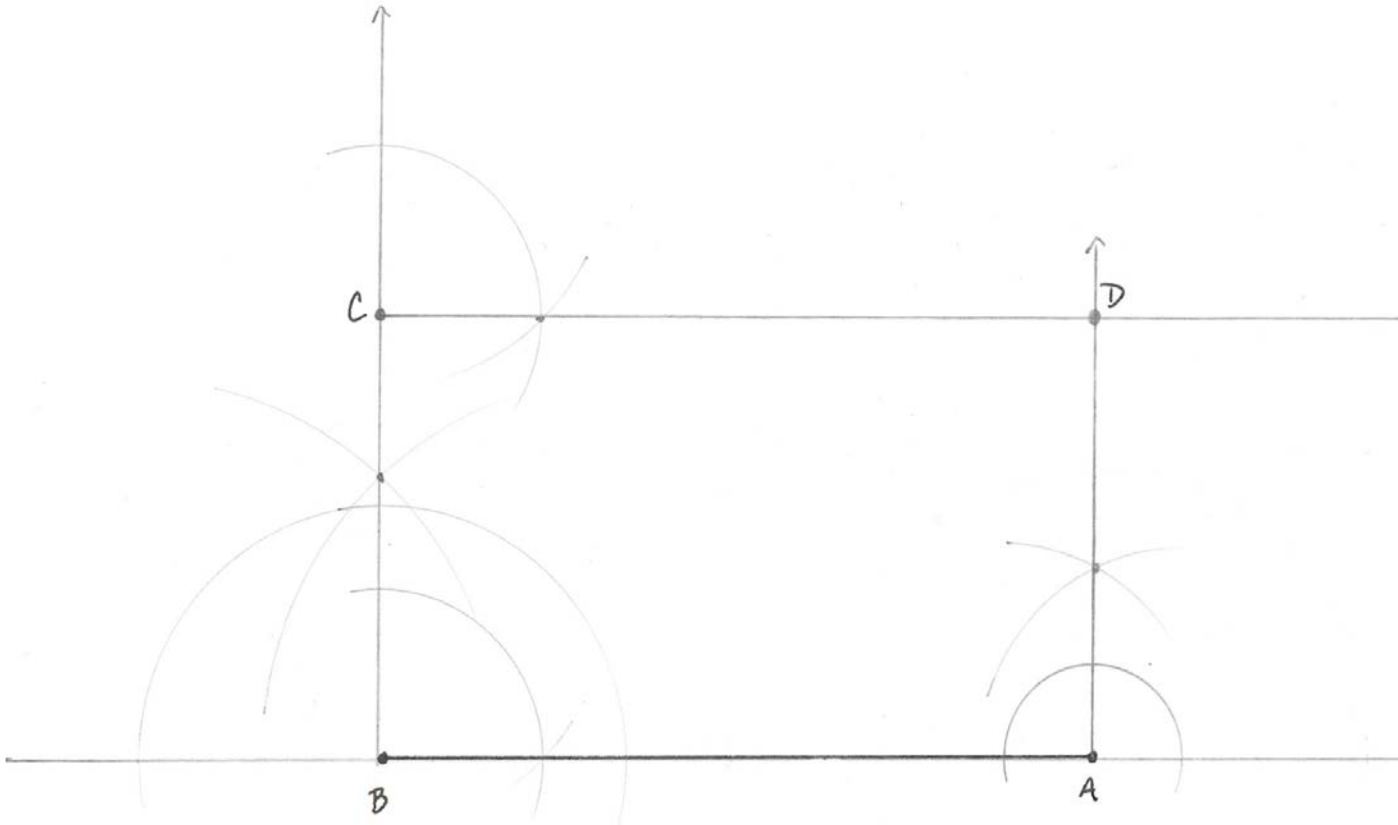
Quadrilateral Definitions

Quadrilateral	a closed four-sided plane figure.
Parallelogram	a quadrilateral that's opposite sides are parallel.
Rectangle	a parallelogram with two pairs of congruent opposite sides and at least one right angle.
Rhombus	a parallelogram with all 4 sides congruent.
Square	a parallelogram with four congruent sides and four right angles.
Trapezoid	a quadrilateral with only two parallel sides.
Isosceles Trapezoid	a trapezoid in which the non-parallel sides are congruent.
Kite	a quadrilateral with exactly two pairs of adjacent congruent sides.

