

## IDENTIFYING FAILURE SCENARIOS IN COMPLEX SYSTEMS BY PERTURBING MARKOV CHAIN MODELS

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## Summary

- **Goal:** To develop scalable modeling tools for monitoring real-world complex systems and predicting catastrophic performance degradations.
- Use Discrete Time Markov Chain (DTMC):
  - Develop detailed *time-inhomogeneous* model of system behavior that can represent evolution from normal conditions to failure states.
  - Perturb DTMC transition probability matrices (TPMs) to simulate alternative system evolutions.
  - → Identify failure scenarios







## **Problem and solution approach**

 To identify failure scenarios in a complex system, it is advantageous to model more extensive range of possible system states—can lead to large, detailed models

<u>**however**</u>, perturbation of large DTMCs may involves search spaces that increase exponentially with model size.

- Solution approach (to be shown): Use minimal s-t cut set analysis on directed graph of DTMC in combination with other techniques:
  - Detailed DTMC, time-inhomogeneous representation (sets of TPMs for different time periods),
  - Model perturbation and
  - Markov simulation modeling

#### in order to.....

→ Identify small parts of the model – <u>critical state transitions</u> – that can be directly perturbed to change system performance. (<u>thus avoiding large search space</u>)



## Outline

- 1. DTMC concepts and model development
- 2. Perturbing a DTMC to identify a failure scenario
- 3. Using minimal s-t cut set analysis to reduce search for failure scenarios
- 4. Summary/Conclusions and future directions



## State model of a cloud computing system

- Large-scale simulation for a cloud computing system [Mi2010]
  - Clouds "rent" compute resources virtual machines or VMs (CPUs, memory, disk)
- Focus: Process of requesting and allocating VMs (computing resources) •
  - *Lifecycle of user requests phases/stages in request process*
  - Each phase is decomposable into detailed states and state transitions
  - Total of 39 states and 139 state transitions
  - **Request Active** (state) grant of VMs (resources) to users; **Failed State** rejection.



## Detailed model allows more precise analysis

#### Lifecycle phases



## Decomposed state model for *cluster estimating* phase

#### Summary of phase

Cloud controller obtains estimates from clusters of ability to provide VMs to satisfy a user request. Partial Grant (*Allocating Minimum*) Full Grant (*Allocating Maximum*)

#### then

- 1. Controller selects cluster to implement
- 2. If cluster successful request (eventually) reaches *Request\_Granted* state.
- 3. Or, if no cluster can  $\rightarrow$  Failed\_State







## **Building a Discrete Time Markov Chain (DTMC) model**

- *DTMCs* are state models where probability of transition from one state to another does not depend on past history:  $Pr(X_{n+1} = x | X_n = x_n, ..., X_1 = x_1) = Pr(X_{n+1} = x | X_n = x_n)$  for sequence of states  $X_n, X_{n+1}, X_{n+2}$ .....
- Probability state *i* transitions to state *j*, **p**<sub>ij</sub>, is the proportion  $p_{ij} = \frac{f_{ij}}{\sum_{k=1}^{n} f_{ik}}$ of total number of transitions from state *i* to other states,  $p_{ij} = \frac{f_{ij}}{\sum_{k=1}^{n} f_{ik}}$



\*where  $p(\epsilon) = 1.0e^{-6}$  without perturbation



## Result is set of TPMs for *m* time periods

Ω

#### Summary TPM -- weighted average of m periods

	1	2	3	4	5	6	7	8	9	10 3	11 :	12 1	13 1	4 15	16	17	18	19	20	21 2	2 23	24	25	26	27	28 2	29 3	0 33	32	33	34	35	36	37
1 Initial	0.995	6 0.005	0	0	0	0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0
2 Thinking	0	0.962	0.038	0	0	0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0
3 Submitting	0	ε	0.873	0.122	0	0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0
4 Transferring_User_Request	0	0	0.022	ε	0.978	0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0
5 Initiating_Request_Session	0	0	3	0	0	1-2ε	0	0	0	0	0	0	0	0 0	з	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0
6 Preparing_Cluster_Estimate_Requests	0	0	ε	0	0	0	1-2ε	0	0	0	0	0	0	0 0	ε	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0
7 Transferring_Estimate_Request	0	0	ε	0	0	0	ε (	.993	0	0	0	0	0	0 0.00	7 0	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0
8 Allocating_Minimum	0	0	ε	0	0	0	0	0 0	.248 0	0.752	0	0	0	3 0	0	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0
9 Allocating_Maximum	0	0	ε	0	0	0	0	0	0	ε 0.	.464 0.	.536	0	3 <sup>0</sup>	0	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0
10 Transferring Failure_Estimate	0	0	З	0	0	0	0	0	0	ε	0	0	0	0 1-2	ε 0	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0
11 Allocating Partial	0	0	ε	0	0	0	0	0	0	ε	0 1	ι-3ε	0	ο ε	0	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0
12 Recording_Allocation	0	0	ε	0	0	0	0	0	0	ε	0	0 1	1-3ε	ο ε	0	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0
13 Transferring_Allocation_Estimate	0	0	ε	0	0	0	0	0	0	0	0	0	0 1-	-2ε ε	0	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0
14 Selecting_Next_Cluster	0	0	ε	0	0	0	0	0	0	0	0	0	0;	ε 0.16	8 0	0.402	0.429	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0
15 Selection Failing	0	0	ε	0	0	0	0	0	0	0	0	0	0 ;	ε ε	1-34	E O	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0
16 Transferring_Failure_Response	0	0	0.952	0	0	0	0	0	0	0	0	0	0	ε 0	0.04	8 0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0
17 Transferring_Implementation_Request (F)	0	0	ε	0	0	0	0	0	0	0	0	0	0 0.0	012 0	0	ε	0	0.133	0 0	.855 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0
18 Transferring Implementation Request (P)	0	0	ε	0	0	0	0	0	0	0	0	0	0 0.0	012 0	0	0	ε	0	0.053	0 0.9	34 0	0	0	0	0	0	0	0 0	0	0	0	0	0	0
19 Queued_for_Implementation (F)	0	0	ε	0	0	0	0	0	0	0	0	0	0	ε 0	0	0	0	ε	0 1	-3ε 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0
20 Queued_for_Implementation (P)	0	0	ε	0	0	0	0	0	0	0	0	0	0	ε 0	0	0	0	0	ε	0 1-3	lε 0	0	0	0	0	0	0	0 0	0	0	0	0	0	0
21 Verifying_Allocation (F)	0	0	ε	0	0	0	0	0	0	0	0	0	0	ε 0	0	0	0	0	0	0 0	0.821	0.061	0	0	0	0	ε	0 0	0.11	8 0	0	0	0	0
2 Verifying_Allocation (P)	0	0	ε	0	0	0	0	0	0	0	0	0	0	ε 0	0	0	0	0	0	0 0	ε	0.684	0	0	0	0	ε	0 0	0.31	3 0	0	0	0	0
23 Launching_Instances (F)	0	0	ε	0	0	0	0	0	0	0	0	0	0	ε 0	0	0	0	0	0	0 0	0	0	0.485	0	0.496	0.018	0	0 0	0	0	0	0	0	0
24 Launching_Instances (P)	0	0	ε	0	0	0	0	0	0	0	0	0	0	ε 0	0	0	0	0	0	0 0	0	0	0	0.317	0	0.587 0.	096	0 0	0	0	0	0	0	0
25 Reallocating_VM_Instances (F)	0	0	З	0	0	0	0	0	0	0	0	0	0 ;	ε 0	0	0	0	0	0	0 0	1-58	0	ε	0	ε	0	ε	0 0	0	0	0	0	0	0
26 Reallocating_VM_Instances (P)	0	0	ε	0	0	0	0	0	0	0	0	0	0 ;	ε 0	0	0	0	0	0	0 0	0	1-4ε	0	ε	0	0	ε	0 0	0	0	0	0	0	0
27 Recording_Launch (F)	0	0	ε	0	0	0	0	0	0	0	0	0	0	ε 0	0	0	0	0	0	0 0	0	0	0	0	0	0	ε 1-	3ε 0	0	0	0	0	0	0
28 Recording_Launch (P)	0	0	ε	0	0	0	0	0	0	0	0	0	0	ε 0	0	0	0	0	0	0 0	0	0	0	0	0	0	ε	0 1-3	ε 0	0	0	0	0	0
29 Rolling_Back_Implementation	0	0	ε	0	0	0	0	0	0	0	0	0	0 ;	ε 0	0	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	1-2	е 0	0	0	0	0
30 Transferring_Implementation_Success (F)	0	0	ε	0	0	0	0	0	0	0	0	0	0	ε 0	0	0	0	0	0	0 0	0	0	0	0	0	0	0	E 0	0	1-3ε	0	0	0	0
Transferring_Implementation_Success (P)	0	0	ε	0	0	0	0	0	0	0	0	0	0 ;	ε 0	0	0	0	0	0	0 0	0	0	0	0	0	0	0	ο ε	0	0	1-3ε	0	0	0
32 Transferring_Implementation_Failure	0	0	ε	0	0	0	0	0	0	0	0	0	0 1-	-2ε 0	0	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	ε	0	0	0	0	0
33 Preparing_Grant (F)	0	0	ε	0	0	0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	1-ε	0	0
34 Preparing_Grant (P)	0	0	ε	0	0	0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0	1-ε	0
35 Transferring_Grant (F)	0	0	0.028	0	0	0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0.077	0	0.89
(6 Transferring, Grant (P)	0	0	0.014	0	0	0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0.038	0
37 Request_Active (F)	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	-1
38 Request_Active (P)	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0
9 Failed State	0	0	0	0	0	0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0 0	0	0	0	0	0	0	0	0 0	0	0	0	0	0	0