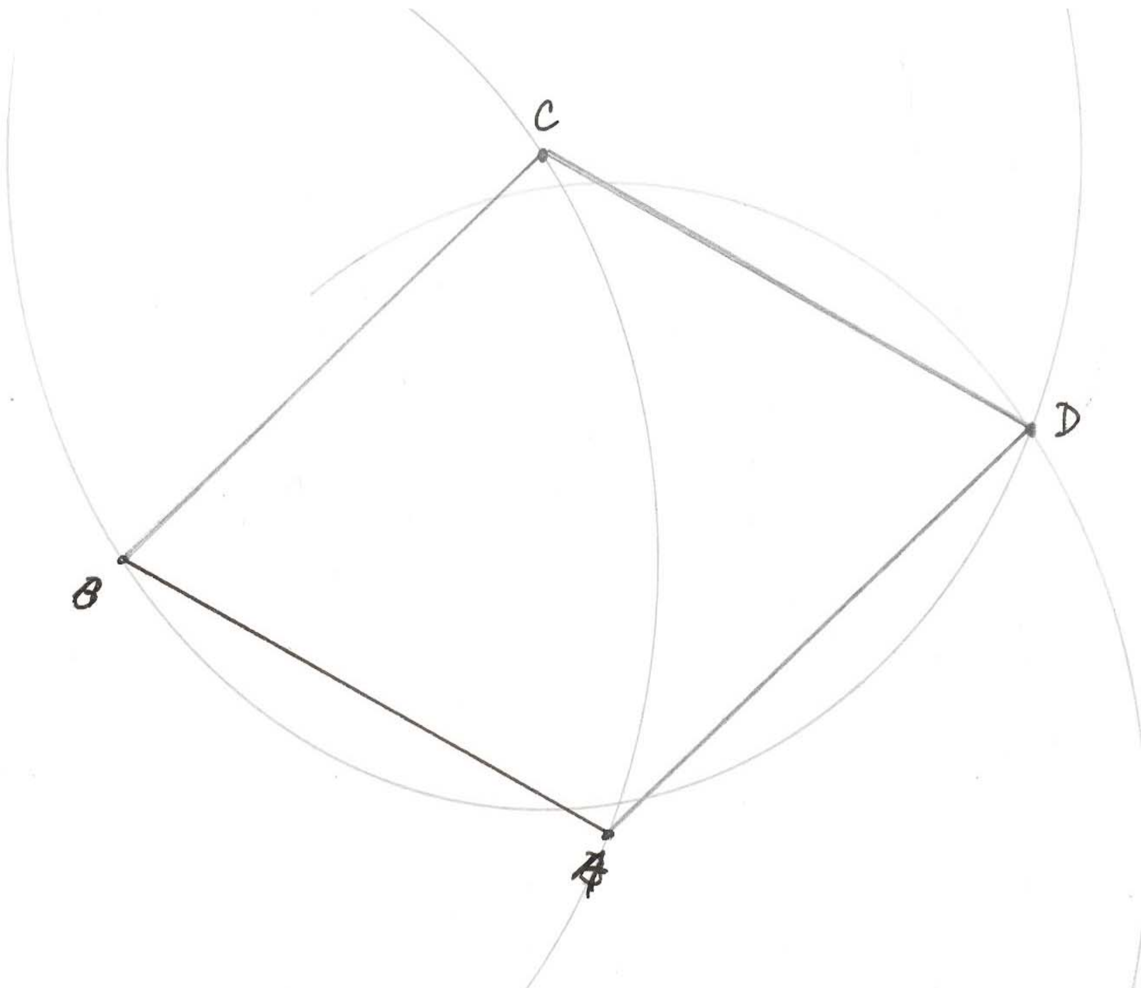
Parallelogram Construction



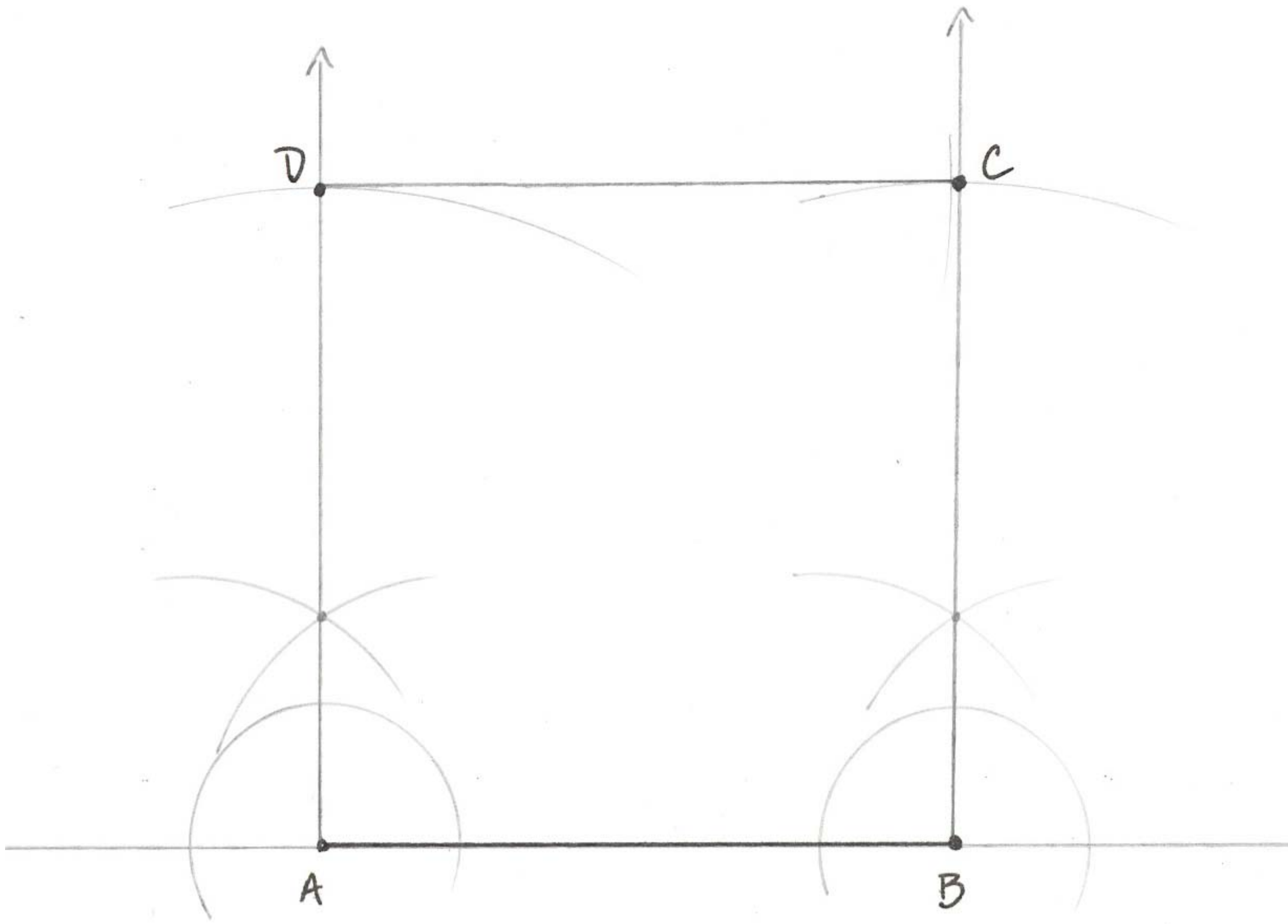
Rectangle Construction



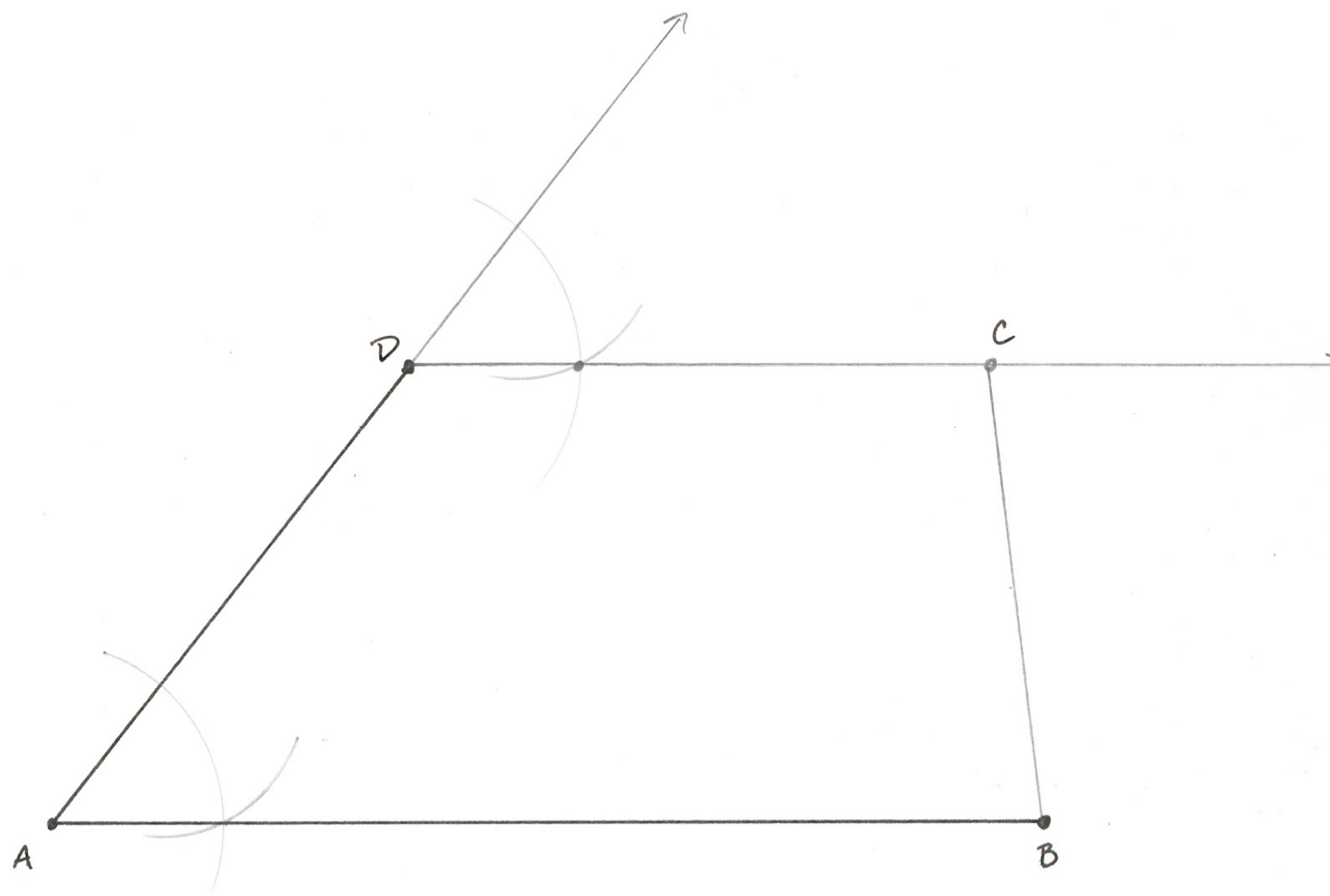
Rhombus Construction



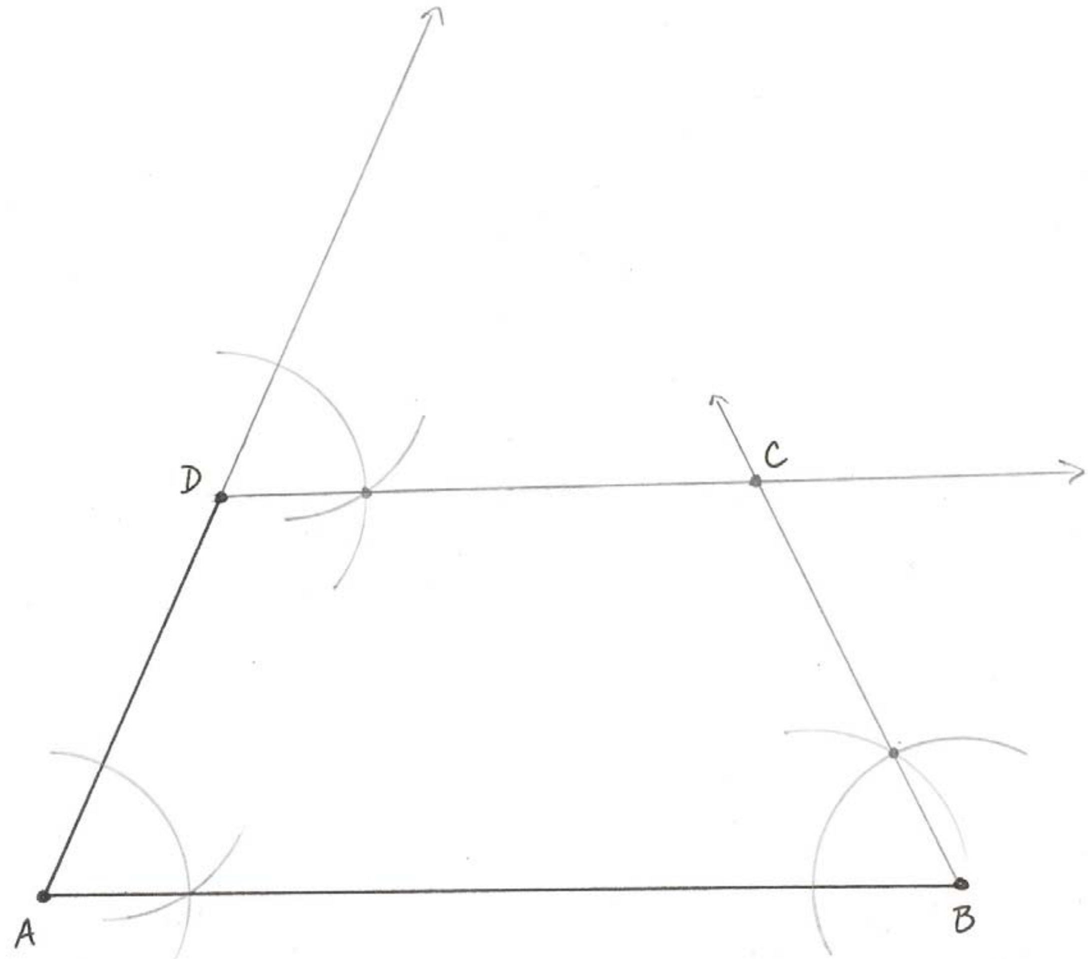
Square Construction



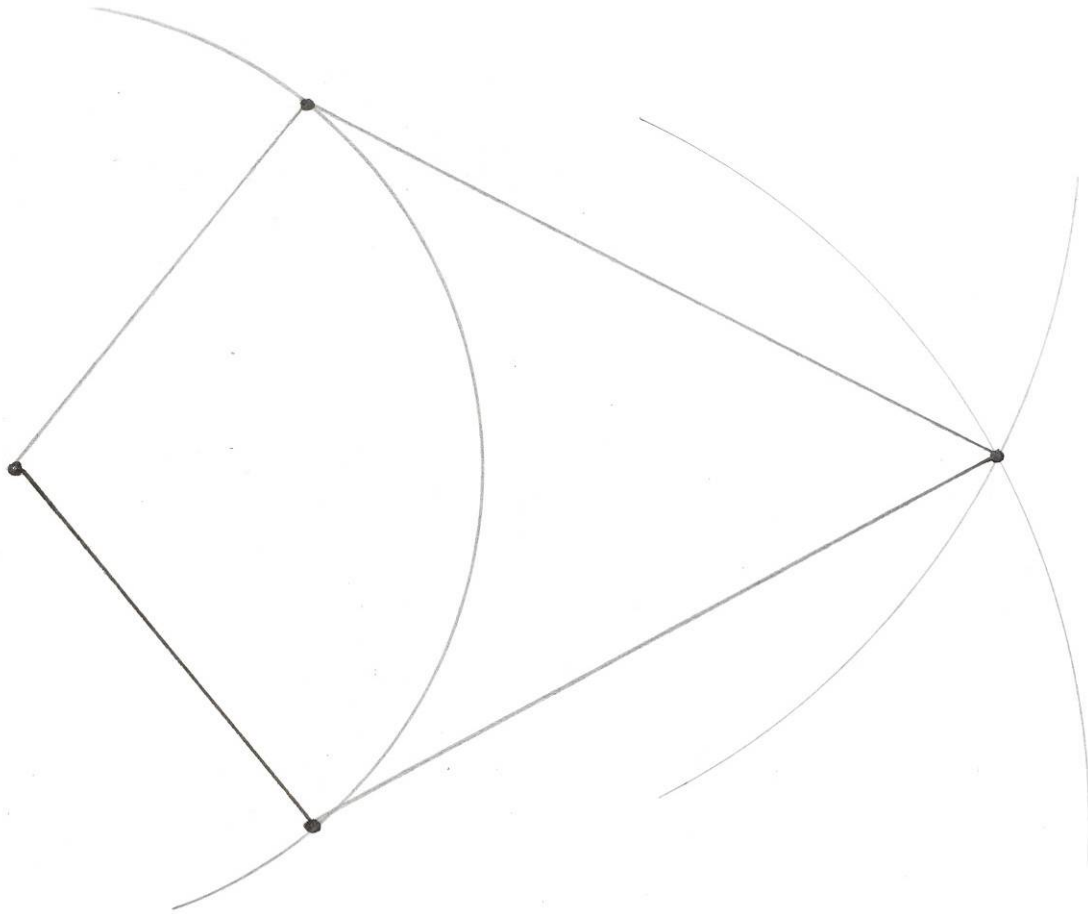
Trapezoid Construction



Isosceles Trapezoid Construction



Kite Construction



Lesson 2: Parallelograms

Brief Overview:

In this lesson, students will use Geometer's Sketchpad to investigate the properties of a parallelogram. They will discover that both pairs of opposite sides are parallel and congruent, both pairs of opposite angles are congruent, same-side interior angles (consecutive interior angles) are supplementary, and the diagonals bisect each other.

Preassessment:

Review the definition of a parallelogram. Review the definition of the diagonals of a polygon and how to find the sum of the interior angles of a quadrilateral.

Launch:

Students will investigate the properties of a parallelogram using Geometer's Sketchpad and Student Resource Sheet 2.1. The directions on Student Resource Sheet 2.1 assume that students are familiar with Geometer's Sketchpad and its features.

Teacher Facilitation:

Lead the investigation of the properties while the students are working on Student Resource Sheet 2.1. When Student Resource Sheet 2.1 is completed, lead a discussion with the students to verify the properties of a parallelogram. Model the results on the overhead using Sketchpad. After the discussion, problems similar to Student Resource Sheet 2.2 can be modeled using Geometer's Sketchpad as well.