- <u>Key Concept</u>: Observation of system over time yields series of TPMs for *m* successive time periods to form *a piecewise homogenous DTMC* [Ro2004]
  - $\rightarrow$  captures change over time.

- Absorbing states (tasks enter and never exit)
  - Requests Active (F/P) &
  - Failed\_State

### → absorbing chain [Ke1976]



## DTMC can simulate evolution of cloud computing system

- Set of TPMs, Q<sub>i</sub>, for successive time periods (3600 s)
- System evolves in discrete time steps (100 s per step)
- Vector v<sub>n</sub> shows system state at any step n:
   consists of 39 elements → one for each state
- Matrix multiplication:  $Q^T \cdot v_n = v_{n+1}$  with  $Q_i$  for related time period.

End system state vector **v**<sub>576</sub> approximates result of large scale simulation, i.e., *Total Grants* or Request\_Active (F/P)





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## **TPM perturbation**

 Modifying state transition probabilities changes behavior and outcome of Markov simulation



		8	9	10
8	Allocating_Minimum	0	0.248	0.752
9	Allocating_Maximum	0	0	3
10	Transferring Failure_Estimate	0	0	3

→ changes proportion of requests that enter absorbing states, Request\_Active (F/P) or Total Grants

(\*Note: parenthesized numbers indicate TPM row number)



## Markov simulation to predict performance degradation

- Simulation of perturbed critical transitions over multiple time periods (time inhomogeneous evolution) drives down performance
- Can be related to failure scenarios:

ex. Cluster databases inaccessible to a software or hardware fault.



→ Predict the performance of the system being modeled.