Student Application:

Students will complete Student Resource Sheet 2.2.

Embedded Assessment:

Teacher will informally monitor progress of the students by reviewing their responses on Student Resource Sheet 2.1 and 2.2

Reteaching/Extension:

Students can verify the properties of vertical angles, linear pairs and alternate interior angles within the parallelogram. Students can investigate the properties of parallelogram if all of the

angles are right angles. This can lead i
properties of a rectangle.

Name ____ Date ______________

Investigating the Properties of Parallelograms

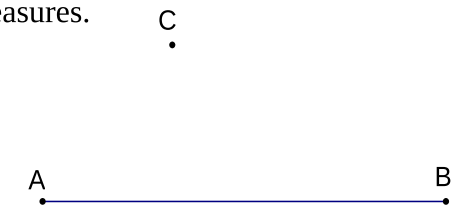
Students will use Geometer's Sketchpad to draw a parallelogram and investigate its properties.

Part A: Drawing a Parallelogram

1. In the edit menu, set the preferences so that the precision of degree measurements is units. Leave hundredths as the precision for all other measures.

2. Draw a segment. Label its endpoints A and B.

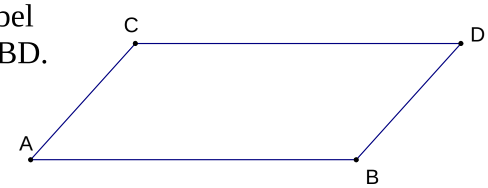
3. Place a point C not on the segment AB.



4. Draw $\overline{AC}$.

5. Construct a line that is parallel to $\overline{AB}$ through C. Then construct a line that is parallel to $\overline{AC}$ through B.

6. Construct a point of intersection for the two lines and label it point D. Hide the lines and draw segments CD and BD.



Part B: Investigating the Properties of a Parallelogram

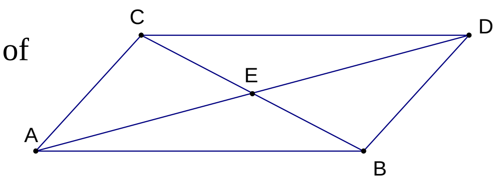
1. Measure the slopes of all four sides of the parallelogram to verify that the opposite sides are parallel.

2. Measure the lengths all four sides. What conjecture(s) can you draw about the relationship between the side lengths of a parallelogram?

3. Measure all four angles. What conjecture(s) can you draw about the relationship between the angles of a parallelogram?

4. Drag one vertex of the parallelogram to change its shape   Do your conjectures from #2 and #3 hold true?

5. Draw diagonals $\overline{AD}$ and $\overline{BC}$. Construct their point of intersection and label it E.



6. Measure the length of each diagonal. Are they congruent?

7. Measure $\overline{AE}$, $\overline{DE}$, $\overline{BE}$, and $\overline{CE}$. What conjecture(s) can you draw about the diagonals of a parallelogram?

1. Drag one vertex of the parallelogram to change its shape. Do your conjectures from #6 and #7 hold true?

If time permits:

9. Measure ∠DCE and ∠EBA.   Why should those angles be equal to each other?

10. Measure the angles at vertex E and record your results. Verify the properties of vertical angles and linear pairs.

11. Drag the vertices of the parallelogram so that all measured angles are 90°. What figure have you created?   Do all of your conclusions about parallelograms still hold true for this figure?

Part C: Conclusions

Based on your investigation, list the properties of a parallelogram.