## Perturbation of combinations of critical transitions

- Multiple critical transitions can be perturbed together to reveal more complicated scenarios
- Failure scenario: impact of multiple (possibly related) software failures





## **Computability of finding critical state transitions**

## Unfortunately, there may be many combinations of perturbations to examine in a large problem.

- Combinations in which the transition probability of one column is raised while the transition probabilities of one or more other non-zero columns in the same row is lowered involves 115 possible perturbations.
- When perturbing different combinations of rows together to find combinations of state transitions in different rows which together are critical, the figure increases by a factor of

$$\binom{n-4}{r}$$

where n = number of states (39) and r = number of rows in combination:

- 5355 perturbations to examine all possible combinations of two rows
- 58, 905 perturbations to examine all possible combinations of three rows

#### → Brute force search over all combinations infeasible



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## Minimal s-t cut set analysis

- A DTMC is a directed graph
- Minimal s-t cut set: edges (transitions) that disconnect all paths from vertex s (*Initial* state) to vertex t --desired absorbing states Request\_Active (F/P)
  - $\rightarrow$  Cut sets contain critical transitions where perturbation

drives down performance





## Applying minimal s-t cut set analysis

- Use of algorithm to enumerate all cut sets in a directed graph [Pr1984]
- Results in 159 cut sets of 1 to 5 transitions in size
  - Ex. 33 cut sets of one and two transitions vs. 115 + 5355
  - 26 cut sets of three transition vs. 58,905
- → Reduces number of perturbation combinations to examine to focus on most critical

 $\rightarrow$  2x magnitude reduction in computation cost

#### Single-transition cuts

	Set of member	Total
	transitions	Probabilty
1-1	{1, 2}	0.001
1-2	{2, 3}	0.025
1-3	{3, 4}	0.124
1-4	{8, 9}	0.264
1-5	{4, 5}	0.978
1-6	{6, 7}	0.978
1-7	{7, 8}	0.990
1-8	{13, 14}	0.991
1-9	{5, 6}	0.995
1-10	{12, 13}	1.000

#### Multiple-transition cuts

	oct of memoer	Number of	Total
	transitions	From States	Probabilty
2-1	{14, 17} {14, 18}	1	0.895
2-2	{9, 11} {9, 12}	1	1.000
<mark>2-</mark> 3	{9, 12} {11, 12}	2	1.395
2-4	{23, 27} {36, 38}	2	1.438
2-5	{23, 27} {31, 34}	2	1.499
2-6	{23, 27} {28, 31}	2	1.507
2-7	{23, 27} {34, 36}	2	1.507
2-8	{35, 37} {36, 38}	2	1.861
2-9	{31, 34} {35, 37}	2	1.922
2-10	{30, 33} {36, 38}	2	1.924
2-11	{28, 31} {35, 37}	2	1.930
2-12	{34, 36} {35, 37}	2	1.930
2-13	{27, 30} {36, 38}	2	1.931
2-14	{33, 35} {36, 38}	2	1.931
2-15	{30, 33} {31, 34}	2	1.985
2-16	{27, 30} {31, 34}	2	1.993
2-17	{31, 34} {33, 35}	2	1.993
2-18	{28, 31} {30, 33}	2	1.993
2-19	{30, 33} {34, 36}	2	1.993
2-20	{27, 30} {28, 31}	2	2.000
2-21	{27, 30} {34, 36}	2	2.000
2-22	{28, 31} {33, 35}	2	2.000
2-23	{33, 35} {34, 36}	2	2.000



# Using minimal s-t cut sets to identify critical transitions and most likely failure scenarios

#### Further reducing 159 minimal s-t cut sets:

- Structural information
  - Ordering by number of transitions
    - $\rightarrow$  fewer transitions more likely to occur
  - Ordering by probability
  - All transitions originate from same state ( $\mathbf{\nabla}$ )
- Use of domain expertise to reduce selection
  - Ex. cut sets with transitions in same system component .

Cloud Controller Cluster Controller Network



→ Narrows down system (10-15) failure scenarios of greatest interest and likelihood.

Sing	le-transition	cuts

	Set of member	Total
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1-1	{1, 2}	0.001
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1-3	{3, 4}	0.124
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1-5	{4, 5}	0.978
1-6	{6, 7}	0.978
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1-10	{12, 13}	1.000

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	2-4	{23, 27} {36, 38}	2	1.438
	2-5	{23, 27} {31, 34}	2	1.499
	2-6	{23, 27} {28, 31}	2	1.507
	2-7	{23, 27} {34, 36}	2	1.507
	2-8	{35, 37} {36, 38}	2	1.861
	2-9	{31, 34} {35, 37}	2	1.922
	2-10	{30, 33} {36, 38}	2	1.924
	2-11	{28, 31} {35, 37}	2	1.930
	2-12	{34, 36} {35, 37}	2	1.930
	2-13	{27, 30} {36, 38}	2	1.931
	2-14	{33, 35} {36, 38}	2	1.931
	2-15	{30, 33} {31, 34}	2	1.985
	2-16	{27, 30} {31, 34}	2	1.993
	2-17	{31, 34} {33, 35}	2	1.993
	2-18	{28, 31} {30, 33}	2	1.993
	2-19	{30, 33} {34, 36}	2	1.993
	2-20	{27, 30} {28, 31}	2	2.000
	2-21	{27, 30} {34, 36}	2	2.000
	2-22	{28, 31} {33, 35}	2	2.000
	2-23	{33, 35} {34, 36}	2	2.000

## Using simulated failure scenarios as a basis for prediction

## Example: Markov simulation and perturbation of *cut set 2-3*:

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- Corresponds to software failure scenario involving multiple faults/attacks.
- Simulation identifies threshold beyond which increased failure incidence causes drastic performance collapse
- $\rightarrow$  Verified by large-scale simulation



<u>Conclusion</u>: Study indicates approach can be used to predict potential for failure and is more tractable than exhaustive search



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## Conclusions

- Results show potential of approach to model system failure scenarios at reduced computation cost
  - Generally 2x less than brute force search
  - Three examples in paper—can be expanded

→ Indicates potential for predictive use

- Approach uses techniques in combination
  - Large, detailed DTMC models and TPMs
  - Time inhomogeneous representation to capture change over time
  - Markov simulation and quantitative performance analysis (thresholds)
  - Minimal s-t cut set analysis

Use of all four in combination not previously reported

- Areas of further work
  - Tractability for large problems
  - Applicability to other domains
  - Investigate other approaches to finding critical transitions [Hu2011, Da2010]



## Tractability and generality of minimal s-t cut set analysis

- Tractability : number of potential cut sets in big graphs poses barriers.
  → Progress in application to larger problems. See [Da2011b]:
  - Developed node contraction algorithm which finds minimal s-t cut sets probabilistically (though not guaranteed to find all)
  - Applied contraction algorithm to four large DTMC TPMs with as many as > 4.22 ×10<sup>8</sup> cut sets
  - Found most of most highly-ranked cut sets also found through cut set enumeration, with some exceptions.

#### • <u>Generality</u>: application to other domains.

- In a smaller grid computing problem (7 states, 18 state transitions), minimal s-t cut set analysis was used to identify <u>all</u> critical state transitions found through brute force search of combinations [Da2011b].
- Applied to domain of network congestion control algorithm modeling. See [Da2010]



# Another issue: understanding effects of perturbation on *distant* states

 While Markov simulation of perturbed TPMs for cloud computing system was reasonably predictive of Total Grants of all Requests (full and partial),

## →Much harder to predict effect of perturbation on full and partial grants separately

 Why? In large-scale simulation (or target real-world system), indirect effects occur between parts of a system that cannot be modeled as states that are in direct transition with each other.

Ex. Failures of messages from cloud controllers to clusters reduces overall performance, but also increases resource availability. This leads to relative increase of full grants partial  $\rightarrow$  hence, full grants decline less than expected.

• Area of current interest and investigation



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