Teacher Resource Sheet 2.1

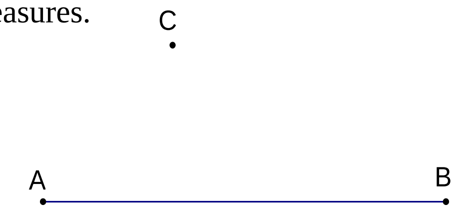
Name __________Answer Key_____________________ Date ______________

Investigating the Properties of Parallelograms

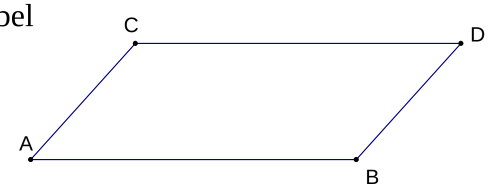
Students will use Geometer's Sketchpad to draw a parallelogram and investigate its properties.

Part A: Drawing a Parallelogram

1. In the edit menu, set the preferences so that the precision of degree measurements is units. Leave hundredths as the precision for all other measures.



2. Draw a segment. Label its endpoints A and B.
3. Place a point C not on the segment AB.
4. Draw $\overline{AC}$.
5. Construct a line that is parallel to $\overline{AB}$ through C. Then construct a line that is parallel to $\overline{AC}$ through B.
6. Construct a point of intersection for the two lines and label it D. Hide the lines and draw segments CD and BD.



Part B: Investigating the Properties of a Parallelogram

1. Measure the slopes of all four sides of the parallelogram to verify that the opposite sides are parallel.
   The slopes of the opposite sides should be equal which verifies that both pairs of opposite sides of a parallelogram are parallel.

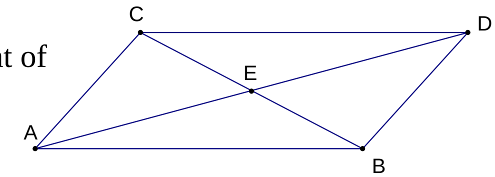
2. Measure the lengths all four sides. What conjecture(s) can you draw about the relationship between the side lengths of a parallelogram?
   The measures of the opposite sides should be equal so students should conclude that both pairs of opposite sides of a parallelogram are congruent.

3. Measure all four angles. What conjecture(s) can you draw about the relationship between the angles of a parallelogram?

The measures of the opposite angles should be equal so students should conclude that both pairs of opposite angles of a parallelogram are congruent.

4. Drag one vertex of the parallelogram to change its shape Do your conjectures from #2 and #3 hold true? **Yes**

5. Draw diagonals $\overline{AD}$ and $\overline{BC}$. Construct their point of intersection and label it E.



6. Measure the length of each diagonal. Are they congruent? **no**

7. Measure $\overline{AE}$, $\overline{DE}$, $\overline{BE}$, and $\overline{CE}$. What conjecture(s) can you draw about the diagonals of a parallelogram?

The diagonals of a parallelogram bisect each other.

8. Drag one vertex of the parallelogram to change its shape. Do your conjectures from #6 and #7 hold true? **yes**

If time permits:

9. Measure ∠DCE and ∠EBA. Why should those angles be equal to each other?

If two parallel lines are cut by a transversal, then alternate interior angles are congruent.

10. Measure the angles at vertex E and record your results. Verify the properties of vertical angles and linear pairs.

Vertical angles are congruent and angles of a linear pair are supplementary.

11. Drag the vertices of the parallelogram so that all measured angles are 90°. What figure have you created? **Rectangle or Square** Do all of your conclusions about parallelograms still hold true for this figure? **yes**

In lesson 3, rectangles and squares will be explored in more detail.

Part C: Conclusions

Based on your investigation, list the properties of a parallelogram.

By definition, both pairs of opposite sides of a parallelogram are parallel.
Both pairs of opposite sides are congruent.
Both pairs of opposite angles are congruent.
Same-side interior angles (consecutive interior angles) are supplementary.

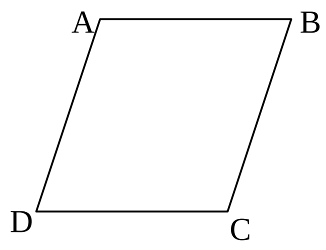
Diagonals bisect each other.

Student Resource Sheet 2.2

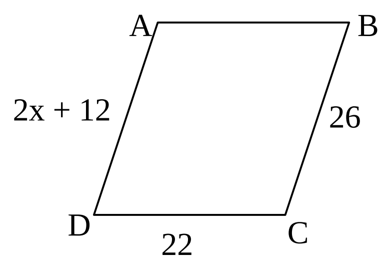
Name ____ Date ________________

Properties of Parallelograms

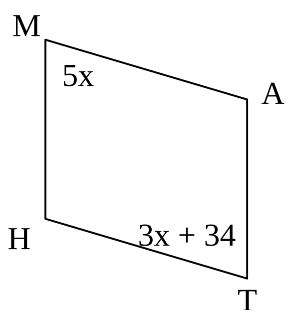
1. Find m∠B, m∠C, and m∠D.



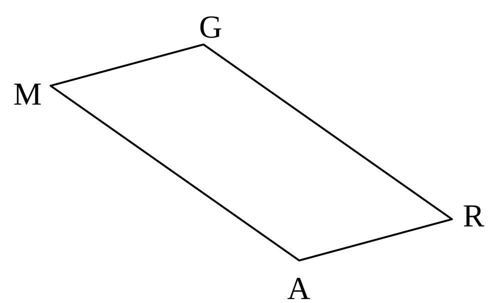
2. Find AB and x.



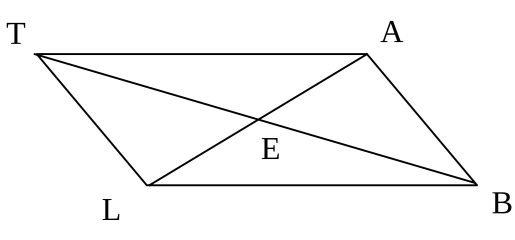
3. Find x.



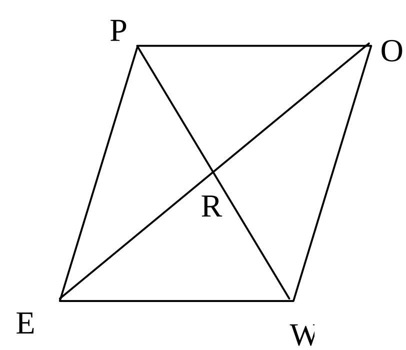
4. Find x.



5. Given m∠ATB = 35° and m∠ABT = 43°, find m∠ATL.



6. Given OR = 8 and, PW = 10, find OE and PR

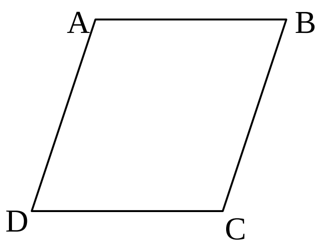


Name ___ANSWER KEY_____________________ Date _______________

Properties of Parallelograms

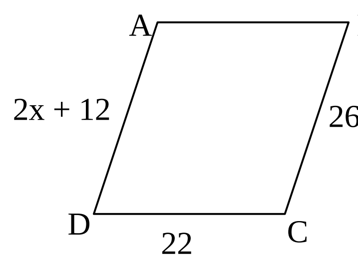
Find the measure(s) in each of the following parallelograms. Use the properties of parallelograms.

1. Find m∠B, m∠C, and m∠D.



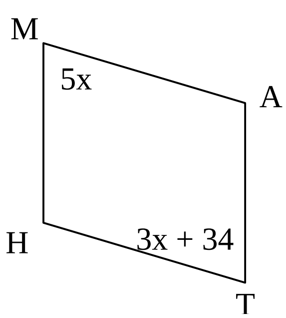
m∠B = 60°, m∠C = 120°, and m∠D = 60°

2. Find AB and x.



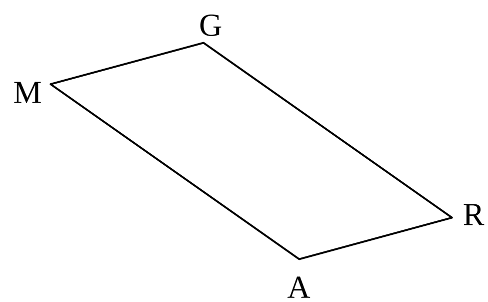
AB = 22 and x = 7

3. Find x.



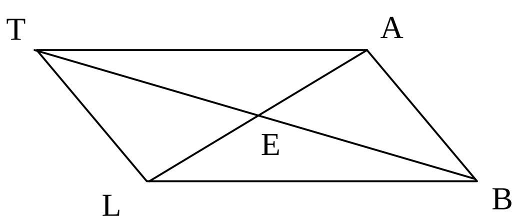
x = 17

4. Find x.

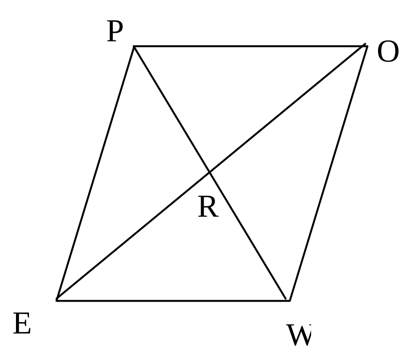


x = 47

5. Given m∠ATB = 35° and m∠ABT = 43°, find m∠ATL.



6. Given OR = 8 and, PW = 10, find OE and PR



m∠ATL = 78°

OE = 16 and PR = 5

Lesson 3: Special Parallelograms

Brief Overview:

In this lesson, students will use Geometer's Sketchpad to investigate the properties of rectangles, rhombi, and squares. They will verify that the properties of parallelograms hold true for these figures. Students will then investigate additional properties for the special parallelograms and discover that the diagonals of a rectangle are congruent, and that the diagonals of a rhombus are perpendicular and also bisect a pair of opposite angles. Finally they will discover that all three of these additional properties are true for squares.

Preassessment/Launch:

Review the properties of parallelograms and the definition of diagonal. Use tiles, index cards, or other manipulatives to model parallelograms, and lead the students in a discussion of parallelogram properties.

Teacher Facilitation/ Student Application:

- Using Sketchpad, students will verify that a rectangle is a parallelogram and that parallelogram properties hold true. Students will investigate additional properties of rectangles and complete questions on Student Resource Sheet 3.1. (Directions assume prior knowledge of Sketchpad and its features.)
- Using Sketchpad and Student Resource Sheet 3.2, students will explore special properties of a rhombus, and summarize their findings.
- Students will use Sketchpad and Student Resource Sheet 3.3 to investigate special properties of a square. Students should discover from their investigation that a square has all the properties of both a rhombus and a rectangle.

Embedded Assessment:

Assess understanding of the concepts by reviewing students' responses on the worksheets and in class discussion. Monitor progress on independent practice.

Reteaching/Extension:

Students are encouraged to repeat the activities with a peer, using a different sketch, if they need additional time working with the concepts.
To extend the lesson, students will investigate and classify the four triangles formed by the diagonals of a rectangle, rhombus, and square. They will then find measures of the angles using the properties of isosceles and right triangles. As another extension, students will explore how the distance, slope, and midpoint formulas might be used to prove the properties of general and special parallelograms.

Name ____ Date _________________

Investigating the Properties of Rectangles

Students will use Geometer's Sketchpad to draw a rectangle and investigate its properties.

Part A: Drawing a Rectangle

1. Under edit, set preferences: change degrees to units. (Leave other measures in hundredths, the default.) Do NOT resize the graph.

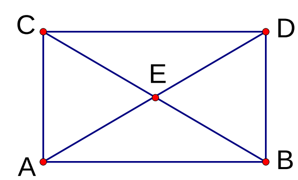
2. Draw a segment. Label it $\overline{AB}$.

3. Construct the line perpendicular to $\overline{AB}$ through point A.

4. Place an arbitrary point on the perpendicular. Label it point C.

5. Construct a line parallel to $\overline{AB}$ through point C.

6. Construct a line parallel to $\overleftrightarrow{AC}$ through point B.



7. Construct the point of intersection for the two lines drawn in steps #5 and #6. Label this point D.

8. Hide the lines: $\overline{AC}$, $\overline{DC}$, and $\overline{BD}$.

9. Draw segments: $\overline{AC}$, $\overline{DC}$, $\overline{BD}$, and the diagonals $\overline{AD}$ and $\overline{CB}$.

10. Construct the point of intersection of the diagonals. Label it point E.

11. Explain why figure ABDC is a rectangle.

(OVER→)

Part B: Investigating the Properties of a Rectangle

1. Demonstrate that the rectangle has all the characteristics of a parallelogram by listing specific measurements and explaining their significance.

2. Measure lengths of the diagonals**.** Are diagonals congruent? Drag one vertex of the rectangle. Does this remain true?

Part C: Conclusion

A rectangle has all the properties of a parallelogram. Based on your investigation, what other characteristic does a rectangle have?

Name ____<u>Answer Key</u>_______________________ Date _______________

Investigating the Properties of Rectangles

Students will use Geometer's Sketchpad to draw a rectangle and investigate its properties.

Part A: Drawing a Rectangle

1. Under edit, set preferences: change degrees to units. (Leave other measures in hundredths, the default.) Do NOT resize the graph.

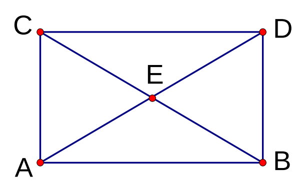
2. Draw a segment. Label it $\overline{AB}$.

3. Construct the line perpendicular to $\overline{AB}$ through point A.

4. Place an arbitrary point on the perpendicular. Label it point C.

5. Construct a line parallel to $\overline{AB}$ through point C.

6. Construct a line parallel to $\overleftrightarrow{AC}$ through point B.



7. Construct the point of intersection for the two lines drawn in steps #5 and #6. Label this point D.

8. Hide the lines: $\overline{AC}$, $\overline{DC}$, and $\overline{BD}$.

9. Draw segments: $\overline{AC}$, $\overline{DC}$, $\overline{BD}$, and the diagonals $\overline{AD}$ and $\overline{CB}$.

10. Construct the point of intersection of the diagonals. Label it point E.

11. Explain why figure ABDC is a rectangle.

    The figure constructed has two pairs of opposite sides parallel, making it a parallelogram. Since in step #3 a perpendicular was constructed, you also have a right angle. Therefore, the figure satisfies the definition of rectangle, which is a parallelogram having one right angle.

(OVER→)

Part B: Investigating the Properties of a Rectangle

1. Demonstrate that the rectangle has all the characteristics of a parallelogram by listing specific measurements and explaining their significance.

   Students' measurements will vary:

   a. **Opposite sides have the same slope; the figure has two pairs of opposite sides parallel.**
   b. **Opposite sides have the same length; there are two pairs of opposite sides congruent.**
   c. **Opposite angles have the same measure; the figure has two pairs of opposite angles congruent.**
   d. **Measures of consecutive angles add up to 180°; consecutive angles are supplementary.**
   e. **Measures of the two halves of each diagonal are equal; the diagonals bisect each other.**

2. Measure lengths of the diagonals**.** Are diagonals congruent? Drag one vertex of the rectangle. Does this remain true?

   Students' measures will vary. Yes, the diagonals will be congruent. After dragging, the diagonals remain congruent.

Part C: Conclusion

A rectangle has all the properties of a parallelogram. Based on your investigation, what other characteristic does a rectangle have?

The diagonals of a rectangle are congruent to each other.

Name ____ Date _______________

Investigating the Properties of a Rhombus

Students will use Geometer's Sketchpad to draw a rhombus and investigate its properties.

Part A: Drawing a Rhombus

1. Draw a segment. Label its endpoints A and B.

2. Rotate $\overline{AB}$ about point A to form a consecutive side of the rhombus.
   - Double click on point A to mark it as the center of rotation
   - Select segment AB and point B
   - From the "transform" menu, select the rotate option
   - Choose an angle measure other than 90^0
   - Choose rotate
   - Label the new endpoint C

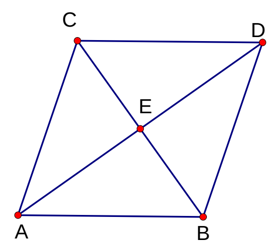
3. Construct a line parallel to segment AC through point B and construct a line parallel to segment AB through point C.

4. Construct the point of intersection and label it D.

5. Hide lines and draw segments CD and BD.

6. Draw diagonals AD and BC. Label the point of intersection E.

7. Explain why ABDC is a rhombus.



Part B: Investigating the Properties of a Rhombus

1. Measure ∠CED. Make a conjecture about the relationship between the diagonals of a rhombus. Measure the slopes of the diagonals. How can you use the slopes of the diagonals to verify your conjectures? Drag one vertex of the rhombus to change the figure. Does your conjecture about the diagonals remain true?

2. Find the measure of ∠CDE and ∠EDB. What appears to be true about the two angles? Without measuring, explain how to determine the measures of∠CAE and ∠EAB? Drag a vertex to change the figure. Does your conjecture about the angles remain true?

3. Measure ∠DCE and ∠ACE. What appears to be true about the measures? Without measuring, find the measures of ∠DBE and ∠ABE.

Part C: Conclusion

By definition a rhombus is a parallelogram and therefore has all the characteristics of a parallelogram. Based on your findings, what are other characteristics of a rhombus?

Name ____Answer Key________________________ Date ______________

Investigating the Properties of a Rhombus

Students will use Geometer's Sketchpad to draw a rhombus and investigate its properties.

Part A: Drawing a Rhombus

1. Draw a segment. Label its endpoints A and B.

2. Rotate $\overline{AB}$ about point A to form a consecutive side of the rhombus.
   - Double click on point A to mark it as the center of rotation
   - Select segment AB and point B
   - From the "transform" menu, select the rotate option
   - Choose an angle measure other than 90^0
   - Choose rotate
   - Label the new endpoint C

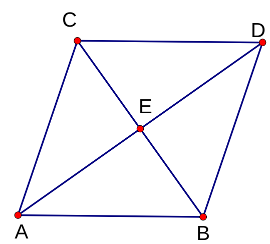
3. Construct a line parallel to segment AC through point B and construct a line parallel to segment AB through point C.

4. Construct the point of intersection and label it D.

5. Hide lines and draw segments CD and BD.

6. Draw diagonals AD and BC. Label the point of intersection E.

7. Explain why ABDC is a rhombus.



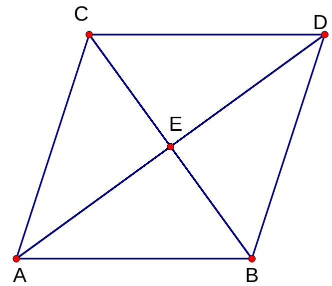
ABDC is a rhombus because it has two pair of opposite sides parallel and all sides are congruent.

Part B: Investigating the Properties of a Rhombus

1. Measure ∠CED. Make a conjecture about the relationship between the diagonals of a rhombus. Measure the slopes of the diagonals. How can you use the slopes of the diagonals to verify your conjectures? Drag one vertex of the rhombus to change the figure. Does your conjecture about the diagonals remain true?

Sample answers: (Student answers will vary.)

$m\angle CED = 90.00°$



Slope $\overline{CB}$ = -1.38

Slope $\overline{AD}$ = 0.72

$(\text{Slope } \overline{CB})\cdot(\text{Slope } \overline{AD}) = -1.00$

Conjecture:

The diagonals of a rhombus are perpendicular.
The product of the slopes is -1 which means that diagonal CB is perpendicular to diagonal AD. (The individual slopes are negative reciprocals. This is easier for students to see when in fraction form.)
Yes, my conjecture remains true when a vertex is dragged.
Changing the size of the rhombus does not change the relationship between the diagonals.

2. Find the measure of ∠CDE and ∠EDB. What appears to be true about the two angles? Without measuring, explain how to determine the measures of ∠CAE and ∠EAB? Drag a vertex to change the figure. Does your conjecture about the angles remain true?

Sample answers: (Students answers will vary.)

$m\angle CDE = 36.00°$

$m\angle EDB = 36.00°$

The measures of the two angles are congruent. The diagonal is an angle bisector.

$\angle$CAE and $\angle$EDB are alternate interior angles and are congruent.
$\angle$EAB and $\angle$CDE are alternate interior angles and are congruent.

$m\angle CAE = 36.00^\circ$

$m\angle EAB = 36.00^\circ$

Yes, the conjecture remains true.

3. Measure $\angle$DCE and $\angle$ACE. What appears to be true about the measures? Without measuring, find the measures of $\angle$DBE and $\angle$ABE.

Sample answers: (Students answers will vary.)

$m\angle DCE = 54.00^\circ$
$m\angle ACE = 54.00^\circ$

The measures of the two angles are congruent.

$m\angle DBE = 54.00^\circ$
$m\angle ABE = 54.00^\circ$

Part C: Conclusion

By definition a rhombus is a parallelogram and therefore has all the characteristics of a parallelogram. Based on your findings, what are other characteristics of a rhombus?

Sample answer:

The diagonals of a rhombus form right angles and are therefore perpendicular. The diagonals bisect a pair of opposite angles.

Student Resource Sheet 3.3

Name ____ Date ______________

Investigating the Properties of a Square

Students will use Geometer's Sketchpad to draw a square and investigate its properties.

Part A: Drawing a Square

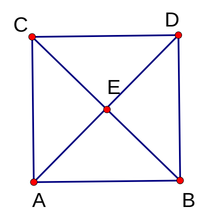
1. Draw a segment. Label its endpoints A and B.

2. Rotate segment AB and point B 90^0 about point A to create segment AC. Rotate segment AC and point A 90^0 about point C to create segment CD.

3. Draw segment DB.

4. Draw diagonals CB and AD. Label the intersection E.

5. Explain why ABDC is a square.



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Part B: Investigating the Properties of a Square

1. Measure the length of the diagonals. Make a conjecture about the relationship between the diagonals of a square? Drag a vertex of the square to change the figure. Does the relationship hold true?

2. Measure the slopes of the diagonals of your square. What other conjecture can you make about the relationship between the diagonals of a square? Drag a vertex of the square to change the figure. Does the relationship hold true?

3. Measure the two adjacent angles at point C. Measure the two adjacent angles at point D. What appears to be true about the measures of these angles? What can you say about the relationship between diagonal CB and $\angle ACD$? Does the same relationship exist between diagonal AD and $\angle CDB$? Drag a vertex to change the figure. Does the relationship hold true?

Part C: Conclusion

By definition a square is a parallelogram and therefore it has all the characteristics of a parallelogram. Based on your findings, what are other characteristics of a square?

Teacher Resource Sheet 3.3

Name ______Answer Key______________________ Date _______________

Investigating the Properties of a Square

Students will use Geometer's Sketchpad to draw a square and investigate its properties.

Part A: Drawing a Square

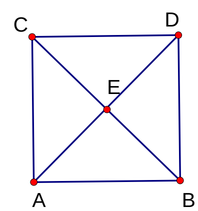
1. Draw a segment. Label its endpoints A and B.

2. Rotate segment AB and point B 90^0 about point A to create segment AC. Rotate segment AC and point A 90^0 about point C to create segment CD.

3. Draw segment DB.

4. Draw diagonals CB and AD. Label the intersection E.

5. Explain why ABDC is a square.



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The figure constructed has two pairs of opposite sides parallel. The sides are congruent and each angle measures 90^0. Therefore the figure satisfies the definition of a square.

Part B: Investigating the Properties of a Square

1. Measure the length of the diagonals. Make a conjecture about the relationship between the diagonals of a square. Drag a vertex of the square to change the figure. Does the relationship hold true?

Sample answers: (Student answers will vary.)

m $\overline{AD}$ = 3.22 cm

m $\overline{CB}$ = 3.22 cm

Conjecture:
The diagonals of a square are congruent. When a vertex is dragged, the relationship still holds true.

2. Measure the slopes of the diagonals of your square. What other conjecture can you make about the relationship between the diagonals of a square? Drag a vertex of the square to change the figure. Does the relationship hold true?

Sample answers: (Student answers will vary.)

Slope $\overline{CB}$ = -0.98

Slope $\overline{AD}$ = 1.02

$(\text{Slope } \overline{CB}) \cdot (\text{Slope } \overline{AD})$ = -1.00

Conjecture:
The diagonals are negative reciprocals; therefore the diagonals are perpendicular.
The relationship still holds true when a vertex is dragged.

3. Measure the two adjacent angles at point C. Measure the two adjacent angles at point D. What appears to be true about the measures of these angles? What can you say about the relationship between diagonal CB and $\angle$ACD? Does the same relationship exist between diagonal AD and $\angle$CDB? Drag a vertex to change the figure. Does the relationship hold true?

Sample answers:

$m\angle DCE = 45.00^\circ$ $m\angle CDE = 45.00^\circ$

$m\angle ACE = 45.00^\circ$ $m\angle BDE = 45.00^\circ$

Conjecture:
The two adjacent angles at point C are congruent.
The two adjacent angles at point D are congruent.
Diagonal CB bisects $\angle$ACD.
Diagonal AD bisects $\angle$CDB
When a vertex is dragged the relationship still holds true.

Part C: Conclusion

By definition a square is a parallelogram and therefore it has all the characteristics of a parallelogram. Based on your findings, what are other characteristics of a square?

Sample answer:

The investigation showed that the diagonals of a square are congruent to

each other, they are perpendicular to each other, and they bisect the angles of the square. (Since a square is a special type of rhombus and also a special type of rectangle, it has all the properties of both a rhombus and a rectangle.)

Name______________________________Date_________________

Assessment following Lesson 3

This activity is sometimes called ***"SOMETIMES"***, or ***"COUNTEREXAMPLE".*** Each condition given below is true in some cases and false in at least one instance. For each item, give one example that shows when the condition is true and one that shows when it is false. Your answer may take the form of a diagram with appropriate markings, an expression, an equation, or a written response.

CONDITION	TRUE	FALSE
1.) A quadrilateral has diagonals that bisect each other.		
2.) The slope of two opposite sides of a quadrilateral is equal.		
3.) A parallelogram with diagonals that are congruent is a square.		
4.) A parallelogram has diagonals that are perpendicular.		
5.) The adjacent sides of a parallelogram are congruent.		
6.) A rhombus has congruent sides and congruent angles.		
7.) Rectangles have perpendicular diagonals.		

8.) In an isosceles trapezoid the shorter base is congruent to the legs.		